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François P. Landes

Viscoelastic Interfaces Driven in Disordered Media

Applications to Friction

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François P. Landes

Viscoelastic Interfaces Driven in Disordered Media

Applications to Friction

Doctoral Thesis accepted by
the Université Paris-Sud, France

Springer

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Supervisors' Foreword

The response of macroscopic systems to a slow but constant energy input can be either linear, with instantaneous dissipation and elastic deformation, or non-linear with stress accumulation followed by sudden and irreversible reorganisations that release energy. The Barkhausen noise measured in ferromagnets, the seismic activity generated by plate tectonics or the jerky flow of granular materials are examples of dynamical systems governed by rapid and dramatic reorganisation, named avalanches.

Although, in all these systems, rich and diverse dynamics are observed, with avalanches displaying peculiar shapes and evolutions, some common and well known features can be identified. One such property is the unpredictability: even with the full track record of the past events it is impossible to predict when, where and of which magnitude, the next avalanche will be. A second important property is that many quantities associated to avalanches (such as the magnitude, the duration or the size of the region involved in the perturbation…) display scale-free statistics.

It is then tempting to consider avalanches as the natural extension to out-of-equilibrium of continuous phase transitions where scale-free behaviour and universality are predicted to occur when approaching the critical point. However avalanche physics is much richer than its equilibrium counterpart. In particular, basic observations such as the presence of aftershocks in earthquakes or the strain localization of granular matter under shear are related to novel non-stationary effects, totally absent in equilibrium critical dynamics.

These effects are considered by François Landes in his thesis, where models for the earthquake statistics are studied. There, the frictional dynamics of two tectonic plates along a fault is modelled as an interface sliding in a heterogeneous medium. Because of the asperities of the medium, the dynamics is jerky and proceeds via sudden and large reorganisations of the interface shape. Such avalanches correspond to earthquakes. In absence of the relaxational effects inherent to friction the classical depinning transition of an elastic interface is recovered. In this limit the avalanche statistics is essentially Poissonian, i.e. avalanches are uncorrelated in time and space. However the relaxational effects allow for a realistic description of seismic activity. In particular, they are responsible for the presence of aftershocks, for the quasi-periodic occurrence of major earthquakes (the so-called seismic cycle) and accounts for a correct Gutenberg-Richter law.

The thesis is introduced with three very substantial chapters, that are actually self-contained reviews of known subjects: the dry friction of sliding solids, the phenomenological laws on the analysis of seismic activity and the model of the depinning of an elastic interface. These reviews are crucial to the good understanding of the main result of the thesis, which, in our opinion, represents an important attempt at bridging the gap between the complex scenario emerging from earthquake data analysis and conventional avalanche models.

France
June 2015

Alberto Rosso
Eduardo Jagla

Abstract

Many complex systems respond to a continuous input of energy by an accumulation of stress over time, interrupted by sudden energy releases called avalanches. Recently, it has been pointed out that several basic features of avalanche dynamics are induced at the microscopic level by relaxation processes, which are neglected by most models. During my thesis, I studied two well-known models of avalanche dynamics, modified minimally by the inclusion of some forms of relaxation.

The first system is that of a viscoelastic interface driven in a disordered medium. In mean-field, we prove that the interface has a periodic behaviour (with a new, emerging time scale), with avalanche events that span the whole system. We compute semi-analytically the friction force acting on this surface, and find that it is compatible with classical friction experiments. In finite dimensions (2D), the mean-field system-sized events become local, and numerical simulations give qualitative and quantitative results in good agreement with several important features of real earthquakes.

The second system including a minimal form of relaxation consists in a toy model of avalanches: the Directed Percolation process. In our study of a non-Markovian variant of Directed Percolation, we observed that the universality class was modified but not completely. In particular, in the non-Markov case an exponent changes of value while several scaling relations still hold. This picture of an extended universality class obtained by the addition of a non-Markovian perturbation to the dynamics provides promising prospects for our first system.

Acknowledgments

This thesis would not have been possible without the help of my supervisor Alberto Rosso, and it would have been much different if I had not had the opportunity to meet my de-facto second supervisor, Eduardo Jagla. I deeply thank them here, as they both played a crucial role in the development of my scientific skills and choices. Of course Alberto remains my main advisor, and all my knowledge of the depinning transition is due to him, as he explained it to me in the softest of the ways: on the blackboard. This was very precious to me, as the depinning framework has been the central axis around which my thesis revolves.

About Argentina and my visits to Bariloche, I am thankful to the ECOS-Sud project, that has made possible my collaboration with Eduardo, and allowed me to visit this wonderful region (Patagonia), twice! It also allowed me to meet Ezequiel Ferrero, Alejandro Kolton and Luis Aragón, whom introduced me to his friends, their home-made beers and the joys of the Asado. I am thankful to Luis and Eiji Kawasaki for proofreading my first chapter, as well as to many people from LPTMS for their help during these 3 years, with respect to proofreading and various degrees of help in rephrasing my thoughts: in particular to Pierpaolo Vivo and Shamik Gupta, but also to Mikhail Zvonarev, Haggai Landa, Martin Trulsson and Kabir Ramola.

Beyond the strict domain of physics, I am thankful for all the intresting discussions about science and about… life, in general, with Arthur, Matthieu, Andrey, Yasar, Pierre-Élie, all the curious things brought to our attention by Ricardo, and in general all the discussions with the new recruits and the ancients, around a lunch or a cigarette.

I want to acknowledge the help of the people who played a decisive part in building my path towards research, and who triggered and directed my interest towards and within Science. Jérôme Perez helped me find my way into fundamental research. Mikko Alava was the first to trust me and allowed me to perform my first research experience, furthermore in a field that was already exciting to me (the “complex systems”). I thank Matti Peltömaki, who proved a very good day-to-day tutor, and friend, during these four months in Finland. In terms of theoretical background, I am indebted towards “Fred” (Frédéric van Wijland), who always had a good piece of advice to find directions (scientifically speaking).

My mastering of the English language is definitely not due to my natural talents, rather it is the sheer result of my visits to good friends of the family: at the Gullifords’ in London (thank you Vincent), then at the Dong's in Connecticut (thank you very much Sophie!).

Contents

Chapter 1
Introduction

There are many natural occurrences of systems that upon a continuous input of energy, react by sudden releases of the accumulated energy in the form of discrete events, that are generally called *avalanches*. Examples are the dynamics of sand piles, magnetic domains inversions in ferromagnets, stress release on the earth crust in the form of earthquakes, and many others. A remarkable characteristic of most of these realizations is the fact that the size distribution of the avalanches may display power-laws, which are a manifestation of the lack of intrinsic spatial scale in these systems (similarly to what happens in *continuous phase transitions* at equilibrium [LL80, Kar07], with the correlation length diverging at criticality). The theoretical analysis is build on the features shared by these various processes, and aims at isolating the minimal set of ingredients needed to explain the common elements of phenomenology. There are numerous models which display critical behaviour and thus power-law avalanche size distributions, however in most cases the exponents characterizing the avalanches can only take a few possible values, corresponding to the existence of a few different *universality classes*.

For almost 20 years, there has been an ongoing effort to understand earthquakes in the framework of these critical and collective out-of-equilibrium phenomena. Several theoretical models are able to reproduce a scale-free statistics similar to that present in seismic events, but miss basic observations such as the presence of aftershocks after a main earthquake or the anomalous exponent of the Gutenberg-Richter law [Sch02]. At a smaller and simpler scale, a general theory for the friction of solids, taking into account the heterogeneities of each surface and the collective displacements, contacts and fractures of the asperities is not yet available [Per00, PT96]. Current theories fail to reproduce some *non-stationary effects* such as the increase of static friction over time or the possibility of the decrease of kinetic friction with increasing velocity.

A first class of models displaying a single well-defined *out-of-equilibrium phase transition* is that of the *depinning* of an extended elastic interface[1] driven over a

[1]The interface can be any manifold, i.e. a line, a surface, a volume, etc.

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disordered (random) energetic landscape [Fis98, Kar98]. While the interface is driven across the disordered environment, it gets alternatively stuck (*pinned*) by the heterogeneities and freed (de-pinned) by the driving force. Despite its locally intermittent character, the overall dynamics of the interface has a stationary regime, which makes various analytical and numerical methods available. Remarkably, one can often disregard the precise details of the microscopic dynamics when considering the large scale behaviour. As a result, the depinning transition successfully represents various phenomena, such as Barkhausen noise in ferromagnets [ABBM90, ZCDS98, DZ00, DZ06], crack propagation in brittle materials [ANZ06, BSP08, BB11] or wetting fronts moving on rough substrates [RK02, MRKR04, LWMR09] (see [Bar95] for notions on fractals, growing surfaces and roughness). Although the framework is also a priori well suited to describe friction and thus earthquakes, the stationary behaviour itself is the ground where major discrepancies arise between theoretical depinning results and real data: the aftershock phenomenon observed in earthquakes, for instance, is clearly not stationary [Sch02].

A second class of such models is that of Directed Percolation (DP) [Ó08, Hin06, Ó04, Hin00], which models the random growth, spatial spread and death of some density of "activity" over time, in the manner of an avalanche. On a lattice, each site can be either active or inactive, and at each time step, each active site tries to activate each of its neighbours, with a probability of success p. When all sites become inactive, the avalanche is over and the state no longer evolves. This inactive state is an "absorbing phase" of the dynamics: the DP transition is an *absorbing phase transition* [HHL08]. There is a critical value of the probability p at which the system reaches criticality, with most stochastic observables distributed as power-laws. Numerous birth-death-diffusion processes share the same critical exponents and scaling functions: the DP class is a wide, robust class. We use the DP process as a toy model of avalanches with Markovian dynamics [vK81].

In this thesis, starting from models of out-of-equilibrium phase transitions with stationary dynamics, we build and study *variants* of these models which still display criticality, but in the same time have *non-stationary* dynamics.

The physical process at the origin of most of our motivation and choices is that of solid on solid, dry friction (i.e. in the absence of lubricants). Actually, during this thesis our concern was initially the application of statistical physics methods to seismic events, however towards the end of the thesis we focused more on laboratory-scaled friction, as it is a much better controlled field. Since this subject is not a common topic in the field of disordered systems, we introduce the problem of friction in Chap. 2. Reviewing the basic phenomenology and the well-established parts of the theory of friction, we are able to identify the main features that any friction model should include. Two points emerge clearly. A first is the need to account for the disordered aspect of the surfaces at play: asperities form a random network of contacts which constantly break and re-form, and the surfaces are heterogeneous so that the contact strengths are randomly distributed. A second is the relevance of some slow mechanisms (plastic creep, in particular) which allow for a strengthening of the contacts over time. The latter point becomes especially relevant at very slow driving speeds, or when there is no motion. We will focus on the slow driving regime, where the non-trivial frictional behaviours appear and which is crucial when considering seismic faults.

The physics of earthquakes is vast and quite complex [Sch02], but presents several points of interest for us. A first is that the sliding of tectonic plates, at first approximation, may be considered as a large-scale manifestation of solid on solid, dry friction. This "application" has been studied quite extensively on its own, and a large amount of data is available, so that seismic faults can be used to test the predictions of friction models. A second point is that due to its importance, the field of geophysics has generated numerous interesting models, which may serve as starting points to understand friction as a collective phenomenon, rather than a simple continuum mechanics problem. This motivates our quick review of seismic phenomena and the related historical models, presented in Chap. 3.

The mapping of an earthquake model onto the problem of elastic depinning naturally introduces our review of the depinning transition in Chap. 4. There, we introduce all the concepts necessary to understand our own modified depinning model, and appreciate its originality. We explain the critical properties of this *dynamical phase transition* (or depinning transition [ZCDS98, RK02, LWMR09, ANZ06]), review the scaling relations and an original approach to the mean field. Even though we notice that the depinning universality class is a robust one, we are forced to acknowledge its inability to account for frictional phenomena.

With the notions presented in the previous chapters, our choice of modification of the depinning problem is quite natural. The starting point of our analysis is to remark that conventional depinning does not allow any internal dynamical effects to take place during the inter-avalanche periods. To address this issue, in Chap. 5 we introduce the model of a *viscoelastic* interface driven in a disordered environment, which allows for a slow relaxation of the interface in between avalanches. The viscoelastic interactions can be interpreted as a simple way to account for the plastic creep, mainly responsible for the peculiarities of friction at low driving velocity. After a qualitative discussion of the novelties of the viscoelastic interface behaviour, we present a derivation of its mean field dynamics. Extending the mean field approach that we presented for the elastic depinning to this new model, we are able to compute the behaviour of the entire system, which is found to be non-stationary, with system-size events occurring periodically. There, we also notice that the addition of the "visco-" part into the elastic interactions is relevant in the macroscopic limit. We compare the mean field dynamics at various driving velocities and find good agreement with experimental results found in fundamental friction experiments (Chap. 2) and observations on earthquakes statistics (Chap. 3). In two dimensions, we are limited to numerical simulations, but we are able to perform them on systems of tremendous sizes (up to 15000×15000 sites on a single CPU), which allows us to unveil some features reminiscent of the mean field behaviour. The various outputs of our simulations (critical exponents, aftershocks patterns, etc.) compare well with the observational results from Chap. 3 (see Sect. 5.6 for a more detailed summary of results). In the comparison with models from various other contexts (amorphous plasticity, granular materials, etc.) we notice similarities in the various models construction, and a shared tendency for global, system-size events.

During this thesis, most of the work was performed on non-stationary variations on the depinning model, with a focus on the applications to seismic events. On the way, we studied a variation of the Directed Percolation, which has to do with non-stationarity, despite being a model completely different from those presented in Chap. 5. The last chapter (Chap. 6) offers the opportunity to consider the bigger picture of avalanche models. In that chapter, we consider the celullar automaton [Wol83] of Directed Percolation (DP). We provide an intuitive link with the problem of interface depinning by showing how much one would need to modify the DP process to let it represent the avalanches of the elastic interface. We introduce a non-Markovian variant of the DP process, in which the probability to activate a site at the first try and the second one are different from those in the ulterior attempts. This provides the system with an implicit memory, making the microscopic dynamics non-stationary. This modified DP displays criticality with some exponents changing continuously with the first and second activation probabilities, while others do not: in particular, only one scaling relation is violated by the new dynamics, so that the new class preserves most of its parent's structure. A long-standing challenge is to find experimental systems belonging to the DP universality class: up to now, there are no such direct examples [Ó04]. Our new model, which includes DP as a particular case, opens the way for possible future experimental work, as we may consider universality classes larger than DP.

As a conclusion, we explain the general path that structures this thesis and draw some directions for future work (Chap. 7).

In each chapter of this thesis, we provide a very quick introduction, which simply details the aim of the chapter and the organization of contents. In the chapters' conclusions, we always carefully summarize the main results, and provide the motivation for the next chapter or some directions for future work. We sometimes refer to the Appendices for technical details or results that are not crucial to our presentation. Although each chapter is a self-contained entity, reading the earlier ones allows to fully understand and appreciate the scope of the latter ones. The articles published during this thesis are [JLR14] and [LRJ12], they essentially correspond to the Chaps. 5 and 6, respectively.

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Chapter 2
Introduction to Friction

In this chapter we aim at giving a short overview of dry friction, i.e. frictional phenomena where the lubricants effect is negligible. We first present the phenomenological laws derived by experimental observations, then present the rudiments of the (incomplete) theory of friction.Excellent references on these topics are [Per00, PT96, Kri02]. In the process, we comment on the existing literature and draw some conclusions about possible directions for future work, especially for the statistical physics community.

2.1 The Phenomenological Laws of Friction

Consider a solid parallelepiped—as depicted in Fig. 2.1—in contact with a large solid substrate over a surface S (supposed to be flat at the macroscopic scale), with a normal load L (for instance due to gravity), being pulled along the surface via a spring k_0, itself pulled at a fixed velocity V_0. The block's velocity is denoted v. The force F_k of frictional effects was[1] claimed to follow these three laws:

- First law: F_k is independent from the surface area S.
- Second law: F_k is proportional to the normal load: $F_k \propto L$.
- Third law: F_k is independent of the sliding velocity v.

This allows to write a phenomenological equation for the friction force:

$$F_k = \mu_k L \tag{2.1}$$

[1]These laws were stated in the 17th century by Amontons for the first two of them, and in the 18th century by Coulomb for the third one.

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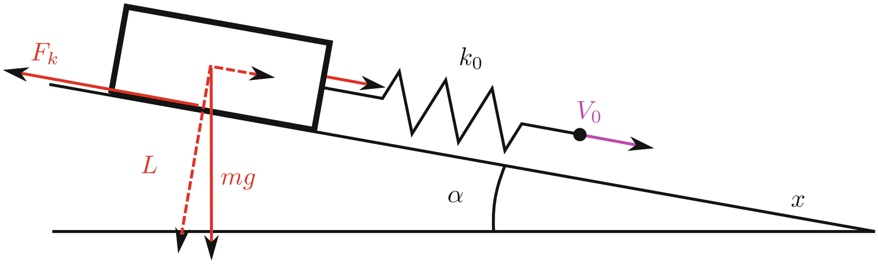

Fig. 2.1 *Solid block* sliding on a solid substrate. Solid parallelepiped sliding on an inclined plane (angle α) at velocity $v = \dot{x}$. The weight can be decomposed in two components, one orthogonal to the surface (the load L), and one parallel to it (which contributes to the pulling). Additional pulling can be provided via a spring k_0, of which the "free" end may be moved at a fixed velocity V_0. The kinetic friction force is denoted F_k

where μ_k is the kinetic (or dynamic) friction coefficient, which depends on the nature of the surfaces in contact along with many other things, but which is here assumed to be independent from S, L and v.

There is one "exception" to the third law which is commonly observed: for the static case ($v = 0$, i.e. when there may be pulling, but without motion) the friction coefficient takes a different value μ_s, larger than the dynamical one: $\mu_s(v = 0) > \mu_k(v > 0)$.

2.1.1 Stick-Slip Motion

Due to the fact that the static ($v = 0$) friction force is higher than the dynamic ($v > 0$) one, a mechanical instability known as "stick and slip motion" can occur, especially when the pulling is provided mainly in a sufficiently flexible way (small k_0) or at sufficiently low driving velocity V_0. As we are going to see, this is something that we experience on a daily basis.

Consider the system pictured in Fig. 2.1, with an angle $\alpha = 0$, for simplicity. The free end of the spring k_0 is denoted w_0 and is driven steadily at a velocity V_0. The spring k_0 can be thought of either as an actual spring through which the driving is performed, or as an effective representation for the bulk rigidity of the solid. As we pull the block from the side, we transmit some shear stress through its bulk. If the solid is driven at constant velocity V_0 directly from a point on its side, the effective stiffness k_0 is proportional to the Young's modulus E and inversely proportional to the height d of the driving point (neglecting torque effects). See Fig. 2.2 for a visual explanation.In the context of a simple table-top experiment as presented here, the solid's stiffness is generally too large for stick-slip to occur, so that the use of an actual spring k_0 to perform driving is useful.

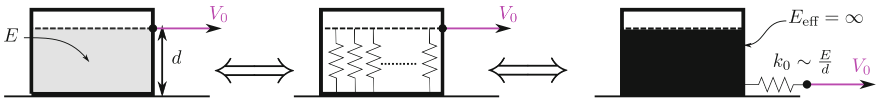

Fig. 2.2 Effective stiffness of the driving spring. *Left* a *solid block* with Young's modulus E is pulled rigidly from some point at a height d, i.e. this point is forced to have the velocity V_0. *Middle* the *solid block* can be pictured as a dense network of springs, related to E. Springs in the *horizontal* directions are not pictured for clarity. *Right* effective modelling by a block with infinitely rigid bulk, pulled by an effective spring $k_0 \sim E/d$

Newton's equations for the center of mass of the block at position x can be written in the dynamic and static cases:

$$m\ddot{x} = k_0(V_0 t - x) - \mu_k L \qquad \text{(dynamic)} \qquad (2.2)$$

$$0 = k_0(V_0 t - x) - F_s \qquad \text{(static)} \qquad (2.3)$$

where the static friction force F_s adapts according to Newton's second law (Law of action and reaction) in order to balance the pulling force, as long as it does not exceed its threshold: $|F_s| < \mu_s L = (F_s)_{\max}$.

We start with $x(t = 0) = 0$, $w_0(0) = 0$, and for $t > 0$ we perform the drive, $w_0 = V_0 t$. As long as $|F_s| < \mu_s L$, the block does not move: we are in the "stick" phase.

At time $t_1 = \frac{\mu_s L}{k_0 V_0}$, the static friction force F_s reaches its maximal value $\mu_s L$ and the block starts to slide. This is the "slip" phase. Thus we have the initial condition $x(t_1) = 0$, $\dot{x}(t_1) = 0$ for the kinetic equation. The solution reads:

$$x(t) = V_0(t - t_1) - \sqrt{\frac{m}{k_0}} V_0 \sin\left(\sqrt{\frac{k_0}{m}}(t - t_1)\right) + \frac{(\mu_s - \mu_k)L}{k_0}\left(1 - \cos\left(\sqrt{\frac{k_0}{m}}(t - t_1)\right)\right). \qquad (2.4)$$

It is natural to take a look at the short-time limit of the solid's position:

$$x(t) \underset{t\sim 0}{\sim} \frac{(\mu_s - \mu_k)L}{2m} t^2 + \frac{k_0 V_0}{6m} t^3 - \frac{(\mu_s - \mu_k)L k_0}{24 m^2} t^4 + o(t^4), \qquad (2.5)$$

which is increasing at short time, as expected, since $\mu_s > \mu_k$.

As x initially increases faster than $V_0 t$, the driving force from the spring, $(k_0(V_0 t - x))$, decreases over time, so that $\dot{x}$ may reach zero again. If at some point $\dot{x} = 0$, the kinetic friction coefficient is replaced by the static one, and oscillations (and any form of further sliding) are prevented. We can compute the times t_2 such that formally, $\dot{x}(t_2) = 0$:

$$t_2 = t_1 + 2\sqrt{\frac{m}{k_0}}\left(p\pi - \arctan\left(\frac{(\mu_s - \mu_k)L}{\sqrt{m k_0} V_0}\right)\right) \qquad (2.6)$$

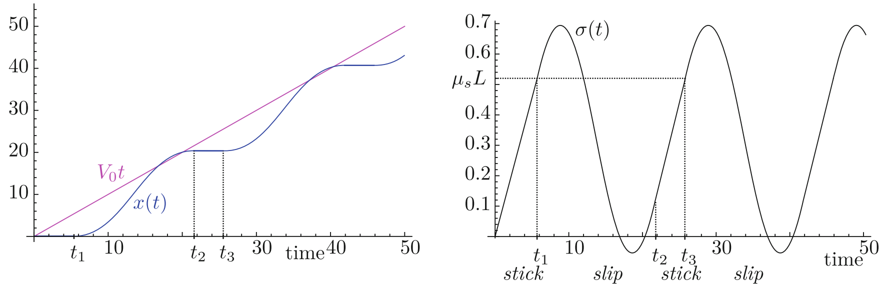

Fig. 2.3 Stick-slip evolution of the block over time. *Left* variations of the center of mass x over time t (*solid blue*) computed from (Eq. 2.4). *Right* saw-tooth evolution of the stress during stick-slip motion. Variations of the stress $\sigma = k_0(V_0t - x)$ (*solid grey line*) computed from (Eq. 2.4). The function V_0t (*dashed purple*) is given for reference. At time t_1, the threshold for the static force is reached and the block starts to move, with a decreased friction force F_k (kinetic). At time t_2, as velocity cancels, one needs to consider the static friction force. Loading then increases until the time t_3 where the threshold of static friction is once again reached. Parameters used for the two figures are: $m = 1$, $V_0 = 1$, $k_0 = 0.1$, $\mu_S L = 0.52$, $(\mu_S - \mu_K)L = 0.2$. Note that the slip phase seems long, but this is due to the parameters used: in particular, with a larger $(\mu_S - \mu_K)$ we get longer stick phases (and—relatively—shorter, sharper slip phases) Here we have a detailed view of the slip phase

where $p \in \mathbb{N}$. The physical solution corresponds to the first positive time that can be obtained, i.e. $p = 1$. At this time, the friction force (that always opposes motion, whichever direction it goes) increases from $\mu_k L$ to $\mu_s L$ and motion stops. The evolution of the block is once again controlled by the static equation of motion (Eq. 2.3), and we are in the "stick" phase.

The system will remain in the stick state until the time t_3 such that $V_0t_3 - x(t_3) = \mu_s L/k_0$. Since the system has no memory (beyond $\dot{x}$), the dynamics at ulterior times is exactly periodic, as shown in Fig. 2.3.

In friction experiments, one usually measures the *total shear stress* or total friction force, which is given by $\sigma = k_0(V_0t - x)$. We present the evolution of $\sigma(t)$ in Fig. 2.3 (right), to be compared with experimental results, e.g. for a mica surface pulled at constant velocity (Fig. 2.4).

The difference between μ_s and μ_k generates a mechanical instability, in which the elastic energy provided by the driving is at times stored (static case, or "stick" phase) and at times released over a short[2] period (kinetic case, or "slip" phase). This is the exact opposite of the more common situation of dissipative forces monotonously increasing with velocity so that a balance between drive and drag naturally yields stable solutions.

[2] Note that in Fig. 2.3, the parameters chosen are such that the stick phase is rather short. For larger $(\mu_S - \mu_K)$ we get longer stick phases, and—relatively—shorter slip phases, since the duration of the slip phase is independent of μ_S, but the loading time grows essentially linearly with it.