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# Theoretical and Observational Consistency of Massive Gravity

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Lavinia Heisenberg

# Theoretical and Observational Consistency of Massive Gravity

Doctoral Thesis accepted by  
the University of Geneva, Switzerland

 Springer

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Quantum Corrections in Massive Gravity:  
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2. C. de Rham, G. Gabadadze, L. Heisenberg, D. Pirtskhalava:  
Non-Renormalization and Naturalness in a Class of Scalar-Tensor Theories:  
Phys. Rev. D 87, 085017 (2013)
3. P. de Fromont, C. de Rham, L. Heisenberg, A. Matas:  
Superluminality in the Bi- and Multi- Galileon:  
JHEP 1307 (2013) 067
4. C. de Rham, L. Heisenberg:  
Cosmology of the Galileon from Massive Gravity:  
Phys. Rev. D 84, 043503 (2011)
5. C. Burrage, C. de Rham, L. Heisenberg:  
De Sitter Galileon:  
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Cosmic Acceleration and the Helicity-0 Graviton:  
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Chronology Protection in Galileon Models and Massive Gravity:  
JCAP 07 (2012) 004
2. J.B. Jimenez, R. Durrer, L. Heisenberg, M. Thorsrud:  
Stability of Horndeski Vector-Tensor Interactions:  
JCAP 1310 (2013) 064

*The Road Not Taken*

*TWO roads diverged in a yellow wood,  
And sorry I could not travel both  
And be one traveler, long I stood  
And looked down one as far as I could  
To where it bent in the undergrowth;*

*Then took the other, as just as fair,  
And having perhaps the better claim,  
Because it was grassy and wanted wear;  
Though as for that the passing there  
Had worn them really about the same,*

*And both that morning equally lay  
In leaves no step had trodden black.  
Oh, I kept the first for another day!  
Yet knowing how way leads on to way,  
I doubted if I should ever come back.*

*I shall be telling this with a sigh  
Somewhere ages and ages hence:  
Two roads diverged in a wood, and I –  
I took the one less traveled by,  
And that has made all the difference.*

—Robert Frost

*Dedicated to the endless memories of Rojden  
and Zera.*

# Supervisor's Foreword

It is with great pride that I look back on the 5 years since Lavinia Heisenberg started her Ph.D. at Geneva University. Committed to work on theoretical aspects of Cosmology, Lavinia joined the field of modified gravity as a tool to explain the late-time acceleration of the Universe and tackle the Cosmological Constant problem.

The Cosmological Constant problem, or why the vacuum energy of particle physics is so small, is a longstanding puzzle existing now for more than seven decades. The recent realizations that our Universe is accelerating and that this acceleration is very well fitted by a Cosmological Constant has revived the old Cosmological Constant problem as well as the search for alternative theories.

At the time when Lavinia started her thesis research, most consistent models of modified gravity in the infrared were driven by higher-dimensional setups and the “Galileon” scalar field had just emerged as an effective description of such theories. During the first year of Lavinia’s Ph.D., the field of modified gravity underwent a remarkable transition and Lavinia bonded with it in an exceptional way.

First, massive gravity (a theory of gravity where the particle responsible for the gravitational force has a mass) was derived as its own consistent theory, an effort which was previously thought to be impossible. From there, theories of bi-gravity and multi-gravity emerged as new ways to understand gravity. These new theories required the development of many new techniques that were so far lacking in the literature and Lavinia literally took the role of a forerunner in developing many of them.

Second, many properties of the Galileon scalar fields were still to be explored, and throughout her research, Lavinia has been devoted to understanding their theoretical and phenomenological properties in-depth. From extending the Galileon symmetry to other maximally symmetric spacetimes, to understanding their quantum behavior and the implications of their superluminal propagation, Lavinia’s work has pushed our understanding of these fields to a new level. These studies became all the more relevant after the realization that the helicity-0 mode of the graviton in massive gravity and bi- or multi-gravity also behaves as a Galileon scalar field in a certain limit.

Motivated by this observation, Lavinia was able to establish the existence of 'degravitating' solutions in massive gravity where a large cosmological constant could effectively be 'eaten' by the graviton mass. Although these solutions cannot degravitate an arbitrarily large cosmological constant without also exciting the helicity-0 mode at an undesirably low scale, this framework still provides a proof-of-principle for circumventing Weinberg's no-go theorem when it comes to tackling the Cosmological Constant problem.

Related to the late-time acceleration of the Universe, Lavinia's thesis also shows how the helicity-0 mode of the graviton can itself 'self-accelerate' the expansion of the Universe without the need for any dark energy.

Based on these results, Lavinia was then able to describe how to derive a proxy theory for massive gravity which captures the essential ingredients of massive gravity without many of its complications. This proxy theory was shown to be a special case of Horndeski Scalar-Tensor theory, which has independently enjoyed a great deal of attention recently. Within the context of this proxy theory, Lavinia was then able to capture the essence of the self-accelerating solution in more depth, taking into account the cosmological history of the Universe and presenting the first elements toward a study of structure formation. A subsequent analysis has established that the self-accelerating solution can only be a transient regime within that proxy theory and can never be a late time attractor.

Furthermore, motivated by the derivation of the proxy theory and their relation with Horndeski scalar-tensor theories, Lavinia also generalized the Horndeski framework to vector fields and established the first most general consistent vector-tensor theory, as well as exploring the phenomenological implications for Cosmology.

Whether it is in massive gravity, in Galileon theories or in more general Horndeski theories, the presence of superluminal modes has been a source of concern and arguments for more than a decade. When Lavinia started her thesis research, it had been established that a certain class of bi-Galileon theories could avoid any superluminality issues while still exhibiting a consistent self-accelerating solution, as well as an active Vainshtein mechanism. This realization attracted much interest and theories of multi-gravity or of gravity embedded in more than one extra dimension were investigated with the hope of avoiding the superluminality issues, since they behave as multi-Galileon theories in some limit. Thanks to Lavinia's work, this issue was re-explored and it was found that the presence of superluminalities is actually unavoidable in all of these classes of theories, even though closed-time-like curves are impossible within the regime of validity of such theories.

When the new theories of modified gravity came out (massive gravity, bi- and multi-gravity, as well as their extensions), two of the most pressing questions were related to their quantum stability and their cosmological phenomenology. It is fair to say that Lavinia's work represented major advances in both directions. The quantum stability was first established in the Galileon limit at all loops, and then beyond that limit at the one-loop level. While a full analysis is yet to be performed, the arguments presented in this thesis go a long way toward understanding the

technical naturalness of the graviton mass and the stability of the allowed interactions in these types of theories.

On the cosmological side, Lavinia's thesis has established many different techniques to explore the phenomenology of these theories, both at early and late-time, and combining these results with data analysis from different cosmological probes. In particular, Lavinia was able to generalize the coupling to matter in these types of theories and provide new directions to study their cosmology.

Much of Lavinia's thesis has now taken on a life of its own and the issues are being explored further. There is no doubt, however, that it gathers together many different facets of the recent developments in the field of massive gravity and will serve as a valuable reference for the field.

Cleveland, USA  
December 2014

Prof. Claudia de Rham

# Abstract

This doctoral thesis encompasses a detailed study of phenomenological as well as theoretical consequences derived from the existence of a graviton mass within the ghost-free theory of massive gravity, the de Rham-Gabadadze-Tolley (dRGT) theory, which incorporates a two-parameter family of potentials. In this thesis we pursue to test the physical viability of the theory. To start with, we have put constraints on the parameters of the theory in the decoupling limit based on purely theoretical grounds, like classical stability in the cosmological evolution. Hereby, we were able to construct self-accelerating solutions which yield similar cosmological evolution to a cosmological constant. Furthermore, we studied the degravitating solutions, which enables us to screen an arbitrarily large cosmological constant in the decoupling limit. Nevertheless, conflicts with observations push the allowed value of the vacuum energy to a very low value rendering the found degravitating solution phenomenologically not viable for tackling the old cosmological constant problem. Next, we constructed a proxy theory to massive gravity from the decoupling limit resulting in non-minimally coupled scalar–tensor interactions as an example of a subclass of Horndeski theories. We explored the self-accelerating and degravitating solutions in this proxy theory in analogy to the decoupling limit and extended the analysis by studying the change in the linear structure formation.

Furthermore, Galileon models are a class of effective field theories that naturally arise in the decoupling limit of theories of massive gravity. We show that the existence of superluminal propagating solutions for multi-Galileon theories is an unavoidable feature.

Finally, we addressed the natural question of whether the introduced parameters in the theory are subject to strong renormalization by quantum loops. Starting with the decoupling limit we have shown how the non-renormalization theorem protects the graviton mass from quantum corrections. Beyond the decoupling limit the quantum corrections are proportional to the graviton mass, proving its technical naturalness in an explicit realization of 't Hooft's naturalness argument. Moreover, we pushed the analysis beyond the decoupling limit by studying the stability of the graviton potential when including matter and graviton loops. One-loop matter

corrections contribute a cosmological constant term leaving the potential unaffected. On the contrary, the one-loop contributions from the gravitons destabilize the special structure of the potential. Nevertheless, we showed that even in the case of large background configuration, the Vainshtein mechanism redresses the one-loop effective action so that the detuning remains irrelevant below the Planck scale.

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I would like to take this chance to raise my deepest gratitude to my thesis advisor, Claudia de Rham, for her impeccable supervision, endless support, patience, and stimulating guidance. I am very thankful that she gave me the opportunity to work with her in an active and interesting research field. I feel extremely fortunate to have had such an active and engaged advisor. I am even more thankful that she stayed committed to my formation as a researcher when she was offered a permanent position at Case Western Reserve University (CWRU) in Cleveland. I especially appreciate the effort that she has made all the time to get things done properly and that my training was not neglected. We have managed to work together on several projects over distance and she made it possible for me to visit her at CWRU for an extended period of time. It is also an appropriate place to express my gratitude to all my colleagues and friends at CWRU who made my stay abroad enjoyable, specially Emanuela Dimastrogiovanni, Matteo Fasiello, David Jacobs, Andrew Matas, Lucas Keltner, Raquel Ribeiro, and Amanda Yoho.

The major part of this thesis has been developed in the Department of Theoretical Physics at the University of Geneva, and thanks to all the facilities the university provided me, and under the Swiss national funding, I was able to attend a large number of conferences and schools. I was able to profit a lot with my visits to the USA and Japan and I am very thankful to the Swiss national funding for providing financial support. Furthermore, I am very thankful to two very special persons at the University of Geneva, to Ruth Durrer, who has been always very supportive and patient and to Michele Maggiore for very useful discussions. I would also like to thank my colleagues and friends at the University of Geneva, especially Jose Beltran Jimenez for the numerous interesting conversations and for being supportive, Guillermo Ballesteros for his persistent questionings which gave rise to so many interesting knowledgeable conversations, and Dani Figuerola for our sushi evenings.

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# Chapter 1

## Introduction

### 1.1 Field Theories in Cosmology

Physics studies Nature, its matter content and its evolution modeled by the laws of physics. Physicists like Newton, Galileo or Kepler successfully postulated the physical laws of the “every day physics scales”, the theory of classical mechanics, which describes the motion of objects with velocities much smaller than the speed of light and with sizes much larger than atoms or molecules. Nevertheless, on the two edges of these scales, i.e. on very large or very small scales and at speeds close to the speed of light, the theory of classical mechanics breaks down. After a significant theoretical and observational efforts pioneered by physicists like Einstein, Planck, Heisenberg, Dirac or Schroedinger, the laws of physics were also extended towards these extreme scales, incorporating concepts of relativity and quantum mechanics to describe physics on atomic and galactic scales. As an outcome, physicists now describe the macroscopic and microscopic world by two simple standard models: the Standard Model of particle physics and the Standard Model of Big Bang cosmology. A dream of every physicist is the unification of these two standard models into a single ultimate “theory of everything” in a consistent way. Thanks to the advances in our understanding of many physical phenomena at a fundamental level, we are witnessing remarkable attempts towards this direction.

The Standard Model of particle physics unifies the electro-weak and strong interactions with an exquisite experimental success. It is a theory consisting of elementary and composite particles described by the robust framework of quantum field theory. In this picture, particles correspond to the excited states of an underlying physical field which can be created/annihilated by local operators given by the irreducible unitary representations of the Poincaré group (Weinberg 2005). It is the  $SU(3) \times SU(2) \times U(1)$  gauge symmetry that defines the Standard model of particle physics. The group  $SU(3)$  corresponds to the color gauge symmetry of Quantum Chromodynamics, whereas  $SU(2) \times U(1)$  is the gauge symmetry of the electro-weak interaction and breaks down spontaneously to  $U(1)$  through the Higgs mechanism, process thanks to which the elementary particles acquire their masses. This symme-

try breaks spontaneously down to  $SU(2) \times U(1)$  which is the remaining symmetry of the electro-weak interactions and to  $SU(3)$  which is the symmetry of the quantum chromodynamics.

The Standard Model consists of fermions and bosons, which differ fundamentally from each other by their spin statistics. The twelve elementary fermions divide into six leptons (electron, muon, tau and the corresponding neutrinos) and six quarks (up, down, charm, strange, top and bottom). The twelve bosons (photon,  $W^\pm$ ,  $Z$ , eight gluons) carry the strong, weak and electromagnetic forces. Interactions between the electrically charged particles mediated by the photons are successfully described by quantum electrodynamics, whereas the interactions between quarks (color charged) and gluons is described by quantum chromodynamics. The weak force mediated by the  $W^\pm$ ,  $Z$  bosons is mathematically merged with the electromagnetic force via electroweak interaction. Last but not least the Higgs boson detected recently at LHC is the crucial ingredient to explain the masses of the particles. All these elementary particles come in with different masses and from a theoretical point of view it is essential to study the interactions of massless and massive spin- $n$  particles. The natural starting point is to find the consistent Lagrangian of the classical field theories with the corresponding Hamiltonian being bounded from below. Once this non-trivial prerequisite is successfully fulfilled, then the classical fields can be quantized using different quantization techniques like canonical quantization or path integral quantization. Even though Standard Model of particle physics provides theoretical robustness of quantum fields and breathtaking experimental predictions, it is still far from being complete since it does not incorporate the theory of general relativity.

Cosmology has progressively developed from a philosophical to an empirical scientific discipline. Given the high precision achieved by the cosmological observations, cosmology is now a suitable arena to test fundamental physics. The challenging task of cosmology is to unite the physics of the large scale structures in the Universe with the physics of the small scale structures in order to describe the dynamics of the Universe successfully. Therefore, cosmology is highly multi-disciplinary and merges together concepts from general relativity, quantum mechanics, field theory, fluid mechanics and statistics. Furthermore, its interplay with high energy and particle physics facilitates the creation of synergies between these different fields.

Observations of the Cosmic Microwave Background (CMB), supernovae Ia (SNIa), lensing and Baryon Acoustic Oscillations (BAO) have led to the cosmological standard model which requires an accelerated expansion of the late Universe, driven by dark energy. The physical origin of the accelerated expansion is still a mystery. In the Standard Model of particles the detection of the missing fundamental particle, the Higgs boson, was a revolutionary event and an unexaggerated merit of Nobel-prize. In a similar way, the missing particles in the Standard Model of cosmology, like the graviton or dark matter, and the resolution to the puzzle of accelerated expansion and its origin would be as revolutionary. There are promising explanatory attempts which fall into three primary categories.

The first solution consists of considering a small cosmological constant  $\lambda$  with a constant energy density giving rise to an effective repulsive force between cosmological objects at large distances (Peebles and Ratra 2003). The Einstein-Hilbert

action is invariant under general diffeomorphisms and a cosmological constant  $\lambda$  can be included to this action without breaking this symmetry.<sup>1</sup> If we assume that the cosmological constant corresponds to the vacuum energy density, then the theoretical expectations for the vacuum energy density caused by fluctuating quantum fields differ from the observational bounds on  $\lambda$  by up to 120 orders of magnitude. This gigantic mismatch between the theoretically computed high energy density of the vacuum and the low observed value has remained for decades as one of the most challenging puzzles in theoretical physics and is called the cosmological constant problem. Indeed the cosmological constant problem is a puzzle concerning both particle physics and cosmology, since it involves quantum field theory techniques applied to cosmology. One of the lines taken in this thesis will be trusting the result from particle physics and tackle the cosmological constant problem from the gravity side, although many of the techniques employed lie at the interface between particle physics and cosmology.

The second solution could for instance consist in introducing new dynamical degrees of freedom by invoking new fluids with negative pressure. Quintessence is one of the important representatives of this class of modifications. The acceleration is due to a scalar field whose kinetic energy is small in comparison to its potential energy, causing a dynamical equation of state with initially negative values (Doran et al. 2001). This class of theories might exhibit fine-tuning problems analogous to the cosmological constant.

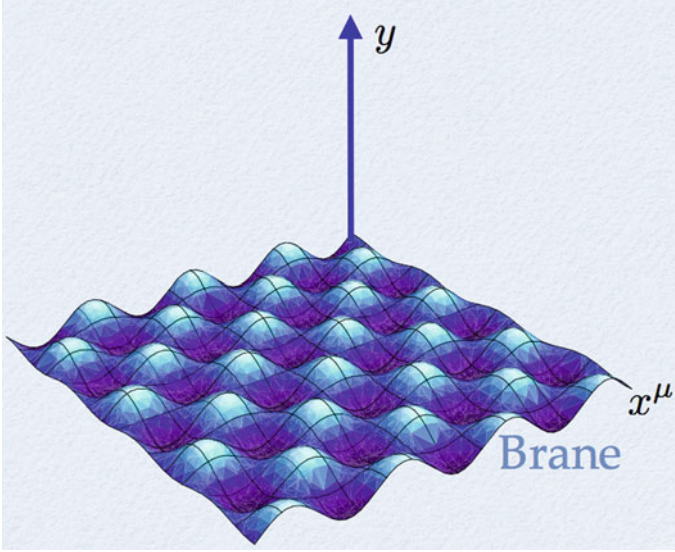
Alternatively, the third solution would correspond to explaining the acceleration of the Universe by changing the geometrical part of Einstein's equations. In particular, weakening gravity on cosmological scales might not only be responsible for a late-time speed-up of the Hubble expansion, but could also tackle the cosmological constant problem. Such scenarios arise in infrared modifications of general relativity like massive gravity or in higher-dimensional frameworks, which will be summarized shortly in the following.

### ***1.1.1 Infrared Modifications of GR***

In this thesis we will consider the first and third categories, namely cosmological constant  $\lambda$  and modified gravity. We will particularly study the infra-red modifications of gravity. One of the important large scale modified theories of gravity in the higher dimensional picture is the Dvali-Gabadadze-Porrati (DGP) model (Dvali et al. 2000). In this braneworld model our Universe is confined to a three-brane embedded in a five-dimensional bulk (Fig. 1.1). On small scales, four-dimensional gravity is recovered due to an intrinsic Einstein Hilbert term given by the brane curvature, whereas on larger, cosmological scales gravity is systematically weaker as the graviton leaks into the extra dimension. The action of the DGP model is given by

---

<sup>1</sup>In fact, it must be included from an effective field theory point of view.



**Fig. 1.1** Could our universe be just a part of higher dimensional space-time?

$$S_{\text{DGP}} = \frac{M_5^3}{2} \int d^5x \sqrt{-g_5} R_5 + \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g_4} R_4 + M_{\text{Pl}}^2 \int d^4x \sqrt{-g_4} K \quad (1.1)$$

where  $M_{\text{Pl}}$  and  $M_5$  respectively correspond to the fundamental Planck scales in the bulk and on the brane and  $K$  is the trace of the extrinsic curvature on the brane. Similarly,  $R_5$  and  $R_4$  are the corresponding Ricci scalars on the bulk and the brane respectively. The brane is positioned at  $y = 0$  where  $y$  denotes the extra fifth dimension and  $x_\mu$  are the four-dimensional coordinates. The crossover scale between 4- and 5-dimensional gravity is given by the ratio of these two Planck scales:  $r_c = 1/m_c$  where  $m_c = M_5^3/M_{\text{Pl}}^2$ .

Using the principle of least action one obtains the modified Einstein equations

$$M_5^3 G_{ab}^{(5)} + M_{\text{Pl}}^2 G_{\mu\nu}^{(4)} \delta_a^\mu \delta_b^\nu \delta(y) = T_{\mu\nu} \delta_a^\mu \delta_b^\nu \delta(y). \quad (1.2)$$

where here  $a, b = 0, \dots, 4$  and  $\mu, \nu = 0, \dots, 3$ . Being a fundamentally higher dimensional theory, the effective four-dimensional graviton on the brane carries five degrees of freedom, namely the usual helicity-2 modes, two helicity-1 modes and one helicity-0 mode. Whilst the helicity-1 modes typically decouple, the helicity-0 one can mediate an extra fifth force. In the limit  $m_c \rightarrow 0$ , one recovers General Relativity (GR) through the Vainshtein mechanism: The basic idea is to decouple the additional modes from the gravitational dynamics via nonlinear interactions of