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Hoai An Le Thi
Tao Pham Dinh
Ngoc Thanh Nguyen *Editors*

Modelling, Computation and Optimization in Information Systems and Management Sciences

Proceedings of the 3rd International Conference on Modelling, Computation and Optimization in Information Systems and Management Sciences - MCO 2015 - Part I

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Preface

This volume contains 86 selected full papers (from 181 submitted ones) presented at the MCO 2015 conference, held on May 11–13, 2015 at University of Lorraine, France.

MCO 2015 is the third event in the series of conferences on Modelling, Computation and Optimization in Information Systems and Management Sciences organized by LITA, the Laboratory of Theoretical and Applied Computer Science, University of Lorraine.

The first conference, MCO 2004, brought together 100 scientists from 21 countries and was a great success. It included 8 invited plenary speakers, 70 papers presented and published in the proceedings, "Modelling, Computation and Optimization in Information Systems and Management Sciences", edited by Thi Hoai An and Pham Dinh Tao, Hermes Sciences Publishing, June 2004, 668 pages, and 22 papers published in the European Journal of Operational Research and in the Journal of Global Optimization. The second conference, MCO 2008 was jointly organized by LITA and the Computer Science and Communications Research Unit, University of Luxembourg. MCO 2008 gathered 66 invited plenary speakers and more than 120 scientists from 27 countries. The scientific program consisted of 6 plenary lectures and of the oral presentation of 68 selected full papers as well as 34 selected abstracts covering all main topic areas. Its proceedings were edited by Le Thi Hoai An, Pascal Bouvry and Pham Dinh Tao in Communications in Computer and Information Science 14, Springer. Two special issues were published in Journal of Computational, Optimization & Application (editors: Le Thi Hoai An, Joaquim Judice) and Advance on Data Analysis and Classification (editors: Le Thi Hoai An, Pham Dinh Tao and Ritter Guntter).

MCO 2015 covered, traditionally, several fields of Management Science and Information Systems: Computer Sciences, Information Technology, Mathematical Programming, Optimization and Operations Research and related areas. It will allow researchers and practitioners to clarify the recent developments in models and solutions for decision making in Engineering and Information Systems and to interact and discuss how to reinforce the role of these fields in potential applications of great impact. It would be a timely occasion to celebrate the 30th birthday of DC programming and DCA, an efficient approach in Nonconvex programming framework.

Continuing the success of the first two conferences, MCO 2004 and MCO 2008, MCO 2015 will be attended by more than 130 scientists from 35 countries. The International

Scientific Committee consists of more than 80 members from about 30 countries all the world over. The scientific program includes 5 plenary lectures and the oral presentation of 86 selected full papers as well as several selected abstracts covering all main topic areas. MCO 2015's proceedings are edited by Le Thi Hoai An, Pham Dinh Tao and Nguyen Ngoc Thanh in Advances in Intelligent Systems and Computing (AISC), Springer. All submissions have been peer-reviewed and we have selected only those with highest quality to include in this book.

We would like to thank all those who contributed to the success of the conference and to this book of proceedings. In particular we would like to express our gratitude to the authors as well as the members of the International Scientific Committee and the referees for their efforts and cooperation. Finally, the interest of the sponsors in the meeting and their assistance are gratefully acknowledged, and we cordially thank Prof. Janusz Kacprzyk and Dr. Thomas Ditzinger from Springer for their supports.

We hope that MCO 2015 significantly contributes to the fulfilment of the academic excellence and leads to greater success of MCO events in the future.

March 2015

Hoai An Le Thi
Tao Pham Dinh
Ngoc Thanh Nguyen

DC Programming and DCA: Thirty Years of Developments

The year 2015 marks the 30th birthday of DC (Difference of Convex functions) programming and DCA (DC Algorithm) which were introduced by Pham Dinh Tao in 1985 as a natural and logical extension of his previous works on convex maximization since 1974. They have been widely developed since 1994 by extensive joint works of Le Thi Hoai An and Pham Dinh Tao to become now classic and increasingly popular.

DC programming and DCA can be viewed as an elegant extension of Convex analysis/Convex programming, sufficiently broad to cover most real-world nonconvex programs, but no too in order to be able to use the powerful arsenal of modern Convex analysis/Convex programming. This philosophy leads to the nice and elegant concept of approximating a nonconvex (DC) program by a sequence of convex ones for the construction of DCA: each iteration of DCA requires solution of a convex program. It turns out that, with appropriate DC decompositions and suitably equivalent DC reformulations, DCA permits to recover most of standard methods in convex and nonconvex programming. These theoretical and algorithmic tools, constituting the backbone of Nonconvex programming and Global optimization, have been enriched from both a theoretical and an algorithmic point of view, thanks to a lot of their applications, by researchers and practitioners in the world, to model and solve nonconvex programs from many fields of Applied Sciences, including Data Mining-Machine Learning, Communication Systems, Finance, Information Security, Transport Logistics & Production Management, Network Optimization, Computational Biology, Image Processing, Robotics, Computer Vision, Petrochemicals, Optimal Control and Automatic, Energy Optimization, Mechanics, etc. As a continuous approach, DC programming and DCA were successfully applied to Combinatorial Optimization as well as many classes of hard nonconvex programs such as Variational Inequalities Problems, Mathematical Programming with Equilibrium Constraints, Multilevel/Multiobjective Programming.

DC programming and DCA were extensively developed during the last two decades. They were the subject of several hundred articles in the high ranked scientific journals and the high-level international conferences, as well as various international research projects, and were the methodological basis of more than 50 PhD theses. More than

90 invited symposia/sessions dedicated to DC programming & DCA were presented in numerous international conferences. The ever-growing number of works using DC programming and DCA proves their power and their key role in Nonconvex programming/Global optimization and many areas of applications.

In celebrating the 30th birthday of DC programming and DCA, we would like to thank the founder, Professor Pham Dinh Tao, for creating these valuable theoretical and algorithmic tools, which have such a wonderful scientific impact on many fields of Applied Sciences.

Hoai An Le Thi
General Chair of MCO 2015

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Part I
Combinatorial Optimization and Applications

A New Variant of the Minimum-Weight Maximum-Cardinality Clique Problem to Solve Conflicts between Aircraft

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Abstract. In this article, we formulate a new variant of the problem of finding a maximum clique of minimum weight in a graph applied to the detection and resolution of conflicts between aircraft. The innovation of the model relies on the cost structure: the cost of the vertices cannot be determined a priori, since they depend on the vertices in the clique. We apply this formulation to the resolution of conflicts between aircraft by building a graph whose vertices correpond to a set of maneuvers and whose edges link conflict-free maneuvers. A maximum clique of minimal weight yields a conflict-free situation involving all aircraft and minimizing the costs induced. We solve the problem as a mixed integer linear program. Simulations on a benchmark of complex instances highlight computational times smaller than 20 seconds for situations involving up to 20 aircraft.

Keywords: Air Traffic Control, Conflict Resolution, Maximum Clique, Mixed Integer Linear Programming.

1 Introduction

Developing advanced decision algorithms for the air traffic control (ATC) is of great importance for the overall safety and capacity of the airspace. Resolution algorithms for the air conflict detection and resolution problem are relevant especially in a context of growing traffic, where capacity and safety become an issue. Indeed, a simulation-based study performed by Lehouillier et al. [1] shows that the controllers in charge of the traffic in 2035, which will have increased by 50%, would have to solve on average 27 conflicts per hour in a busy sector.

Maintaining separation between aircraft is usually referred to as the air conflict detection and resolution (CDR) problem. A conflict is a predicted loss of separation, i.e., when two aircraft are too close to each other regarding predefined horizontal and vertical separation distances of 5NM and 1000ft respectively. To solve a conflict, the controllers issue maneuvers that can consist of speed, heading or altitude changes. Given the current position, speed, acceleration and the

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predicted trajectory of a set of aircraft, the CDR problem corresponds to identifying the maneuvers required to avoid all conflicts while minimizing the costs induced.

The CDR problem is one of the most widely studied problems in air traffic management. For a comprehensive coverage of the existing literature, the reader may refer to the review in Martìn-Campo thesis [2]. Exact methods include optimal control, which can be associated with nonlinear programming. However, these methods suffer from the sensitivity to the starting point of the resolution and the high computational time. Mixed integer linear and nonlinear programming (MILPs and MINLPs) techniques are often considered. Omer and Farges [3] present a time-discretization of optimal control. Omer [4] also develops a space discretization using the points of interest for the conflict resolution. Pallottino et al. [5] develop MILPS solving the problem with speed changes and constant headings or with heading changes and constant speeds. Alonso-Ayuso et al. [6] develop a MILP that considers speed and altitude changes. However, MINLPs suffer from high computational times and do not give any optimality guarantee in finite time. Besides, the hypotheses made in MILPs to have linear constraints may not work in all situation. Several heuristics were developed to find a solution rapidly. Examples of techniques developed include ant colony algorithms like in Durand and Alliot [7], variable neighborhood searches (see Alonso-Ayuso et al. [8]). Other fast methods include particle swarm optimization, prescribed sets or neural networks. Heuristics find a solution rapidly, but the hypotheses can be restrictive and the convergence is not guaranteed. Graph theory is seldom used in ATC. Generally, conflicts between aircraft are modeled by a graph whose vertices represent the different aircraft and whose edges link pairs of conflicting aircraft, like in Vela [9]. Barnier and Brisset [10] assign flight levels to aircraft with intersecting routes by looking for maximum cliques in a graph where a proper coloring of the vertices defines an assignment of all aircraft to a set of flight levels.

The model presented in this article uses the concept of a clique in a graph, which is a subset of the vertices where each pair of elements is linked by an edge. Finding a maximum clique in an arbitrary graph is a well-known optimization problem that is $\mathcal{NP}$-hard. The problem has been thoroughly studied and several methods, both exact and heuristic, have been developed. For a comprehensive coverage on the subject, one can refer to Bomze et al. [11] and Hao et al. [12].

We formulate the air conflict detection and resolution problem as a new variant of the problem of finding a maximum clique of minimum weight in a graph. To this end, we build a graph whose vertices represent a set of possible maneuvers and where a clique yields a conflict-free solution involving all the aircraft. On the one hand, our model is innovative due to the cost structure for the vertices. With this model, we can maintain a reasonable size for the graph built, hence reducing the computational time. On the other hand, our model significance relies on its flexibility: a modification of the problem constraintes or objective function do not jeopardize the validity of the mathematical framework developed.

Being flexible is critical in ATC: in addition to being able to cover more ground, it will allow meaningful comparisons with existing models in the literature.

2 Problem Formulation

2.1 Modeling Aircraft Dynamics

To model the flight dynamics, we use the three-dimensional point-mass model presented in the BADA user manual [13]. Aircraft follow their planned 4D trajectory, which is a sequence of 4D points requiring time and space accuracy, leaving the remainder of the trajectory almost unconstrained. The non-compliance with this contract costs penalty fees to companies. As a consequence, an aicraft needs to recover its initial 4D trajectory after performing a maneuver. We assume that the planned speed for an aircraft corresponds to its nominal speed, i.e., the speed minimizing the fuel burn rate per distance unit traveled using the model described in [13].

Maneuvers are performed dynamically as described in [14], where the author states that the typical acceleration during a speed adjustment is in the order of 0.4kn/s. Heading changes are approximated by a steady turn of constant rate and radius. The changes of flight level are performed with a vertical speed, whose computation is detailed in [13], as a function of the thrust, drag, and true airspeed.

2.2 On Cliques and Stables

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected, simple graph with a vertex set $\mathcal{V}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$.

A *clique* in graph $\mathcal{G}$ is a vertex set $\mathcal{C}$ with the property that each pair of vertices in $\mathcal{C}$ is linked by an edge:

$$\mathcal{C} \subseteq \mathcal{V} \text{ is a clique} \Leftrightarrow \forall (u, v) \in \mathcal{C} \times \mathcal{C}, (u, v) \in \mathcal{E} \tag{1}$$

A *maximum* clique in $\mathcal{G}$ is a clique that is not a subset of any other clique in $\mathcal{G}$. The cardinality of a maximum clique of $\mathcal{G}$ is called *clique number* and is denoted by $w(\mathcal{G})$. Let $c : \mathcal{V} \to \mathbb{R}$ be a vertex-weight function associated with $\mathcal{G}$. A *maximum* clique of *minimum-weight* in $\mathcal{G}$ is a maximum clique $\mathcal{C}$ that minimizes $\sum_{v \in \mathcal{C}} c(v)$.

A *stable set* $\mathcal{S} \subseteq \mathcal{V}$ is a subset of vertices no two of which are adjacent. A *bipartite* graph is a graph whose vertices can be partitionned into two distinct stable sets $\mathcal{V}_1$ and $\mathcal{V}_2$. Each edge of the graph connects one vertex of one stable to a vertex in the other stable. This concept is extended to $k-$*partite* graphs, where the vertex set is partitionned into k distinct stable sets.

2.3 Graph Construction

In this subsection, we introduce the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ used to model the CDR problem.

Defining the Vertices. Let $\mathcal{F} = [\![1; n]\!]$ denote the set of the considered aircraft. We define $\mathcal{M} = \cup_{f=1}^{n} \mathcal{M}_f$ as the set of the possible maneuvers, $\mathcal{M}_f$ being the set of maneuvers for aircraft $f \in \mathcal{F}$. We consider both horizontal and vertical maneuvers of the following types:

- *NIL* refers to the *null* maneuver, i.e., when no maneuver is performed;
- H_θ corresponds to a heading change by an angle $\theta \in [-\pi; \pi]$;
- S_δ corresponds to a relative speed change of $\delta\%$;
- $V_{\delta h}$ denotes a change of δh flight levels.

A maneuver $m \in \mathcal{M}$ is described as a triplet $(\delta\chi_m, \delta V_m, \delta FL_m)$ corresponding to the heading, speed and flight level changes induced by m. The set of vertices is defined as $\mathcal{V} = [\![1; |\mathcal{M}|]\!]$, where $|\mathcal{M}|$ is the cardinality of set $\mathcal{M}$. We note $\mathcal{V}_f$ the set of vertices corresponding to aircraft f.

In emergency scenarios where the feasibility of the problem can be an issue, it is possible to introduce n vertices corresponding to costly emergency maneuvers to ensure the feasibility of the problem. However, since the feasibility was not an issue for the tested instances, those vertices were not considered in this article. The weight of the vertices correspond to the fuel consumption induced by the corresponding maneuvers. We give further detail in Subsection 2.3.

Defining the Edges. Let $(i, j) \in \mathcal{V} \times \mathcal{V}$ be a pair of vertices representing maneuvers $(m_i, m_j) \in \mathcal{M} \times \mathcal{M}$ of aircraft $(f_i, f_j) \in \mathcal{F} \times \mathcal{F}$. For $i \neq j$, we write $m_i \square m_j$ when no conflict occurs if aircraft f_i follows maneuver m_i while aircraft f_j performs maneuver m_j. The set of edges $\mathcal{E}$ corresponds to the pairs of maneuvers performed by two different aircraft without creating conflicts:

$$\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V}, i \neq j : m_i \square m_j\} \tag{2}$$

It is important to note that there is no edge between two different maneuvers of a given aircraft, which yields Proposition 1.

Proposition 1. *For all $f \in \mathcal{F}, \mathcal{V}_f$ is a stable set, i.e there is no edge linking two distinct vertices of $\mathcal{V}_f$. Hence, the graph $\mathcal{G}$ is $|\mathcal{F}|$-partite.*

Let $(i, j) \in \mathcal{V} \times \mathcal{V}$ be a pair of vertices representing maneuvers $(m_i, m_j) \in \mathcal{M} \times \mathcal{M}$ of aircraft $(f_i, f_j) \in \mathcal{F} \times \mathcal{F}$. The methodology used to compute if the edge (i, j) is added to $\mathcal{G}$ is described with the following notations:

- $\mathcal{T}$: time horizon for the conflict resolution;
- $\boldsymbol{p}_{f_i}(t) \in \mathbb{R}^3$: position vector of aircraft f_i at time t. $p_{f_i,x}(t)$ $p_{f_i,y}(t)$ and $p_{f_i,z}(t)$ denote respectively the abscissa, ordinate and altitude components of the position vector;
- $\boldsymbol{s}_{f_i}(t) \in \mathbb{R}^3$: speed vector of aircraft f_i at time t. $s_{f_i,x}(t)$ $s_{f_i,y}(t)$ and $s_{f_i,z}(t)$ denote respectively the abscissa, ordinate and altitude components of the speed vector;
- $\boldsymbol{a}_{f_i}(t) \in \mathbb{R}^3$: acceleration vector of aircraft f_i at time t. $a_{f_i,x}(t)$ $a_{f_i,y}(t)$ and $a_{f_i,z}(t)$ denote respectively the abscissa, ordinate and altitude components of the acceleration vector;
- $\boldsymbol{p}_{f_j}(t), \boldsymbol{s}_{f_j}(t)$ and $\boldsymbol{a}_{f_j}(t)$ are also defined following the same notations.

The definition of the maneuvers m_i and m_j applied to f_i and f_j is used to project the aircraft trajectory over time. Aircraft f_i and f_j are said to be separated at time t if and only if at least one of constraints (3) and (4) holds. In this paper we choose $D_{\text{h,min}} = 5\text{NM}$ and $D_{\text{v,min}} = 1000\text{ft}$.

$$d^h_{f_if_j}(t)^2 = (p_{f_i,x}(t) - p_{f_j,x}(t))^2 + (p_{f_i,y}(t) - p_{f_j,y}(t))^2 \geq D^2_{\text{h,min}} \quad (3)$$

$$d^v_{f_if_j}(t)^2 = (p_{f_i,z}(t) - p_{f_j,z}(t))^2 \geq D^2_{\text{v,min}} \quad (4)$$

At any time $t \in \mathcal{T}$, either none, one or both aircraft are maneuvering. $\mathcal{T}$ can thus be divided into intervals where both f_i and f_j have a constant acceleration. For each interval, we compute the time at which the aircraft are the closest to verify if the separation constraints hold. Let $\mathcal{T}_k$ be one of these intervals. Consider f_i and $t_0 \in \mathcal{T}$ be the starting time of maneuver m_i. If we assume that maneuver m_i is applied with a constant acceleration, we obtain the position and the speed vector of f_i at time $t_0 + t$ with t such that $t - t_0 \leq |\mathcal{T}_k|$:

$$\boldsymbol{p}_{f_i}(t_0 + t) = \boldsymbol{p}_{f_i}(t_0) + (t - t_0)\boldsymbol{s}_{f_i}(t_0) + \frac{(t - t_0)^2}{2}\boldsymbol{a}_{f_i}(t_0) \quad (5)$$

$$\boldsymbol{s}_{f_i}(t_0 + t) = \boldsymbol{s}_{f_i}(t_0) + (t - t_0)\boldsymbol{a}_{f_i}(t_0) \quad (6)$$

Let $\boldsymbol{p}^h_{f_if_j}$ (respectively $\boldsymbol{s}^h_{f_if_j}$, $\boldsymbol{a}^h_{f_if_j}$) denote respectively the horizontal position, the speed and the acceleration of aircraft f_j relatively to aircraft f_i. We define

$$\begin{aligned} d^h_{f_if_j}(t + \tau) &= ||\boldsymbol{p}^h_{f_if_j}(t + \tau)|| \\ &= ||\boldsymbol{p}^h_{f_if_j}(t) + \tau\boldsymbol{s}^h_{f_if_j}(t) + \frac{\tau^2}{2}\boldsymbol{a}^h_{f_if_j}(t)|| \end{aligned}$$

where $\tau \geq 0$.
Let $\tau_{f_if_j} \in \underset{\tau \geq 0}{\text{argmin}}\, d^h_{f_if_j}(t + \tau)^2$, and $t^h_{f_if_j} \in \underset{t \in \mathcal{T}}{\text{argmin}}\, d^h_{f_if_j}(t)^2$.

We have: $t^h_{f_if_j} = \begin{cases} 0 & \text{if } \tau_{f_if_j} = 0 \\ |\mathcal{T}| & \text{if } \tau_{f_if_j} \geq |\mathcal{T}_k| \\ \tau_{f_if_j} & \text{otherwise} \end{cases}$

Aircraft f_i and f_j are horizontally separated during interval $\mathcal{T}$ if and only if (7) holds:

$$d^h_{f_if_j}(t^h_{f_if_j})^2 \geq D^2_{\text{h,min}} \quad (7)$$

By a similar reasoning, aircraft f_i and f_j are vertically separated during interval $\mathcal{T}$ if and only if (8) holds:

$$d^v_{f_if_j}(t^v_{f_if_j})^2 \geq D^2_{\text{v,min}} \quad (8)$$

If either (7) or (8) holds when aircraft f_i and f_j apply maneuvers m_i and m_j, then an edge is created between i and j. As explained in 2.1, it is important that every aircraft initiates a safe return towards its initial trajectory once the conflict is avoided. For each edge, we compute the minimum time necessary before one or both aircraft can recover their initial trajectories. The cost of the recovery of a trajectory is detailed in Subsection 2.3.

Application to the CDR Problem. As mentioned in Section 1, given the current position, speed, acceleration and the planned trajectories of a set of aircraft, solving the CDR problem consists in finding a conflict-free set of maneuvers that minimizes the costs. Proposition 2 links the cliques in $\mathcal{G}$ to the CDR problem:

Proposition 2. *Let $\mathcal{C}$ be a clique in graph $\mathcal{G}$. Then $\mathcal{C}$ represents a set of conflict-free maneuvers for a subset of $\mathcal{F}$ of cardinality $|\mathcal{C}|$.*

Proposition 2 shows that finding a set of conflict-free maneuvers for $\mathcal{F}$ is equivalent to finding a clique of $\mathcal{G}$ of cardinality $|\mathcal{F}|$. We derive the following theorem:

Theorem 1. *If a conflict-free solution exists, then $\omega(\mathcal{G}) = |\mathcal{F}|$. Otherwise, $\omega(\mathcal{G})$ is the maximum number of flights involved in a conflict-free situation.*

We define the problem $\text{CDR}_{\mathcal{M}}$ as the restriction of the CDR problem to the set of maneuvers $\mathcal{M}$. Using both Proposition 2 and Theorem 1, we can state anew the $\text{CDR}_{\mathcal{M}}$ problem as follows: solving the $\text{CDR}_{\mathcal{M}}$ problem consists in finding a clique of maximum cardinality and minimal cost in graph $\mathcal{G}$. In fact, we consider a new variant of a clique problem where the weight associated with a vertex is not known a priori and rather depends on the edges induced by the clique. Indeed, the cost associated with a maneuver depends on the duration that this maneuver will be performed before returning towards the planned trajectory. Because this duration depends on the maneuvers selected for the other aircraft, it cannot be determined a priori and must be computed as the maximum duration needed to avoid a loss of separation with all other aircraft given their chosen maneuvers. To handle such vertex costs, we first define edge costs.

Computing the Cost of the Edges. The cost measure chosen for this article corresponds to the extra fuel consumption induced by the maneuvers, i.e., the additional fuel required to return to the 4D trajectory after the maneuver is performed. We use the model given in [13]. For a jet commercial aircraft f, the fuel consumption by time and distance unit is given by (9) and (10):

$$C_{t,f}(t) = c_{1,f}\left(1 + \frac{V_f(t)}{c_{2,f}}\right) F_{T,f}(t) \tag{9}$$

$$C_{d,f}(t) = \frac{C_{t,f}(t)}{V_f(t)} \tag{10}$$

where $c_{1,f}$ and $c_{2,f}$ are numerical constants depending on the type of aircraft f.

We compute the cost of an edge $e = (i, j)$ linking two vertices representing two maneuvers of aircraft f_i and f_j, denoted m_i and m_j, as a pair constituted of the extra fuel costs for both f_i and f_j, denoted $C_i^{(i,j)}$ and $C_j^{(i,j)}$. The additional consumed fuel corresponds to the performed maneuver along with the fuel required to recover the inital 4D trajectory. After a change of speed of $\delta\%$ during a period δt, the aircraft recovers its 4D trajectory by making the opposite change of speed during δt. After a change of direction $\delta\chi$ during a period δt, the aircraft performs a turn with an angle θ_r in order to recover its physical trajectory along with a change of speed to retrieve the 4D trajectory. The cost induced is the extra fuel burnt when the aircraft flies at the recovery speed and the fuel burnt on the extra distance induced by the maneuver. For a flight level change, we compute the extra cost as the difference of consumption between the different flight levels, along with the cost of changing twice of flight level. The distance flown is also longer, and this extra distance is also accounted for.

Computing the Cost of the Vertices. Several techniques can be followed in order to determine the vertices cost. The basic one would be to discretize the duration of the maneuver, and to create the vertices accordingly. In this situation, computing the costs would be straight-forward. However, the drawback of this method is that the graph built is huge, which could result in a difficult resolution. We choose to follow another structure of cost because it is more compact in terms of graph size.

Let us consider a vertex i which corresponds to a maneuver m_i for an aicraft f_i. The cost of each edge linking i to one of its neighbors j, associated to a maneuver m_j for aircraft f_j, corresponds to f_i applying m_i during a time t_i^j, which depends on m_j. Time t_i^j is the minimum time during which f_i must apply m_i in order to avoid any conflict if one or both aircraft return to their initial trajectory. Following maneuver m_i for a duration t_i^j induces a cost $C_i^{(i,j)}$. If i is part of the maximum clique $\mathcal{C}$ to be determined, we need to establish the time t_i during which maneuver m_i is actually applied in order to determine its cost c_i. t_i is obtained by:

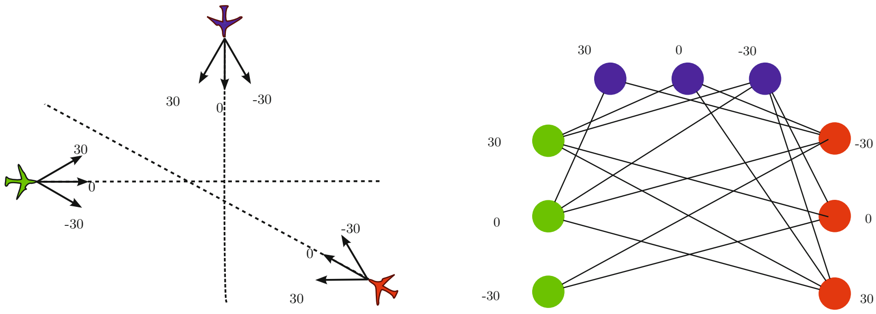
$$t_i = \max_{j \in \mathcal{V} \cap \mathcal{C}} t_i^j \tag{11}$$

As a consequence, we have that c_i is the cost of aircraft f_i applying m_i during t_i. If i is not part of the maximum clique $\mathcal{C}$, then no constraint is imposed on the cost c_i. As detailed in Section 3, the optimization model will automatically force the value of c_i to 0. To conclude, we have that for any $i \in \mathcal{V}$:

$$c_i = \begin{cases} \max_{j \in \mathcal{V} \cap \mathcal{C}} C_i^{(i,j)} & \text{if } i \in \mathcal{C} \\ 0 & \text{otherwise} \end{cases}$$

2.4 Illustrative Example

For the sake of clarity, an illustrative example with three aircraft is given in Figure 1. If each aircraft follows its planned trajectory, conflicts will happen between the blue aircraft and the two others. For this example, we assume that, in addition to the null maneuver, only two heading changes ($\pm 30°$) are allowed. We build the CDR graph shown in Figure 1(b). The graph is 3-partite, as the vertex set is partitionned into 3 stable sets of 3 vertices each. Solving the CDR is then equivalent to searching for a minimum-weight clique of 3 vertices, i.e., a triangle.



(a) Illustrative example (b) Resulting graph $\mathcal{G}$

Fig. 1. Illustrative example with three aircraft

3 Methodology

Determining the cost of a vertex i is very specific, since it is correlated to whether or not i belongs to a maximum clique $\mathcal{C}$. As a consequence, the algorithms usually used in existing libraries dedicated to graph theory cannot be used for our model. We formulate the problem as a mixed-integer linear program using the following variables:

- $x_i = \begin{cases} 1 & \text{if vertex } i \text{ is part of the maximum clique} \\ 0 & \text{otherwise} \end{cases}$
- $c_i \in \mathbb{R}_+$ is the cost of vertex i.

We describe the clique search by the following linear integer program:

$$\text{minimize} \sum_{i \in \mathcal{V}} c_i \tag{12}$$

$$\text{subject to } x_i + x_j \leq 1, \forall (i,j) \notin \mathcal{E} \tag{13}$$

$$\sum_{i \in \mathcal{V}} x_i = |\mathcal{F}| \tag{14}$$

$$c_i \geq C_i^{(i,j)}(x_i + x_j - 1), \forall (i,j) \in \mathcal{E} \tag{15}$$

$$x_i \in \{0;1\}, \forall i \in \mathcal{V} \tag{16}$$

$$c_i \in \mathbb{R}_+, \forall i \in \mathcal{V} \tag{17}$$

The objective function (12) minimizes the cost of the maneuvers. (13) are clique constraints, and constraint (14) exploits Theorem 1 defining the cardinality of the maximum clique. Constraints (15) are used to compute the cost of the vertices: if a vertex is in the maximum clique, then its cost must be greater than its cost on all edges connecting it to other vertices in the clique.

4 Results

All tests were performed on a computer equipped with an Intel Core i7-3770 processor, 3.4 GHz, 8-GB RAM. The algorithms were implemented in C++ and using CPLEX 12.5.1.0[1].
The headings of the tables presented in these section are given as follows:

- *case*: case configuration;
- $|\mathcal{F}|$: number of aircraft;
- $|\mathcal{V}|$: number of vertices;
- $|\mathcal{E}|$: number of edges;
- $d = \frac{2|\mathcal{E}|}{|\mathcal{V}|(|\mathcal{V}|-1)}$: graph density;
- n: number of variables;
- m: number of constraints;
- z_{ip}: optimal value for the problem;
- *nodes*: number of branch-and-bound nodes;
- t_{lp}: time (in seconds) to continuous relaxation of the MILP;
- t_{ip}: time (in seconds) to obtain the z_{ip} value;

4.1 Benchmark Description

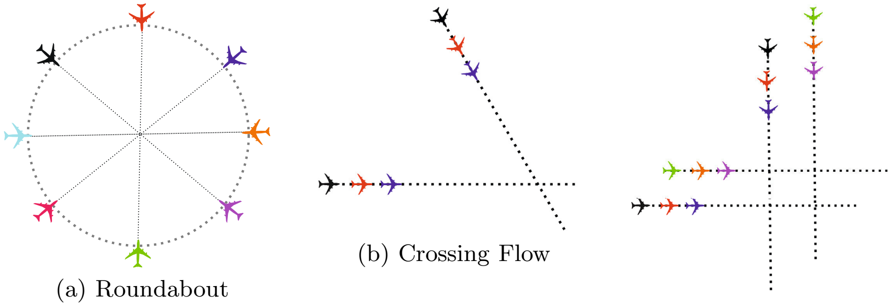
The benchmark used for this study gathers three types of instances. The first set is roundabout instances $\mathcal{R}_n$, where n aircraft are distributed on the circumference of a 100NM radius and fly towards the center at the same speed and altitude. The second set is crossing flow instances $\mathcal{F}_{n,\theta,d}$, where two trails of n aircraft separated by d nautical miles intersect each other with an angle θ. The last type of instance is a grid $\mathcal{G}_{n,d}$ constituted of two crossing flow instances $\mathcal{F}_{n,\frac{\pi}{2},d}$ with a 90° angle, one instance being translated 15NM North-East from the other. An example of these instances is given on Figure 2.

4.2 Computational Results

The first set of simulations considers only horizontal maneuvers, with relative speed changes of ±3% and ±6% and heading changes of $\pm5°, \pm10°, \pm15°$. The graph remains small when one considers this set of maneuvers, and their small magnitude makes them less costly. Nevertheless, if these values were to be inefficient to solve all the conflicts, we could introduce maneuvers of larger magnitude.

Table 1 gathers information about the graph $\mathcal{G}$, the MILP and the main computational results. The solution time for the continuous relaxation is very small, but the quality of the relaxation is mediocre. Indeed, the fractional solution of the linear relaxation chooses two maneuvers for each aircraft with a value of 0.5. Constraints (15) force the cost of each vertex to be 0, yielding an optimal value of 0 and a gap of 100%. Results also display short solution times: problems known to be complex with 20 aircraft are solved to optimality in less than 15 seconds. This result is very satisfying since the density of the graph is high.

[1] See the IBM-ILOG CPLEX v12.5. User's manual for CPLEX.



(a) Roundabout (b) Crossing Flow (c) Grid

Fig. 2. Examples

Table 1. Dimensions of the instances and computational results

Instance type	Case	Graph $\mathcal{G}$ $\lvert\mathcal{F}\rvert$	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}\rvert$	d	MILP m	n	Resolution z_{ip}	*nodes*	t_{lp}	t_{ip}
Roundabout	$\mathcal{R}_2$	2	22	90	0.39	44	225	3.71	6	0	0.02
	$\mathcal{R}_4$	4	44	492	0.52	88	1073	14.98	73	0	0.02
	$\mathcal{R}_6$	6	66	1194	0.56	132	2521	22.7	0	0.01	0.15
	$\mathcal{R}_8$	8	88	2184	0.57	176	4545	31.05	47	0.01	0.53
	$\mathcal{R}_{10}$	10	110	3430	0.57	220	7081	112.7	208	0.05	1.56
	$\mathcal{R}_{12}$	12	132	4944	0.57	264	10153	189.27	581	0.09	3.41
	$\mathcal{R}_{14}$	14	154	6720	0.57	308	13749	224.75	183	0.1	6.98
	$\mathcal{R}_{16}$	16	176	8896	0.57	352	18145	261.44	162	0.15	9.5
	$\mathcal{R}_{18}$	18	198	11358	0.58	396	23113	636.7	257	0.21	12.1
	$\mathcal{R}_{20}$	20	220	14027	0.58	440	28461	740.6	210	0.27	3.2
Flows	$\mathcal{F}_{5,30,10}$	10	110	4522	0.75	220	9265	49.08	405	0.02	1.5
	$\mathcal{F}_{5,45,10}$	10	110	4518	0.75	220	9257	41.29	535	0.02	1.52
	$\mathcal{F}_{5,60,10}$	10	110	4478	0.75	220	9177	34.49	238	0.02	1.39
	$\mathcal{F}_{5,75,10}$	10	110	4492	0.75	220	9205	30.66	496	0.02	1.34
	$\mathcal{F}_{5,90,10}$	10	110	4528	0.76	220	9277	28.28	269	0.02	1.41
Grids	$\mathcal{G}_{2,3,10}$	12	132	6645	0.78	264	13555	57.65	564	0.01	3.64
	$\mathcal{G}_{2,5,10}$	20	220	19724	0.82	440	39889	121.92	2740	0.2	12.7

In the second simulation set, we introduce altitude maneuvers: aircraft are allowed to move to an adjacent flight level. Table 2 reports the main results. The values of the optimal solutions for the roundabout instances remain the same, highlighting that it is optimal to make simple turns instead of changing flight levels. For the crossing flows and the grid instances, it is more efficient for some aircraft to change their flight level instead of turning or changing their speed. As a consequence, the solutions are less expensive. Solution times tend to slightly increase, but the solution can still be computed in a short time. These results are promising since the instances tested are denser than real-life instances.

Table 2. Dimensions of the instances and computational results

		Graph $\mathcal{G}$				MILP		Resolution			
Instance type	Case	$\lvert\mathcal{F}\rvert$	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{E}\rvert$	d	m	n	z_{ip}	*nodes*	t_{lp}	t_{ip}
Roundabout	$\mathcal{R}_2$	2	26	116	0.36	52	285	3.71	7	0	0.05
	$\mathcal{R}_4$	4	52	832	0.63	104	1769	14.98	272	0	0.41
	$\mathcal{R}_6$	6	78	2076	0.69	156	4309	22.7	498	0.01	0.25
	$\mathcal{R}_8$	8	104	3840	0.72	208	7889	31.05	328	0.01	1.12
	$\mathcal{R}_{10}$	10	130	6080	0.73	260	12421	112.7	499	0.05	1.41
	$\mathcal{R}_{12}$	12	156	9096	0.75	312	18505	189.27	798	0.09	3.42
	$\mathcal{R}_{14}$	14	182	12208	0.74	364	24781	224.75	532	0.1	6.88
	$\mathcal{R}_{16}$	16	208	16416	0.76	416	33249	261.44	467	0.15	11.45
	$\mathcal{R}_{18}$	18	234	20772	0.76	468	42013	636.7	682	0.21	14.12
	$\mathcal{R}_{20}$	20	260	25760	0.77	520	52041	740.6	845	0.27	8.12
Flows	$\mathcal{F}_{5,30,10}$	10	130	5102	0.75	260	9956	43.37	784	0.02	1.58
	$\mathcal{F}_{5,45,10}$	10	130	5098	0.75	260	9874	38.45	889	0.02	2.13
	$\mathcal{F}_{5,60,10}$	10	130	5059	0.75	260	9845	29.78	645	0.02	1.96
	$\mathcal{F}_{5,75,10}$	10	130	5134	0.75	260	9899	30.11	897	0.02	1.78
	$\mathcal{F}_{5,90,10}$	10	130	5199	0.76	260	10078	23.01	540	0.02	1.45
Grids	$\mathcal{G}_{2,3,10}$	12	156	7320	0.79	312	15087	45.18	945	0.01	4.12
	$\mathcal{G}_{2,5,10}$	20	260	20945	0.83	520	45878	99.73	3847	0.2	16.7

5 Conclusions

A new variant of the problem of finding a maximum clique of minimum weight in a graph and its application to aircraft conflict resolution have been presented. The innovation of the model comes from the cost structure: the costs of the vertices cannot be determined a priori since they depend on the vertices in the clique. As a consequence, we model the problem as a MILP. The model performs well, since complex instances involving up to 20 aircraft are solved to optimality in near real-time. The design of the model is flexible, meaning that tuning some parameters of the model will allow meaningful comparisons with existing models.

These conclusions validate the model as a basis for further research. For instance, techniques reducing the size of the graph are of interest. Adding uncertainties is also a meaningful extension of the model. Additional benchmarks including real-life instance and random scenarios will be necessary in order to challenge the model.

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