**Operations Research/Computer Science Interfaces Series** 

# **Kevin Tierney**

# Optimizing Liner Shipping Fleet Repositioning Plans





# **Operations Research/Computer Science Interfaces Series**

Volume 57

Series Editors: Ramesh Sharda Oklahoma State University, Stillwater, Oklahoma, USA

Stefan Voß University of Hamburg, Hamburg, Germany

More information about this series at http://www.springer.com/series/6375

Kevin Tierney

# Optimizing Liner Shipping Fleet Repositioning Plans



Kevin Tierney Decision Support & Operations Research Lab University of Paderborn Paderborn, Germany

ISSN 1387-666X Operations Research/Computer Science Interfaces Series ISBN 978-3-319-17664-2 ISBN 978-3-319-17665-9 (eBook) DOI 10.1007/978-3-319-17665-9

Library of Congress Control Number: 2015938698

Springer Cham Heidelberg New York Dordrecht London © Springer International Publishing Switzerland 2015

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made.

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www. springer.com)

# Contents

1	Intro	oduction	1			
	1.1	Approach and Contributions	3			
	1.2	Outline	5			
2	Cont	tainerized Shipping	7			
	2.1	Containers	8			
	2.2	Liner Shipping Networks	9			
		2.2.1 Services 1	0			
		2.2.2 Network Structure 1	2			
	2.3	Vessels 1	4			
		2.3.1 Fuel Consumption and Slow-Steaming 1	4			
		2.3.2 Stowage 1	.6			
	2.4	Carrier Alliances 1	.6			
	2.5	Ports and Container Terminals 1	.7			
3	Line	r Shipping Fleet Repositioning	21			
	3.1	Repositioning Overview	22			
	3.2	Phase-Out and Phase-In				
	3.3	Repositioning Activities	25			
		3.3.1 Sailing and Slow-Steaming 2	25			
		3.3.2 Sail-on-Service 2	26			
		3.3.3 Inducement and Omission 2	29			
		3.3.4 Cargo Demands 2	29			
		3.3.5 Equipment 3	30			
		3.3.6 Flexible Visitations	31			
	3.4	Asia-CA3 Case Study 3	31			
	3.5	Related Problems 3	32			
		3.5.1 Shipping Problems 3	32			
		3.5.2 Vehicle Routing Problems 3	34			
		3.5.3 Airline Disruption Management 3	34			
	3.6	Chapter Summary 3	\$4			

4	Meth	odologi	cal Background	35
	4.1	Autom	ated Planning	35
	4.2	Partial	-Order Planning	37
	4.3	Linear	and Mixed-Integer Programming	40
		4.3.1	LP Duality	41
		4.3.2	Mixed-Integer Programming	41
	4.4	Constr	aint Programming	45
		4.4.1	CSPs and COPs	45
		4.4.2	Solving CP Problems	46
	4.5	Metah	euristics	49
		4.5.1	Local Search	50
		4.5.2	Simulated Annealing	51
5	Line	· Shippi	ng Fleet Repositioning Without Cargo	53
	5.1	Datase		54
	5.2	A PDI	DL Model of Fleet Repositioning	55
		5.2.1	PDDL Model	55
		5.2.2	Planners	58
		5.2.3	POPF	59
		5.2.4	PDDL Model Computational Evaluation in POPF	65
	5.3	Tempo	ral Optimization Planning	67
	0.0	5.3.1	Linear Temporal Optimization Planning	70
		5.3.2	Domain Independent Heuristic Cost Estimation	72
		5.3.3	LTOP Model	73
		5.3.4	Fleet Repositioning Specific Heuristics	74
		5.3.5	LTOP Computational Evaluation	74
	5.4	ed-Integer Programming Model of Fleet Repositioning	76	
		541	Graph and MIP Description	76
		542	MIP Model Computational Evaluation in CPLEX	78
	5 5	A Con	straint Programming Model of Fleet Repositioning	79
	5.5	5 5 1	Model Description	79
		5.5.2	CP Model Computational Evaluation in G12	85
	5.6	Chapte	er Summary	87
6	Line	Shinni	ng Fleet Repositioning with Cargo	89
U	6.1	Graph	Construction	90
	0.1	611	Phase-Out	91
		612	Phase-In	01
		613	Flexible Visitations	02
		614	Sail-on-Service	92 Q2
		615	Sailing Cost	92 07
		616	Graph Formalization	94 04
	62	$\Delta re Fl$	ow Model	94 06
	0.2	6 2 1	Parameters	90 06
		0.2.1		90

		6.2.2	Variables	 97
		6.2.3	Objective and Constraints	 97
	6.3	Path-Ba	sed Model	 99
		6.3.1	Master Problem	 100
		6.3.2	Sub-problem	 101
		6.3.3	Reduced Graph	 101
	6.4	LSFRP	with Inflexible Visitations	 102
		6.4.1	Preprocessing	 105
		6.4.2	Equipment as Flows	 105
		6.4.3	Equipment as Demands	 107
	6.5	Heuristi	c Approaches	 108
		6.5.1	Simulated Annealing	 108
		6.5.2	Late Acceptance Hill Climbing	 110
		6.5.3	Solution Representation	 111
		6.5.4	Initial Solution Generation	 111
		6.5.5	Neighborhoods	 114
		6.5.6	Objective Evaluation	 115
	6.6	Comput	ational Complexity	 118
	6.7	Comput	ational Evaluation	 118
		6.7.1	Dataset	 119
		6.7.2	Arc, Node and Path Flow Approach Evaluations.	 120
		6.7.3	SA and LAHC Implementations	 123
		6.7.4	Initial Solution Heuristics Comparison	 127
		6.7.5	Neighborhood Analysis	 128
		6.7.6	SA and LAHC Results	 130
		6.7.7	Reference Instance Performance	 137
	6.8	Chapter	Summary	 138
-	C	•		1 / 1
/		USION		 141
	/.1	Outlook	• • • • • • • • • • • • • • • • • • • •	 142
A	No Ca	argo LSF	RP PDDL Domain	 143
	A.1	PDDL	Model	 143
		A.1.1	Predicates	 143
		A.1.2	Functions	 145
		A.1.3	Time	 147
		A.1.4	Initial and Goal States	 147
		A.1.5	Actions	 148
	A.2	Forward	PDDL Domain	 154
	A.3	Reverse	d PDDL Domain	 159

B	An L	TOP Mo	del of Fleet Repositioning 165
	<b>B</b> .1	Consta	nts
	<b>B</b> .2	State V	Variables
	<b>B.3</b>	Optimi	zation Variables
	<b>B.4</b>	Initial a	and Goal States
	<b>B.5</b>	Action	s167
		<b>B.5.1</b>	Phase-Out
		B.5.2	Phase-In
		B.5.3	Sailing
		B.5.4	Sailing with Equipment
		B.5.5	Sail-on-Service
Ref	ference	s	
Ind	lex		

## Chapter 1 Introduction

Situated at the heart of global trade, liner shipping networks transported over 1.5 billion tons of containerized cargo on over 5,000 seagoing container vessels in 2013 [156]. The world is becoming more and more connected through liner shipping networks. Indeed, according to the liner shipping connectivity index [69, 156], the average connectivity between nations has risen over 43 % between 2004 and 2013 alone. Container shipment volumes have also been steadily increasing ever since the container's invention in the 1950s [98, 156], despite a short period of decline in 2009. In fact, year-over-year global container throughput growth has been around 10 % for nearly two decades [65] with few exceptions. These trends show no signs of stopping as liner shipping continues to be responsible for more and more world trade [156].

Liner shipping networks consist of regularly scheduled, cyclical routes between ports. Some routes are short, visiting only several ports in a confined area such as the North Sea, while other routes span the entire globe. Figure 1.1 illustrates a service from Seago Line's 2014 network in the Mediterranean Sea. A defining feature of liner shipping compared to other forms of shipping, such as tramp shipping, is its periodicity. Liner routes have published schedules in which ports are visited at a regular frequency (e.g., weekly or bi-weekly).

Shipping lines primarily transport containers, which are steel boxes of one of several standardized dimensions that are filled with goods for shipment. All manner of cargo is carried over the seas, including everything from household goods to heavy machinery and even perishables like fruit and meat. Containers offer shippers security for their cargo as well as specialized features for cargo that needs to be cooled or cared for along its journey.

As the use of containers has increased, so too has the challenge of getting containers to their destinations on time and on budget. Modern shipping lines have networks that span the entire globe and use hundreds of container ships to move millions of containers. The liner carrier Maersk Line, for example, had enough capacity in 2014 on its ships to handle over 2.8 million 20-foot containers at a single given time [4]. Furthermore, the liner shipping market is competitive in many areas of



Fig. 1.1: The weekly Mediterranean Sea service offered by Seago Line in 2014 [131]

the world, requiring carriers to make their operations as efficient as possible to stay in business. In order to deal with the complexity and scale of the challenges they are facing, shipping lines are increasingly turning to algorithmic approaches and decision support systems that assist their employees in making difficult decisions. However, systems performing optimization of a cost or revenue based objective are still rare in the liner shipping industry. Indeed, the same aspects of such combinatorial problems that make them difficult for humans to solve also make them hard for computers to solve. Recent progress from computer scientists and operations researchers into liner shipping optimization has begun to make solving such problems possible, allowing a new generation of systems to emerge.

The research community has identified and modeled several well-known optimization problems that attempt to alleviate the challenges that the liner shipping industry faces (see, e.g., [26, 27]). At the strategic level, shipping lines must create the routes in their network to satisfy their customers' demands, a problem that despite a number of years of research is still too difficult to solve for most realistic sized problems [5, 121]. At the tactical level, container ships are assigned to services in an existing shipping network in order to minimize the sailing costs while keeping customer's demands in consideration [119]. And, finally, at the operational level, shipping lines must determine vessels' sailing speeds [43, 161], which vessels carry which containers, where to stow containers on vessels, and manage disruptions [20].

Through our collaboration with Maersk Line, currently the world's largest container shipping line [4], we found that a key operational problem has not yet been addressed in the liner shipping literature. Liner shipping networks are regularly adjusted in order to handle fluctuations in seasonal demand, as well as to adjust the network to changing macroeconomic conditions. These adjustments can involve adding new routes to the network, removing unprofitable routes, or expanding/contracting existing routes. Container ships must be *repositioned* within the network to facilitate these changes. Repositioning involves sailing vessels between routes in a liner shipping network in order to minimize sailing costs, port fees, and disruptions to container flows. During repositioning, ships may undertake a number of cost saving activities, such as bringing empty containers to places where they are needed or temporarily taking over operations on existing services. This problem is sometimes known in industry parlance as the *phase-in/phase-out* problem, which refers to the process of removing and inserting a repositioning vessel into normal operations, respectively.

Repositioning a ship can cost upwards of a million US dollars, and hundreds of ships are repositioned each year by the world's shipping lines. Finding plans of activities that minimize the cost of repositioning, while avoiding the disruption of cargo flows in the network, has the potential to save shipping lines significant amounts of money. Currently, ship repositioning plans are created by hand by employees of shipping lines. Furthermore, since one of the main cost components of repositioning ships is bunker fuel, optimizing vessel activities can also help shipping lines reduce their  $CO_2$  output and become more sustainable. This monograph considers the following core question:

Can an algorithm be developed to create real-world realizable liner shipping fleet repositioning plans within a reasonable amount of CPU time?

Fleet repositioning involves a number of side constraints. We aim to model key constraints that determine a plan's real-world feasibility, such as constraints on the sailing speed and capacity of ships, and ensure that the routes determined for ships adhere to a number of liner shipping specific constraints. Our goal is to generate repositioning plans in under an hour in order to allow repositioning coordinators to use our algorithms in an interactive fashion in a decision support system. Since even a small improvement in the objective function can translate to tens of thousands of dollars in cost savings or increased revenue, our algorithms should find repositioning plans that are within a few percent of the optimal solution.

We show that fleet repositioning problems can be solved in many cases to optimality with or without cargo flows on a dataset based on real-world data from our industrial collaborator. In addition, we compare our solution approaches for fleet repositioning to a real world scenario from Maersk Line in 2011. Our solution methods are able to find a repositioning plan with a profit of \$32.1 million,<sup>1</sup> which is roughly \$14 million higher than the profit of the actual repositioning that was carried out by Maersk Line.

#### **1.1 Approach and Contributions**

We approach fleet repositioning problems from a variety of perspectives, both in terms of what level of abstraction we take in our models, as well as in the solution procedures used to generate repositioning plans. We call the general problem of repositioning the liner shipping fleet repositioning problem (LSFRP). We model this problem with and without cargo flows, which represents two different views

<sup>&</sup>lt;sup>1</sup> All monetary units in this monograph are US dollars.

of operational goals for a shipping line. We test both of these models on datasets based on real scenarios from our industrial collaborator and crafted scenarios using industrial data.

We first model the problem without cargo flows in order to focus on minimizing the cost of the activities chosen during repositioning, and call this problem the no-cargo LSFRP (NCLSFRP). This problem has a number of simplifications of the overall problem with cargo flows that we present in detail in Chap. 5. A number of activities, such as sailing a vessel, have time-dependent task costs, i.e., the cost of performing an activity is dependent on the action duration. The NCLSFRP is difficult to solve due to these costs as well as because of the interactions of vessels in choosing which activities each vessel should perform. Since the problem contains both scheduling and routing components, it is not clear a priori which types of solution method will solve it most effectively.

We present four different solution approaches and describe the trade-offs between them, starting with an automated planning model using the planning domain definition language (PDDL). Since existing automated planning techniques have difficulties with time-dependent task costs, we introduce a novel planning technique called linear temporal optimization planning (LTOP), which combines the wellknown partial-order planning paradigm with linear programming. We model the NCLSFRP in the LTOP framework and show that planning and optimization need not be mutually exclusive endeavors. We also provide a mixed-integer programming (MIP) model and constraint programming (CP) model of the NCLSFRP, showing that while the MIP has a difficult time with the scheduling aspects of the problem, CP is able to outperform all other approaches, albeit with the least extensible model. Our PDDL, LTOP and MIP models were first published in [150], and our CP model in [88].

We then move to a model of the LSFRP with cargo and equipment flows. In addition to the challenge of optimizing activities with time-dependent task costs, we incorporate a multi-commodity flow problem to handle cargo and equipment. We also include a more realistic view of certain problem activities than in the NCLSFRP. We create a graph based representation of the problem that includes a number of key problem constraints and activities, such as sail-on-service opportunities, within the graph itself. We use this graph in all of our solution methods for the LSFRP with cargo and equipment flows.

We first model the problem as a MIP using an arc flow formulation and show that, while small instances can be solved to optimality, larger instances prove challenging for the MIP. We also introduce a node flow model for a special case of the LSFRP in which all activities have a fixed start and end time. On those instances that conform to this simplification of the LSFRP, we are able to find optimal solutions to nearly every instance in our dataset, including several instances that the arc flow model cannot solve.

Furthermore, we introduce a path based model which is solved using a column generation procedure. This approach solves even more problems that neither the arc flow or node flow models can solve. This model iterates between a master problem which selects vessel paths through the graph, and a sub problem that generates paths. Since column generation can only be used for solving linear programs (rather than mixed-integer programs), the path based model is only guaranteed, from a theoretical perspective, to find a lower bound to the overall LSFRP. However, our path based model finds solutions with integer values for 95 % of our dataset, meaning it finds the optimal solution.

Finally, we also introduce two heuristic approaches based off of simulated annealing (SA) [90] and late acceptance hill climbing (LAHC) [21], respectively. Although our heuristic approaches are not always able to find optimal solutions, they are effective at providing good solutions quickly. We presented our work on the arc flow model of the LSFRP in [152], the node flow model in [153], and on solving the LSFRP with heuristics in [148] and [147].

The contributions of this monograph are as follows:

- 1. A planning model of the NCLSFRP.
- 2. A formalism of temporal optimization planning and its linear instantiation, linear temporal optimization planning.
- 3. A linear temporal optimization planning model of the NCLSFRP.
- 4. A mixed-integer programming model of the NCLSFRP.
- 5. A constraint programming model of the NCLSFRP.
- 6. A graph construction and arc flow mixed-integer programming model of the LS-FRP with cargo flows.
- 7. A simulated annealing and late acceptance hill climbing approach to the LSFRP with cargo flows.
- 8. A node flow mixed-integer programming model of a specialized version of the LSFRP with cargo flows.

We therefore answer the core question of this monograph with "yes"; that is, an algorithm can be developed to create liner shipping fleet repositioning plans for a decision support system within a reasonable amount of CPU time.

#### 1.2 Outline

The outline of this monograph is as follows.

- Chapter 2 We provide background on containerized shipping both at sea and on land. We describe the components relevant to fleet repositioning, inter-terminal transportation and the capacitated *k*-shift problem, including liner shipping networks, container ships, and port operations.
- Chapter 3 In this chapter, we describe liner shipping fleet repositioning in detail. We introduce all of the cost saving (and revenue earning) activities present in fleet repositioning, as well as describe a case study performed with our industrial collaborator.
- Chapter 4 We provide background information for the several different solution approaches used in this monograph, including automated planning, mixedinteger programming and constraint programming.

- Chapter 5 We model and solve a simplification of fleet repositioning that ignores cargo flows, but focuses on other difficult aspects of the problem, such as time-dependent task costs. We use automated planning, an automated planning and linear programming hybrid called linear temporal optimization planning, mixed-integer programming and constraint programming to find optimal solutions on a dataset of instances modeled on a real repositioning scenario.
- Chapter 6 This chapter examines the LSFRP including cargo and equipment flows using a specialized graph. We solve the problem by formulating models based on an arc flow, path generation, and a node flow. In addition, we present two heuristic solution approaches, simulated annealing and late acceptance hill climbing, in order to solve problems that are too large for the optimal approaches.

### Chapter 2 Containerized Shipping

Before Malcolm McLean invented the shipping container in 1956, transporting cargo by sea was a long and arduous process. Longshoremen unloaded trucks and trains and packed ships full of boxes and bags of goods, and then unloaded them again at their destination, a process that could take several days for just one ship. Shipping goods was not only expensive, but cargo often had a tendency to go missing or be damaged along the way to its destination [37, 98]. Containers, which are large steel boxes in which goods are placed, revolutionized seaborne trade, allowing cargo to be securely, safely and quickly handled and transported to destinations around the world. With containers, cargo can be quickly transferred not only to and from ships, but also between various modes of transportation, such as between trains, trucks and ships. Containers have significantly lowered the costs of maritime trade, and costs continue to decline with the increasing size and efficiency of seagoing vessels [156].

*Liner* shipping encompasses most of the world's seaborne containerized shipping, in which specially built seagoing vessels carry thousands or even tens of thousands of containers on a regular schedule between ports. The "liner" in liner shipping refers to the way vessels follow one another in a line-like fashion along a route. The fixed schedule of liner shipping is a key selling point for the liner shipping industry, and is what differentiates it from *tramp* or *industrial* shipping. A common analogy to understand the difference between liner shipping and tramp shipping is that the regular schedule of liner shipping is similar to that of a bus or rail network, except the periodicity is generally weekly, rather than every few minutes or hours. In contrast, in tramp shipping vessels act more like taxis, carrying cargo without a fixed schedule or route [28, 124]. In both tramp and liner shipping, the goal of shipping firms is to maximize their profit, whereas in industrial shipping the goal is to minimize costs.

Liner shipping has been growing steadily since its inception, with a small pullback during the economic crisis of 2008–2009 (see Fig. 2.1). The liner shipping industry was responsible for carrying roughly 60 % of global seaborne trade in 2007, and is credited with providing 13.5 million direct and related jobs [73]. Numerous liner carriers compete for customers' cargo, with the top three carriers accounting for almost 40% of the total vessel (owned and chartered) capacity in 2014 according to Alphaliner [4]. There are over 100 liner carriers currently in operation, although many serve specialized markets, such as the Deutsche Afrika-Linien (DAL), which only operates several vessels along a route from Germany to Africa. Meanwhile other carriers serve particular regions, such as the Unifeeder company in the North, Baltic and Mediterranean seas. Still other carriers carry cargo globally, like Maersk Line, which calls hundreds of ports with its network.



Fig. 2.1: Growth of liner shipping in millions of tons of containers (Source: [156])

In the remainder of this chapter, we will discuss the innovation that makes containerized shipping possible, the container, followed by an overview of liner shipping networks, which connect the ports of the world. We next analyze liner carrier alliances, and finally, we look at the components of a container port, the interface between the land and the sea.

#### 2.1 Containers

The centerpiece of containerized shipping is the intermodal container, a rectangular steel box with castings on all of its corners to allow the safe stacking of multiple containers. Standard containers have a width of 8 ft, a height of 8 ft 6 in. and are either 20 or 40 ft long, as depicted in Fig. 2.2. A variety of variations on these standard containers exist, including containers with an extra foot of height (high cubes) and containers that are 45 ft long.

Quantities of containers are measured in either the *twenty-foot equivalent unit* (TEU) or the *forty-foot equivalent unit* (FEU or FFE). A TEU represents a single twenty foot container, and one FFE is equal to two TEU (i.e. two twenty-foot containers or one forty-foot container). Even though there are a number of different sizes of containers, not all of which have a length divisible by 20, their quantities are measured in TEU or FFE nonetheless.



Fig. 2.2: A 20 ft and a 40 ft container (Adapted from [114])

Numerous types of containers exist to fit the varied needs of shippers, such as tank containers for liquids, open top containers for tall cargo, and refrigerated containers for cargo that needs to be chilled, to name a few. Some non-standard containers, such as those carrying dangerous/explosive cargo, require special handling and storage procedures to ensure that their contents are not damaged and to ensure the safety of the vessel and its crew.

This monograph focuses primarily on *dry* and *reefer* (i.e. refrigerated) containers. Dry containers are standard 20 or 40 ft containers requiring no special handling and can be stored anywhere on a vessel. Reefer containers, which can also be either 20 or 40 ft in length, have an integrated refrigeration unit and therefore require access to an electrical outlet on board a vessel. Most vessels only have a limited number of locations where an outlet is available, meaning the number of reefer containers that can be transported by a vessel tends to be less than the number of dry containers. Reefer containers can be easily spotted in terminals and on ships as they are colored bright white in order to help keep their contents cool.

#### 2.2 Liner Shipping Networks

The goal of a *liner shipping network* is to facilitate the transport of containers between ports. A liner shipping network is defined by a published, periodic schedule, that determines when vessels visit ports. The periodicity of the schedule is central to liner shipping, in that seagoing vessels visit ports on a weekly or bi-weekly basis, at the same time each week. Liner shipping networks can span the entire globe or be constrained to a specific geographic region, depending on the shipping line and its customers. The overall structure of a liner shipping network takes the form of either a hub-and-spoke, a direct routing approach or a mix of hub-and-spoke and direct routing [74, 111].

#### 2.2.1 Services

In order to achieve a weekly or bi-weekly regularity, vessels are sent on cyclical routes called *services*.<sup>2</sup> Services are cyclical routes that visit ports on a regular, usually weekly, schedule. A weekly service has as many vessels as the number of weeks a vessel requires to complete a single rotation of the service. That is, a service with a rotation duration of 5 weeks requires 5 vessels in order to maintain a weekly frequency. Each vessel is assigned a *slot* on the service, thus, there are as many slots on the service as there are vessels. A slot can be viewed as containing the specific schedule for each vessel. In other words, while the schedule for a service specifies that a particular port is visited, for example, on Mondays at 17:00, the schedule for a slot provides the exact day that the vessel in that slot should visit the port (i.e., April 22, 2013 at 17:00). For a service with a weekly regularity, the vessel in the subsequent slot would visit the port on April 29, and the vessel in the previous on April 15. Each port visited on a service slot is referred to as a *call*. Shipping lines assign each call an arrival and departure time. Between the arrival and departure time, the vessel sailing on the service loads and unloads containers. Each slot contains multiple *legs*, and each leg begins at a port call and ends at the next scheduled call.



Fig. 2.3: An example liner shipping service connecting several nations of Northern Europe with the duration of a single rotation lasting 3 weeks. The position of each of the three vessels required for the service in (**b**), represented by *red*, *blue*, *green arrows*, corresponds to week 3 in (**a**)

 $<sup>^2</sup>$  In shipping parlance, services are sometimes called "strings". We avoid this term to prevent confusion with programming terminology.