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Matt Allen · Randall L. Mayes · Daniel J. Rixen *Editors*

Dynamics of Coupled Structures, Volume 4

Proceedings of the 33rd IMAC, A Conference and Exposition on Structural Dynamics, 2015

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Preface

Dynamics of Coupled Structures represents one of ten volumes of technical papers presented at the 33rd IMAC, A Conference and Exposition on Balancing Simulation and Testing, 2015, organized by the Society for Experimental Mechanics, and held in Orlando, Florida, February 2–5, 2015. The full proceedings also include volumes on Nonlinear Dynamics; Dynamics of Civil Structures; Model Validation and Uncertainty Quantification; Sensors and Instrumentation; Special Topics in Structural Dynamics; Structural Health Monitoring & Damage Detection; Experimental Techniques, Rotating Machinery & Acoustics; Shock & Vibration Aircraft/Aerospace, Energy Harvesting; and Topics in Modal Analysis.

Each collection presents early findings from experimental and computational investigations on an important area within Structural Dynamics. Coupled structures or, substructuring, is one of these areas.

Substructuring is a general paradigm in engineering dynamics where a complicated system is analyzed by considering the dynamic interactions between subcomponents. In numerical simulations, substructuring allows one to reduce the complexity of parts of the system in order to construct a computationally efficient model of the assembled system. A subcomponent model can also be derived experimentally, allowing one to predict the dynamic behavior of an assembly by combining experimentally and/or analytically derived models. This can be advantageous for subcomponents that are expensive or difficult to model analytically. Substructuring can also be used to couple numerical simulation with real-time testing of components. Such approaches are known as hardware-in-the-loop or hybrid testing.

Whether experimental or numerical, all substructuring approaches have a common basis, namely the equilibrium of the substructures under the action of the applied and interface forces and the compatibility of displacements at the interfaces of the subcomponents. Experimental substructuring requires special care in the way the measurements are obtained and processed in order to assure that measurement inaccuracies and noise do not invalidate the results. In numerical approaches, the fundamental quest is the efficient computation of reduced order models describing the substructure's dynamic motion. For hardware-in-the-loop applications, difficulties include the fast computation of the numerical components and the proper sensing and actuation of the hardware component. Recent advances in experimental techniques, sensor/actuator technologies, novel numerical methods, and parallel computing have rekindled interest in substructuring in recent years leading to new insights and improved experimental and analytical techniques.

The organizers would like to thank the authors, presenters, session organizers, and session chairs for their participation in this track.

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Contents

Chapter 1
Robust Stability and Performance Analysis for Multi-actuator Real-Time Hybrid Substructuring

Rui M. Botelho and Richard E. Christenson

Abstract Real-time hybrid substructuring (RTHS) is a relatively new method of vibration testing for characterizing the system-level performance of physical components or substructures. With RTHS, the coupled system is partitioned into physical and numerical substructures and interfaced together in real-time as cyber-physical system similar to hardware-in-the-loop testing. Control actuation and sensing is used to enforce the compatibility and equilibrium conditions between the physical and numerical substructures. Since RTHS involves a feedback loop, the frequency-dependent magnitude and inherent time delay of the actuator dynamics can introduce inaccuracy and instability. This paper presents a robust stability and performance analysis method for multi-actuator RTHS based on robust stability theory for multiple-input-multiple-output (MIMO) feedback control. This analysis method involves casting the actuator dynamics as a multiplicative uncertainty and applying the small gain theorem to derive the sufficient conditions for robust stability and performance. The attractive feature of this robust stability and performance analysis method is that it accommodates linearized modeled or measured frequency response functions for both the physical substructure and actuator dynamics.

Keywords Experimental structural dynamics • Real-time hybrid testing • Dynamic substructuring • Hardware-in-the-loop testing • Robust stability • Feedback control systems

1.1 Introduction

Real-time hybrid substructuring (RTHS), also called real-time hybrid simulation or real-time dynamic substructuring, is a relatively new method of vibration testing in experimental structural dynamics. RTHS allows a coupled system to be partitioned into separate physical and numerical components or substructures. The substructures that are well understood are simulated in real-time using analytical or numerical models, while those that are highly complex or of particular interest are physically tested using physical specimens. The physical and numerical substructures are interfaced together as cyber-physical system similar to hardware-in-the-loop (HWIL) testing. In a RTHS test, the numerical and experimental substructures communicate together in real-time by transferring displacement and force signals through a feedback loop using controlled actuation and sensing. The physical substructure is usually the experimental component of interest, while the numerical substructure is typically an analytical or numerical model of the remaining system incorporating various complexities that may be difficult to represent physically. RTHS has recently become more feasible due to advances in numerical computing power, digital signal processing, and high-speed servo-hydraulic actuation.

Early developments of RTHS include Horiuchi et al. [1, 2], Nakashima and Masaoka [3], and Darby et al. [4]. RTHS is also a major element of the Network for Earthquake Engineering Simulation (NEES). For example, Christenson and Lin [5] describe a large-scale RTHS test setup at the University of Colorado Boulder NEES facility to examine the system-level performance of multiple 200 kN MR fluid dampers in a three-story building structure. Jiang et al. [6] describe recent RTHS testing at the Lehigh University NEES facility of full-scale MR dampers attached to a bridge structure, and Friedman et al. [7] describe RTHS tests at the Lehigh University NEES facility using the same MR dampers installed in a seismically excited steel moment resisting frame.

Figure 1.1 illustrates the block diagram of a typical RTHS test. At a given time-step, numerical loading is applied to the numerical substructure to simulate the numerical displacements, x_n, to be imposed on the physical substructure in real-time. The numerical displacements are then fed into a controller to apply the command displacement, x_c, to the actuator transfer

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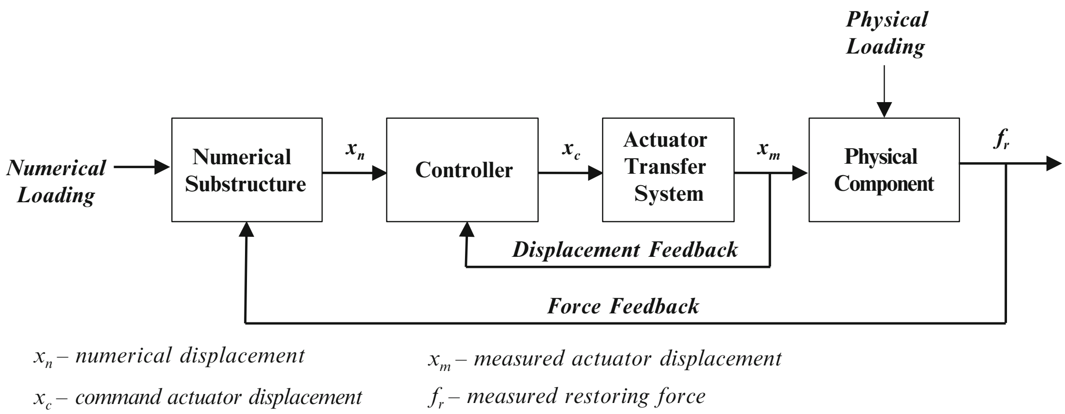

Fig. 1.1 Block diagram of a typical RTHS test

system, which is usually a set of hydraulic actuators controlled by a proportional-integral-derivative (PID) servo-controller using the measured displacement, x_m, as the feedback signal. The measured restoring force, f_r, of the physical substructure is then fed back into the numerical substructure to compute the numerical displacements for the next time step. RTHS also accommodates physical loading of the physical substructure from external shaker forcing of the physical specimen. This closed-loop testing method makes RTHS very similar to HWIL testing, whereby a real-time virtual model is directly interfaced with actual physical hardware forming a cyber-physical system.

Since RTHS involves a feedback loop, the inherent time delay of the actuator transfer system can lead to inaccuracy in the actuator tracking and potential instability during closed-loop testing. The effect of time delay on RTHS testing was initially considered by Horiuchi et al. [1], who showed that actuator delay can cause an increase in the total energy, which is equivalent to introducing negative damping into the system. When the negative damping is larger than the total system damping, the RTHS test will become unstable. Previous RTHS tests without actuator delay compensation had been performed for systems with very low natural frequencies and high damping to ensure stability. There are also other sources of time delay in a RTHS test, including communication delays of the various electrical signals and computational delays for solving the numerical substructure. These time delays are generally much smaller than the inherent time delay of the actuator transfer system.

To improve the closed-loop stability and performance of RTHS, researchers have developed a variety of techniques for compensating the time delay or more generally the frequency dependent dynamics of the actuator transfer system. The techniques range from polynomial extrapolation in Horiuchi et al. [1] and inverse compensation in Chen and Ricles [8] to reduce the actuator delay as well as adaptive techniques in Chen and Ricles [9] and Chae et al. [10]. Carrion and Spencer [11] used a controls approach to develop model-based feedforward-feedback control to compensate the frequency-dependent magnitude and phase of the actuator dynamics. Phillips and Spencer [12] extended this approach with a more accurate feedforward inverse of the actuator dynamics and adding linear-quadratic Gaussian (LQG) feedback control. Christenson and Lin [5] employed virtual coupling to balance closed-loop stability and performance in RTHS testing of large-scale MR dampers. Gao et al. [13] recently developed an H-infinity robust loop-shaping controller to compensate the actuator dynamics for RTHS testing of lightly damped steel frame structures. The ultimate goal of these compensation techniques is to provide effective displacement tracking of the actuator transfer system over the desired frequency range of the RTHS test, called the control band.

Stability and performance analysis is an important tool for understanding the effect of actuator time delay on the stability behavior and accuracy of RTHS. This information is especially useful in guiding the compensation design of the actuator transfer system to reduce its inherent time delay and provide closed-loop stability and performance. Wallace et al. [14] studied the effect of actuator time delay using delay differential equation (DDE) modeling of a single degree of freedom (SDOF) RTHS system comprised of a physical stiffness coupled to an analytically modeled mass-spring oscillator. This approach yields a governing characteristic equation with an exponential delay term whose solution has an infinite number of roots. The purely imaginary roots define the critical frequencies at which switches in the stability behavior of the system occur and can be used to identify the critical time delays of the RTHS system. Kyrychko et al. [15] applied a similar approach to identify the critical time delays for RTHS of a physical pendulum coupled to an analytical mass-spring oscillator, but using neutral DDE's.

Mercan and Ricles [16] applied a pseudodelay technique to solve the DDE for a SDOF RTHS system and identified the critical time delays in terms of the mass, damping, stiffness parameters of the test structure. Mercan and Ricles [17] extended the pseudodelay technique to evaluate multiple sources of time delay including those for multi-actuator RTHS. Fudong et al. [18] employed a Padé rational fraction approach to approximate the exponential delay term and used a root locus technique to investigate the relationship between the closed-loop poles and substructure partitioning parameters on the stability conditions of a SDOF RTHS system. Botelho et al. [19] presented an exact stability analysis technique for single-actuator RTHS of 1-DOF and 2-DOF mass-spring systems to quantify their critical time delays. Although these stability analysis techniques provide insight into the stability behavior of RTHS, they assume pure time delay for the actuator dynamics and are limited to lumped parameter descriptions of the numerical and physical substructures.

While progress has been made in the development of stability analysis techniques for RTHS, performance analysis has received less attention. Maghareh et al. [20] recently proposed a predictive performance indicator for SDOF RTHS to assess the effect of actuator time delay as well as computational and communication delays on the accuracy of RTHS. The predictive performance indicator also provides insight on the effect of mass, stiffness, and damping partitioning of the physical and numerical substructures. This performance indicator can be used in conjunction with a stability switch criterion based on the exact solution for the critical time delays of the RTHS system to derive acceptance criteria for conducting successful RTHS tests.

This paper presents a new robust stability and performance analysis method for multi-actuator RTHS based on the concepts from robust stability theory for multiple-input-multiple-output (MIMO) feedback control systems. This method involves casting the actuator dynamics of the RTHS feedback loop as a multiplicative uncertainty and then applying the small gain theorem to derive sufficient conditions for robust stability and performance for RTHS. Gawthrop et al. [21] was first to consider robust stability in RTHS, in which they used the robust stability criterion in Goodwin et al. [22] to study the stability of single-actuator RTHS, but assumed a pure time delay for the actuator transfer system. Recognizing the versatility of this approach, this paper presents an extension of robust stability analysis to consider robust performance and multi-actuator RTHS. Unlike previous stability analysis techniques which assume pure time delay, this method accommodates the linearized modeled or measured frequency-dependent magnitude and phase of the actuator dynamics as well as linearized modeled or measured frequency response functions of the physical substructure.

1.2 Theory

Multi-actuator RTHS applied to a notional structural dynamic system partitioned into separate numerical and physical substructures is illustrated in Fig. 1.2. The corresponding RTHS feedback loop, which is essentially a multi-input multi-output (MIMO) feedback control system, is shown in Fig. 1.3. For the numerical substructure, the transfer function $N_{XnFn}(s)$ relates input numerical forces, f_n, to output displacement responses, x_n, and the transfer function $N_{XnFr}(s)$ relates input restoring forces, f_r, to output displacement responses, x_n. For the physical substructure, the transfer function $P_{FrFe}(s)$ relates external physical forces, f_e, to measured restoring forces, f_r, and the transfer function $P_{FrXa}(s)$ relates input actuator displacements, x_a, to measured restoring forces, f_r. Although this system illustrates a two interface degrees-of-freedom (DOFs), the subsequent reformulation of dynamic substructuring as a feedback control problem is general and can be applied to cases with many interface DOFs.

To enforce force equilibrium, the measured restoring forces from the physical substructure are fed back to the numerical substructure and applied as equal and opposite input forces. To enforce displacement compatibility, the actuator transfer system with compensation is used to impose the displacement response of the numerical substructure onto the physical substructure. The dynamics of the actuator transfer system with compensation is represented by the actuator transfer function $\hat{A}(s)$. It should be noted that the above block diagram assumes that the sensor dynamics for the measured restoring force of the physical substructure are negligible. With appropriate selection of force sensors with constant magnitude and little phase distortion at low frequencies, this assumption is reasonable for the control band below 100 Hz of a typical RTHS test.

From the above block diagram, the closed-loop response for the numerical displacement is

$$\begin{aligned}\{x_n(s)\} = &\left[I + N_{XnFr}(s)P_{FrXa}(s)\widehat{A}(s)\right]^{-1}[N_{XnFn}(s)]\{f_n(s)\}\\ &-\left[I + N_{XnFr}(s)P_{FrXa}(s)\widehat{A}(s)\right]^{-1}[N_{XnFr}(s)]\ [P_{FrFe}(s)]\{f_e(s)\}\end{aligned} \tag{1.1}$$

The closed-loop response for the measured restoring force is

$$\begin{aligned}\{f_r(s)\} = &\left[I + P_{FrXa}(s)\widehat{A}(s)N_{XnFr}(s)\right]^{-1}[P_{FrXa}(s)]\ \left[\widehat{A}(s)\right][N_{XnFn}(s)]\{f_n(s)\}\\ &+\left[I + P_{FrXa}(s)\widehat{A}(s)N_{XnFr}(s)\right]^{-1}[P_{FrFe}(s)]\{f_e(s)\}\end{aligned} \tag{1.2}$$

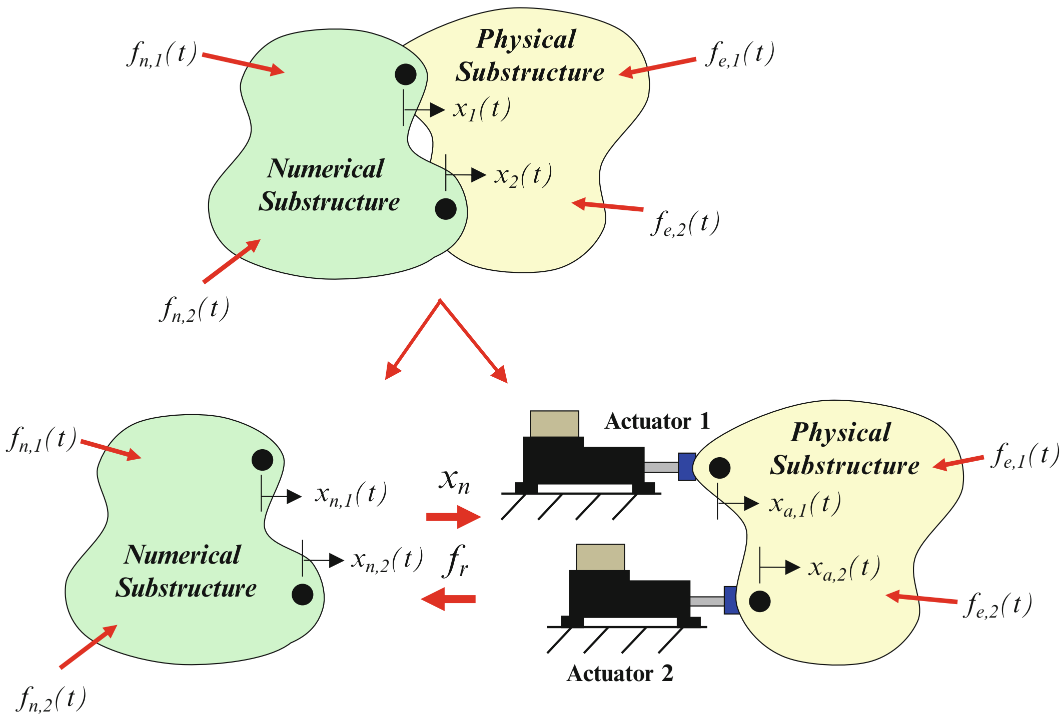

Fig. 1.2 Multi-actuator RTHS applied to a notional structural dynamic system

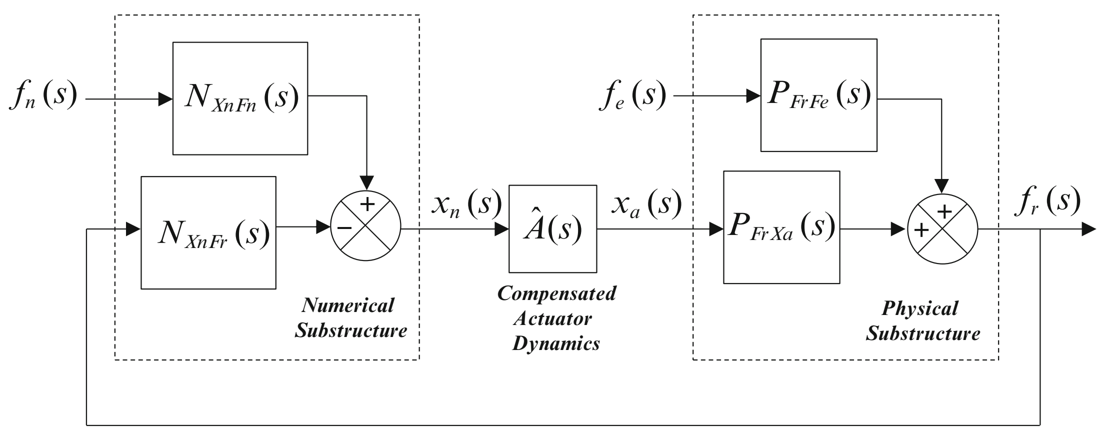

Fig. 1.3 Feedback loop for multi-actuator RTHS

Note that the above expression has a common factor, which is the sensitivity matrix

$$[S(s)] = [I + L(s)]^{-1} \tag{1.3}$$

where, $L(s) = P_{FrXa}(s)\widehat{A}(s)N_{XnFr}(s)$ is the loop gain for RTHS.

The complimentary sensitivity matrix is then defined as

$$[T(s)] = [I + L(s)]^{-1}[L(s)] \tag{1.4}$$

Since the actuator dynamics is in the feedback path, the presence of time delay of the actuator transfer system will have a destabilizing effect on the RTHS closed-loop response. Using concepts from robust stability theory for MIMO feedback control, as in Goodwin et al. [22] and Skogestad and Postlethwaite [23], we cast the compensated actuator dynamics as a multiplicative uncertainty as illustrated in Fig. 1.4.

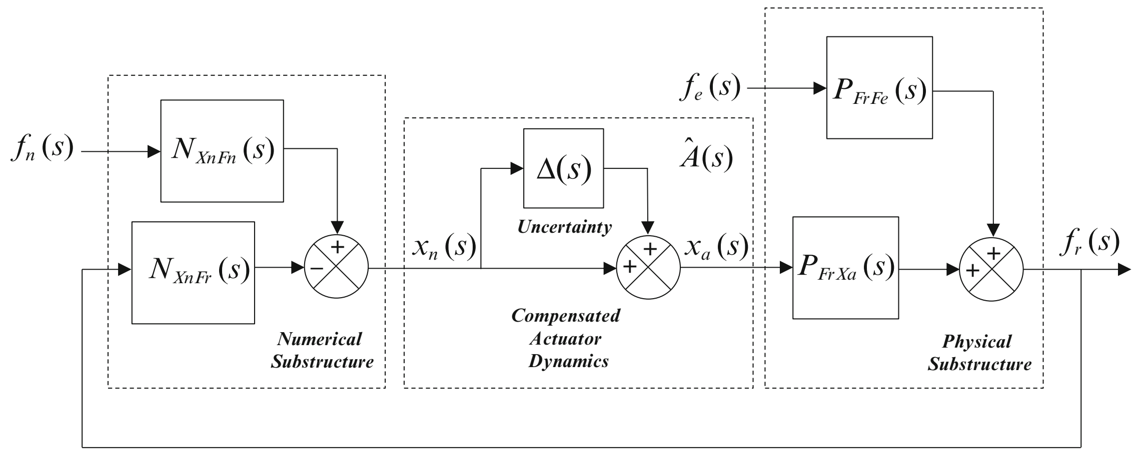

Fig. 1.4 RTHS feedback loop with compensated actuator dynamics cast as multiplicative uncertainty

The compensated actuator dynamics matrix is related to the uncertainty matrix by

$$\left[\widehat{A}(s)\right] = [\Delta(s)] + [I] \tag{1.5}$$

Substituting (1.5) into (1.4), the complimentary sensitivity matrix becomes

$$[T(s)] = [I + P_{FrXa}(s)\,(\Delta(s) + I)\,N_{XnFr}(s)]^{-1}\,[P_{FrXa}(s)]\ [\Delta(s) + I]\ [N_{XnFr}(s)] \tag{1.6}$$

Expanding as

$$[T(s)] = [I + P_{FrXa}(s)N_{XnFr}(s) + P_{FrXa}(s)\Delta(s)N_{XnFr}(s)]^{-1}\,[P_{FrXa}(s)]\ [\Delta(s) + I]\ [N_{XnFr}(s)] \tag{1.7}$$

Factoring $[I + P_{FrXa}(s)N_{XnFr}(s)]^{-1}$ leads to

$$[T(s)] = [I + T_o(s)\Delta(s)]^{-1}\,[T_o(s)]\ [\Delta(s) + I] \tag{1.8}$$

where the nominal complimentary sensitivity matrix is defined as

$$[T_o(s)] = [I + P_{FrXa}(s)N_{XnFr}(s)]^{-1}\,[P_{FrXa}(s)]\ [N_{XnFr}(s)] \tag{1.9}$$

Note that the presence of actuator dynamics introduces the term $T_o(s)\Delta(s)$ in the denominator of (1.8). As this term approaches -1, the RTHS system will go unstable. By employing the small gain theorem, the sufficient condition for robust stability is

$$\|[T_o(s)]\ [\Delta(s)]\|_\infty < 1 \tag{1.10}$$

Where, $\|\,\|_\infty$ denotes the maximum singular value over the RTHS control band.

The above robust stability criterion provides a conservative measure for ensuring that the RTHS system has robust stability in the presence of actuator dynamics. Since multi-actuator RTHS is a MIMO feedback control system, the robust stability criterion involves a singular value decomposition of the nominal complimentary sensitivity matrix multiplied by the uncertainty matrix. The nominal complimentary sensitivity matrix for RTHS is described by (1.9), while the uncertainty matrix is obtained by rearranging (1.5) as

$$[\Delta(s)] = \left[\widehat{A}(s)\right] - [I] \tag{1.11}$$

For single-actuator RTHS, the robust stability criterion simplifies to

$$|T_o(s)\Delta(s)| < 1 \tag{1.12}$$

Where, $\left|\ \ \right|$ denotes the maximum magnitude over the RTHS control band.

The robust stability criterion alone does not ensure robust performance of the RTHS system in the presence of actuator dynamics. Goodwin et al. [22] suggests one approach for robust performance is to ensure that the actual sensitivities are close to the nominal sensitivities by

$$\|[T_o(s)]\,[\Delta(s)]\|_\infty << 1 \tag{1.13}$$

The above robust performance criterion is similar but more restrictive than the robust stability criterion.

The above sufficient conditions for robust stability and performance analysis assume that the dynamic system without actuator dynamics has nominal stability. A dynamic system is said to have nominal stability if its closed-loop poles are on the right-side of the complex plane. The closed loop response of the dynamic system without actuator dynamics can be found by setting the uncertainty matrix to zero, which yields

$$\begin{aligned}\{x_o(s)\} = {} & [I + N_{XnFr}(s)P_{FrXa}(s)]^{-1}\,[N_{XnFn}(s)]\,\{f_n(s)\} \\ & - [I + N_{XnFr}(s)P_{FrXa}(s)]^{-1}\,[N_{XnFr}(s)]\ \ [P_{FrFe}(s)]\,\{f_e(s)\}\end{aligned} \tag{1.14}$$

The above response represents the case of perfect actuation with no time delay of the actuator transfer system. The nominal stability of this system can be characterized by determining its closed-loop poles.

The sufficient conditions in (1.10) and (1.13) provide the basis for a new robust stability and performance analysis method for both single and multi-actuator RTHS. The robust stability and performance criterion for RTHS are similar to those for MIMO feedback control with plant uncertainty in Goodwin et al. [22] and Skogestad and Postlethwaite [23]. The differences are in the expressions for the nominal complimentary sensitivity and uncertainty. In most cases, it is more practical to perform the robust stability and performance analysis in the frequency domain instead of the Laplace domain in order to directly utilize measured frequency response functions for the actuator dynamics and physical substructure. The proposed robust stability and performance analysis involves a singular value decomposition of the nominal complimentary sensitivity matrix multiplied by the uncertainty matrix at each frequency. Unlike previous stability analysis techniques which assume pure time delay and lumped parameter descriptions, this method better captures the actual frequency dependence of the magnitude and phase lag of the actuator dynamics as well as accommodates more complex representations of the numerical and physical substructures.

1.3 Conclusions

This paper presents a new analysis method for evaluating the robust stability and performance of multi-actuator RTHS. This MIMO analysis method requires a singular value decomposition of the nominal complimentary sensitivity matrix multiplied by the uncertainty matrix at each frequency. For robust stability, the maximum singular values across frequency should be less than 1 over the RTHS control band. For robust performance, the maximum singular values should be much less than 1 over the control band. These sufficient conditions for robust stability and performance were derived by casting the actuator dynamics in the RTHS feedback loop as a multiplicative uncertainty and applying the small gain theorem. The proposed robust stability and performance analysis method provides a useful tool for pre-test planning and post-test diagnostics of RTHS tests involving multiple actuators. This analysis method can be used to evaluate the robust stability and performance of the actuator transfer system and the compensation approach for RTHS. The attractive feature of this method is that it accommodates linearized modeled or measured frequency response functions for both the physical substructure and actuator dynamics. This presents a major advancement over previous RTHS stability analysis techniques which assume pure time delay for the actuator dynamics and lumped parameter descriptions or the numerical and physical substructures.

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Chapter 2
Effect of Actuator Delay on Real-Time Hybrid Simulation Involving Multiple Experimental Substructures

Cheng Chen, Frank Sanchez, and Maryam Khan

Abstract Real-time hybrid simulation provides an economical and effective experimental technique for seismic performance evaluation of civil engineering structures in size limited laboratories. Servo-hydraulic dynamics pose challenges for synchronizing restoring forces between substructures. Actuator tracking therefore contributes most detrimental error to real-time hybrid simulation. Post-experiment reliability assessment through actuator tracking assessment is critical to appropriately interpret real-time hybrid simulation results for the performance of structures under investigation. This study analyzes the effect of actuator delays on the accuracy of real-time hybrid simulations involving multiple experimental substructures. Unlike previous studies focusing on the stability of real-time hybrid simulation, this study acknowledges the fact that delay compensation methods are capable of reducing the tracking errors. The modal analysis technique is evaluated for real-time hybrid simulation involving both time delay and nonlinear structural behavior.

Keywords Real-time hybrid simulation • Actuator delay • Modal analysis • Frequency-domain analysis

2.1 Introduction

Experiments are important for earthquake engineering research. Real-time hybrid simulation (RTHS) splits the structure into experimental substructure(s) and analytical substructure(s), which allows researchers to observe the behavior of critical elements at large or full scale when subjected to dynamic loading [1–3]. The structural response under external excitation is calculated by solving the dynamic equations of motion using an integration algorithm. The desired responses are then imposed onto the experimental substructure(s) using servo-hydraulic actuators and their measured restoring forces are fed back for next step displacement calculation. Real-time hybrid simulation provides an efficient and effective way to evaluate seismic performance of large- or full-scale civil engineering structures in size limited laboratories. Since this experiment is conducted in real-time manner, the RTHS is more approving for rate-dependent seismic devices [4, 5]. After years of development, the RTHS has become a viable alternative to the more well-established shaking table testing method and the pseudo-dynamic testing method [6–9].

However, due to inherent servo-hydraulic dynamics, the actuator has an inevitable time delay in response to the displacement command. Previous researches showed that the time delay would lead to inaccurate test results and even destabilize the entire simulation if not compensated properly [10–12]. Various compensation methods have been proposed to minimize the effect of actuator delay. These compensation methods are either based on constant time delay assumption [13–15] or formulated using adaptive control theory [16, 17]. The advance in delay compensation method has helped reduce the actuator tracking error. Experimental studies however indicated that actuator tracking errors cannot be completely eliminated even when a sophisticated compensation technique is used [18]. This brings concern on how reliably the real-time hybrid simulation results have replicated the actual structural responses under earthquakes. In other words, it poses a great challenge for reliability assessment of real-time hybrid simulation results with the presence of tracking errors since true structural responses are often not available for an immediate comparison.

Various techniques have been used to evaluate the actuator delay induced tracking error, including the maximum tracking error (MTE), root-mean-square (RMS) of the tracking error, tracking indicator (TI) [19] and energy error (EE) [20, 21]. These variables provide qualitative but not quantitative assessment on the effect of actuator tracking. Guo et al. [22] proposed a frequency evaluation index (FEI) method to interpret amplitude error and phrase error existing in actuator tracking, which divides test error into. Chen et al. [23] further verified this method for RTHS tests of a single-story steel moment

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resisting frame with elastomeric damper. The FEI method utilizes the command displacements sent to the servo-hydraulics and the measured displacements measured from experimental substructures. When a real-time hybrid simulation involves multiple experimental substructures associated with multi-degree-of-freedom (MDOF), the FEI method application cannot differentiate between different structural modes, therefore cannot provide quantitative information on the amplitude and phase error for each structural mode. This study evaluates the effect of actuator delay on real-time hybrid simulation involving multiple experimental substructures and explores the application of FEI method on MDOF structure to assess the effect of actuator delay on multiple structural modes.

2.2 Real-Time Hybrid Simulation of Linear Elastic MDOF Structure

For a linear elastic MDOF structure, the equation of motion can be represented as:

$$\underline{\mathrm{M}} \cdot \underline{\ddot{\mathrm{x}}} + \underline{\mathrm{C}} \cdot \underline{\dot{\mathrm{x}}} + \underline{\mathrm{K}} \cdot \underline{\mathrm{x}} = \underline{\mathrm{F}} \tag{2.1}$$

where $\underline{\mathrm{M}}$, $\underline{\mathrm{C}}$ and $\underline{\mathrm{K}}$ are the mass, viscous damping and stiffness matrices, respectively; $\underline{\ddot{\mathrm{x}}}$, $\underline{\dot{\mathrm{x}}}$ and $\underline{\mathrm{x}}$ are the acceleration, velocity and displacement response, respectively; and $\underline{\mathrm{F}}$ is the external excitation force vector. For the purpose of analysis, a two degree-of-freedom (DOF) structure is analyzed in this paper, where the mass matrix, stiffness matrix, damping matrix and equations of motion can be expressed as $\underline{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $\underline{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$, $\underline{K} = \begin{bmatrix} \mathrm{k}_1 + \mathrm{k}_2 & -\mathrm{k}_2 \\ -\mathrm{k}_2 & \mathrm{k}_2 \end{bmatrix}$, $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\underline{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$, where m_1 and m_2 are the mass of the first and second story of the 2DOF structure, respectively; k_1 and k_2 represent the story stiffness; c_{11}, c_{12}, c_{21} and c_{22} represent the viscous damping inherent to the structure; F_1 and F_2 represent the external excitation force on the structure; and x_1 and x_2 represent the displacement response of the first and second story, respectively. The equation of motion for each story can also be written as

$$\mathrm{m}_1 \cdot \ddot{\mathrm{x}}_1 + \mathrm{c}_{11} \cdot \dot{\mathrm{x}}_1 + \mathrm{c}_{12} \cdot \dot{\mathrm{x}}_2 + \mathrm{k}_1 \cdot \mathrm{x}_1 + \mathrm{k}_2 \cdot (\mathrm{x}_1 - \mathrm{x}_2) = \mathrm{F}_1 \tag{2.2a}$$

$$\mathrm{m}_2 \cdot \ddot{\mathrm{x}}_2 + \mathrm{c}_{21} \cdot \dot{\mathrm{x}}_1 + \mathrm{c}_{22} \cdot \dot{\mathrm{x}}_2 + \mathrm{k}_2 \cdot (\mathrm{x}_2 - \mathrm{x}_1) = \mathrm{F}_2 \tag{2.2b}$$

Assume that structural components associated with k_1 and k_2 are considered as experimental substructures and actuator delays exist in a real-time hybrid simulation, Eq. (2.1) can be modified as

$$\begin{bmatrix} \mathrm{m}_1 & 0 \\ 0 & \mathrm{m}_2 \end{bmatrix} \begin{Bmatrix} \ddot{\mathrm{x}}_1 \\ \ddot{\mathrm{x}}_2 \end{Bmatrix} + \begin{bmatrix} \mathrm{c}_{11} & \mathrm{c}_{12} \\ \mathrm{c}_{21} & \mathrm{c}_{22} \end{bmatrix} \begin{Bmatrix} \dot{\mathrm{x}}_1 \\ \dot{\mathrm{x}}_2 \end{Bmatrix} + \begin{bmatrix} \mathrm{k}_1 + \mathrm{k}_2 & -\mathrm{k}_2 \\ -\mathrm{k}_2 & \mathrm{k}_2 \end{bmatrix} \begin{Bmatrix} \mathrm{x}_1 (\mathrm{t} - \tau_1) \\ \mathrm{x}_2 (\mathrm{t} - \tau_2) \end{Bmatrix} = \begin{Bmatrix} \mathrm{F}_1 (\mathrm{t}) \\ \mathrm{F}_2 (\mathrm{t}) \end{Bmatrix} \tag{2.3}$$

where τ_1 and τ_2 are the delays due to the servo-hydraulic actuators attached to the experimental substructures in the first and second story, respectively. The actuator delays are observed to be coupled for the 2DOF.

Figure 2.1 shows the comparison of the story displacements with and without actuator delay. The ground motion is the 1994 Northridge earthquake recorded by USC in Beverly Hills with a peak ground acceleration of 0.4158 g. The delay incorporated is 3 ms and 3 ms for the actuators attached to the experimental substructures in the first and second story, respectively. The structure has the natural frequencies of 3.88 rad/s and 10.17 rad/s for the first and second mode, respectively. The structure is assumed to have 2 % Rayleigh viscous damping for both the first and second mode. It can be observed that due to the actuator delay, story displacement from real-time hybrid simulation will deviate from the exact solution. Using the definition of MAX error in Eqs. (2.4a) and (2.4b), MAX error in Fig. 2.1 is 20.5 % for the first story and 16.7 % for the second story. This indicates again that small actuator delays can have major effects on the accuracy of RTHS results.

$$\mathrm{MAX}_1 = 100 \times \frac{\max(\mathrm{abs}(\mathrm{x}_{1c} - \mathrm{x}_{1m}))}{\max(\mathrm{abs}(\mathrm{x}_{1c}))} \tag{2.4a}$$

$$\mathrm{MAX}_2 = 100 \times \frac{\max(\mathrm{abs}(\mathrm{x}_{2c} - \mathrm{x}_{2m}))}{\max(\mathrm{abs}(\mathrm{x}_{2c}))} \tag{2.4b}$$

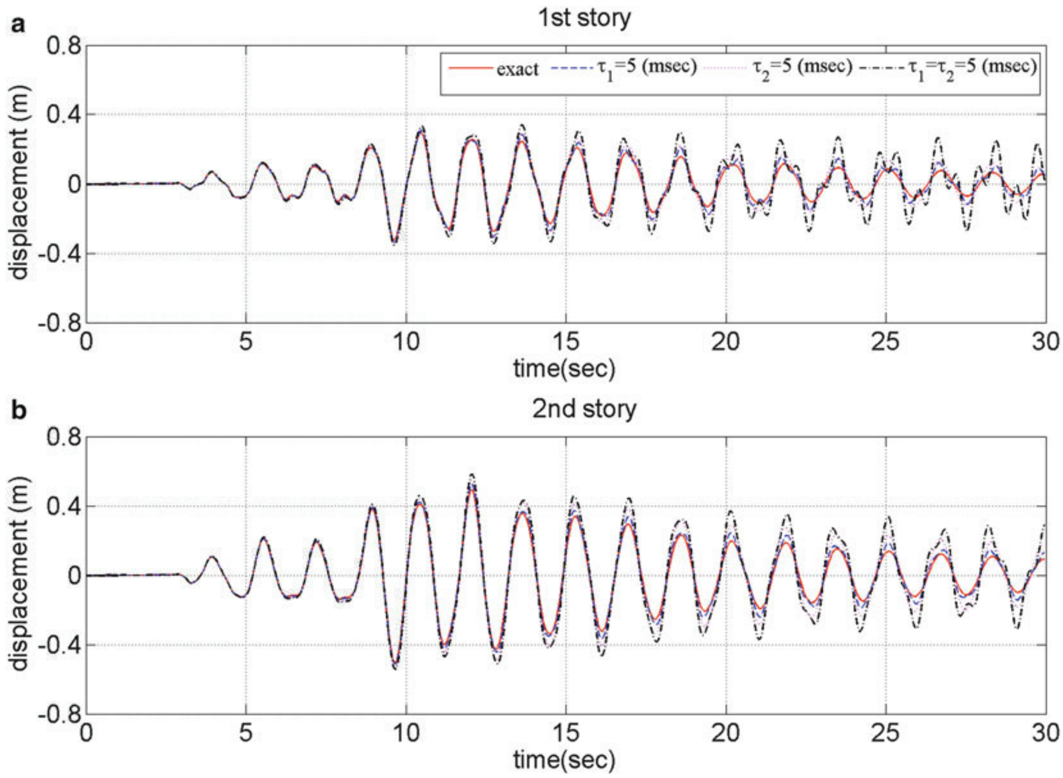

Fig. 2.1 Comparison of story displacements for real-time hybrid simulation of a linear elastic MDOF structure with and without actuator delay

Fig. 2.2 MAX error for real-time hybrid simulation with actuator delay

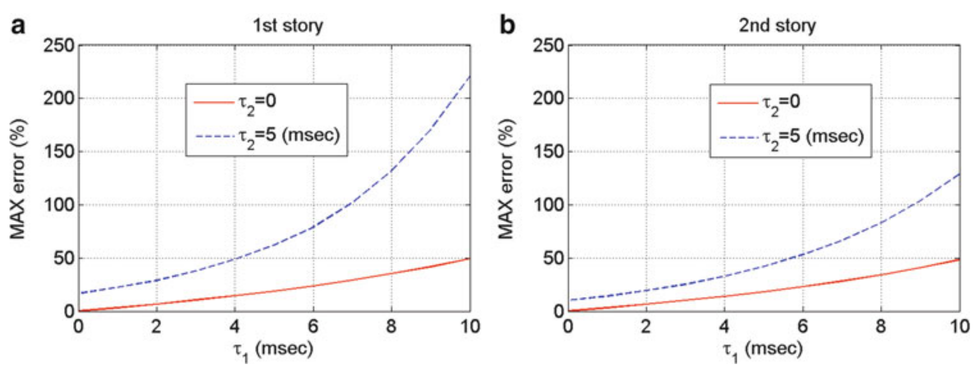

Figure 2.2 presents the increase of MAX errors with respect to the actuator delay associated with the experimental substructures in the first and second story. MAX errors for story displacements are observed even when the associated actuators have no delay. This implies that individual actuator delay affects the all the experimental substructures. For the same actuator delay, the MAX errors are also observed to be different. This indicates that the error in story displacement should not be estimated based on the time delay existing in the associated actuator.

2.3 Modal-Based Frequency-Domain Analysis

The FEI method utilizes the command displacements sent to the servo-hydraulics and the measured displacements measured from experimental substructures. In a more general form, the FEI method can be described as following: (1) Identify the input and output signals and apply Fast Fourier Transform (*FFT*) [24] to the input and output signals. Guo et al. [22] indicated that a Hanning Window technique [19] is necessary to minimize the effect of spectrum leakage. (2) Calculate the frequency evaluation index (*FEI*) between the window transformed input and output signals using Eq. (2.5a). (3) Calculate