Frontiers in Applied Dynamical Systems:
Reviews and Tutorials 3

C. Eugene Wayne
Michael I. Weinstein

Dynamics of Partial Differential Equations

Springer

Frontiers in Applied Dynamical Systems: Reviews and Tutorials

Volume 3

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Frontiers in Applied Dynamical Systems: Reviews and Tutorials

The Frontiers in Applied Dynamical Systems (FIADS) covers emerging topics and significant developments in the field of applied dynamical systems. It is a collection of invited review articles by leading researchers in dynamical systems, their applications, and related areas. Contributions in this series should be seen as a portal for a broad audience of researchers in dynamical systems at all levels and can serve as advanced teaching aids for graduate students. Each contribution provides an informal outline of a specific area, an interesting application, a recent technique, or a "how-to" for analytical methods and for computational algorithms, and a list of key references. All articles will be refereed.

C. Eugene Wayne • Michael I. Weinstein

Dynamics of Partial Differential Equations

Review 1: C. Eugene Wayne: Dynamical Systems and the Two-dimensional Navier-Stokes Equations

Review 2: Michael I. Weinstein: Localized States and Dynamics in the Nonlinear Schrödinger/Gross-Pitaevskii Equation

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Preface to the Series

The subject of dynamical systems has matured over a period more than a century. It began with Poincaré's investigation into the motion of the celestial bodies, and he pioneered a new direction by looking at the equations of motion from a qualitative viewpoint. For different motivation, statistical physics was being developed and had led to the idea of ergodic motion. Together, these presaged an area that was to have significant impact on both pure and applied mathematics. This perspective of dynamical systems was refined and developed in the second half of the twentieth century and now provides a commonly accepted way of channeling mathematical ideas into applications. These applications now reach from biology and social behavior to optics and microphysics.

There is still a lot we do not understand and the mathematical area of dynamical systems remains vibrant. This is particularly true as researchers come to grips with spatially distributed systems and those affected by stochastic effects that interact with complex deterministic dynamics. Much of current progress is being driven by questions that come from the applications of dynamical systems. To truly appreciate and engage in this work then requires us to understand more than just the mathematical theory of the subject. But to invest the time it takes to learn a new subarea of applied dynamics without a guide is often impossible. This is especially true if the reach of its novelty extends from new mathematical ideas to the motivating questions and issues of the domain science.

It was from this challenge facing us that the idea for the *Frontiers in Applied Dynamics* was born. Our hope is that through the editions of this series, both new and seasoned dynamicists will be able to get into the applied areas that are defining modern dynamical systems. Each chapter will expose an area of current interest and excitement, and provide a portal for learning and entering the area. Occasionally, we will combine more than one paper in a volume if we see a related audience as

we have done in the first few volumes. Any given paper may contain new ideas and results. But more importantly, the papers will provide a survey of recent activity and the necessary background to understand its significance, open questions, and mathematical challenges.

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Preface

Natural processes can often be modeled by partial differential equations. In many applications, it is the emergence of spatially localized solutions that is of particular interest: examples are localized light pulses in photonic crystals, vortices in fluid flow, and large-scale circulation events in meteorological and climate systems. Dynamical-systems theory provides a number of techniques that can be utilized to study the existence of these coherent structures and to investigate their local and global stability properties. The way in which these techniques are used depends fundamentally on the nature of the underlying partial differential equations: the analysis of dissipative equations such as the Navier-Stokes equations differs drastically from analyses of dispersive equations that typically conserve an energy functional.

In this volume, Eugene Wayne and Michael I. Weinstein illustrate the applicability of dynamical-systems approaches in the context of dissipative and dispersive partial differential equations, respectively. Wayne reviews recent results on the global dynamics of the two-dimensional Navier-Stokes equations. This system exhibits self-similar, explicitly computable, vortex solutions. By combining classical techniques from dynamical systems theory, such as Lyapunov functions and invariant manifold theorems, one can prove that any solution of the equations for integrable initial vorticity will asymptotically approach one of these vortices - in other words, they are globally stable. However, both numerical investigations and experimental results show that in addition to the viscous time scale over which the stability of these vortices manifests itself, there are additional time scales on which important transient phenomena become evident. Wayne also surveys recent results on these metastable phenomena using analysis which originated in kinetic theory. Weinstein considers the dynamics of localized states in nonlinear Schrödinger/Gross-Pitaevskii equations play a central role in the mathematical study of nonlinear optical phenomena as well as macroscopic quantum systems, e.g. Bose-Einstein condensation. In this contribution, Weinstein reviews recent results on the bifurcation of solitary waves, their linear and nonlinear stability properties, as well as nonlinear scattering results where a conservative dissipation mechanism, radiation damping of energy to spatial infinity, plays an important role. The chapters,

written independently, are combined in one volume as the Editors-In-Chief believed it would be of interest to the audience of this volume to showcase the tools of dynamical systems theory at work in explaining qualitative phenomena associated with two classes of partial differential equations with very different physical origins and mathematical properties.

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