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# Heavy-Tailed Distributions and Robustness in Economics and Finance



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*(To Saniya)*

Саниягэ

*M. I.*

*(To my parents)*

Эти-Эниемэ

*R. I.*

*To Nella, Tintin, Felix, and Theo*

*J. W.*



# Foreword

For a long time, the presence of (heavy-tailed) power laws, also referred to as Pareto distributions, has been observed in data covering all fields of science and applications. What has been much less studied is the important question: “What are the economic consequences of this observation?” Based on several of the authors’ publications, it is precisely this question which is addressed in this interesting book. Heavy-tailed models typically induce a kind of regime switching (non-robust) behavior as a function of the tail-decay parameter; this leads to a fundamental rethinking of important questions like portfolio diversification or the (re)insurance of catastrophic risks. Extreme heavy-tailed behavior (possibly infinite-mean models) should imply risk management caution on behalf of the end-user, decision maker. The authors carefully explain where the pitfalls are in this economically counterintuitive landscape, how best they can be avoided, but also how to optimally structure products and markets in such an environment. Potential applications go well beyond finance, economics, and insurance. Not only financial crises and crashes have catapulted “thinking about extremes, the worst that can happen” to the forefront of the political and regulatory agendas, but also discussions about global warming and the occurrence of natural disasters call for similar methodology.

This book adds economic thinking to statistical modeling, and as such is most highly welcome!

RiskLab, ETH Zurich  
Zurich, Switzerland

Paul Embrechts



# Preface

The purpose of this book is to provide a fairly detailed introduction to the analysis and applications of heavy-tailed distributions in a number of important problems in economics, finance, risk management, and insurance. The target audience for the book is graduate students in economics, finance, risk management, probability, statistics, and insurance, although the book may also be of value for advanced undergraduate students who have completed a graduate course in probability. It should also be useful for professionals in the financial and insurance industries, risk managers, and for regulators and policy makers concerned with modeling the effects of crises, large fluctuations and extreme values of financial returns, foreign exchange rates, and other key economic and financial indicators and variables. More broadly, we hope that anyone who is interested in a self-contained treatment of the topic at a post graduate level will find this book useful.

The book is based on several published articles, but much effort has been invested into making it self-contained. Specifically, notation has been made consistent across chapters, and we have also adjusted the language, making it less technical than in its original form to make the results easier to digest. The cost of these modifications is a slight loss in rigor. Especially, most proofs have been excluded. The interested reader is referred to the original publications.

The book consists of three chapters. The first chapter provides a brief introduction to heavy-tailed distributions, and their presence and applications within finance, insurance, and economics. The chapter begins with a literature review, followed by a definition of what it means for a risk to have a heavy-tailed distribution. It then discusses the empirical evidence for the presence of heavy-tailed risk distributions in practice in the aforementioned fields. Finally, the chapter discusses the main point of this book, namely that there are limitations to diversification with such risk distributions. Specifically, whereas diversification is preferred by risk-averse agents when risks are thin-tailed (the traditional case that has been extensively studied), it may actually be hurtful for agents to diversify when risks are heavy-tailed (the nontraditional case that this book focuses on). Two examples of heavy-tailed distributions are discussed, namely Cauchy and Lévy distributed risks, to show the limits of diversification in the heavy-tailed case.

Chapter 2 focuses on the theory of diversification with heavy-tailed risks, and the implications for economics, finance, and insurance. The chapter first introduces the important concept of majorization, which allows for a general analysis of portfolio diversification. Specifically, several classes of risk distributions are introduced and analyzed with respect to whether diversification decreases risk (as in the traditional setting) or increases it (as in the nontraditional setting). The chapter then studies the implications for econometric and statistical inference. Finally, it introduces several models to analyze the implications of the results.

First, a model of a reinsurance market for catastrophe insurance is analyzed. The distributions of aggregate losses due to catastrophic events are known to be very heavy-tailed. It is shown in the model that this may explain why it has been challenging to develop well-functioning markets for risk-sharing of catastrophic risks. Specifically, in such markets, a coordination problem may exist where many entities need to agree to sell insurance policies against catastrophic events for a market to start functioning. In the outcome where no insurance is sold, there is a so-called nondiversification trap. We show that nondiversification traps may arise when risk distributions have heavy left tails and insurance providers have limited liability. When they are present, there may be a coordination role for a centralized agency, e.g., government or a regulatory authority, to ensure that risk sharing takes place.

We next introduce a model of financial intermediaries, in which so-called diversification disasters can occur. Specifically, if there are negative externalities to society if multiple financial intermediaries default on their obligations at the same time, then risk-sharing, i.e., diversification, among these intermediaries may be suboptimal. We suggest that historical legislation, e.g., in form of the Glass–Steagall act, may have had a role in avoiding such outcomes.

As a third example, we study the problem of optimal bundling for a multiproduct monopolist providing goods in auctions or for profit-maximizing prices to consumers with heavy-tailed private valuations for these goods. We show that several results in the literature that hold in the traditional setting under thin-tailed valuations are reversed under heavy-tailed valuations. Finally, we considered growth models for firms investing into information about their markets, again showing that several standard results in the literature are reversed when distributions of variables entering their assumptions are heavy-tailed.

The main conclusion of the results in Chap. 2 is that the presence of heavy tails can either re-enforce or reserve the properties of many important models in economics, finance, risk management, insurance, econometrics, and statistics, depending on the degree of heavy-tailedness. This further emphasizes the importance of having robust econometrically and statistically justified inference methods under heavy-tailedness.

Chapter 3 deals with robust inference methods under heavy-tailedness. The chapter discusses widely used approaches to inference on the degree of heavy-tailedness and their main asymptotic properties. The asymptotic analysis provides the key to developing econometrically and statistically justified correct standard

errors (evaluation of the degree of uncertainty, so to speak) and correct confidence intervals for the degree of heavy-tailedness.

The chapter further provides applications of the inference approaches for the analysis of whether and how heavy-tailedness properties of emerging and developing markets such as markets for foreign exchange differ from those in developed economies. Finally, the chapter discusses recently developed general approaches to inference in economic and financial models under heterogeneity, dependence, and heavy-tailedness of largely unknown form.

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