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Vibrations and Stability of Complex Beam Systems

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Vibrations and Stability of Complex Beam Systems

First author: *To My Family*

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ISSN 2195-9862 ISSN 2195-9870 (electronic)
Springer Tracts in Mechanical Engineering
ISBN 978-3-319-13766-7 ISBN 978-3-319-13767-4 (eBook)
DOI 10.1007/978-3-319-13767-4

Library of Congress Control Number: 2014956102

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media
(www.springer.com)

Preface

The progress in vibration analysis in the previous period has been made as a consequence of the global advances in technology. The development of powerful and fast computers which can be used for computational techniques has brought about a numerical revolution in validation of complex mathematical models and analytically obtained results. A significant progress has been made in linear large-order systems. Indeed, one of the most significant advances in recent years has been the finite element method, a method developed originally for the analysis of complex structures. Proper knowledge of these two areas (the knowledge of analytical theory of vibrations and the knowledge of numerical techniques with FEM implementation) prepares researches for investigating new phenomena in vibrations, some of which are presented here.

This book contains the obtained results within the author's research during the preparation of the doctoral dissertation, and as such it is primarily intended for postgraduate students in the field of theory of vibrations. Detailed theoretical investigations have yielded original results in linear vibrations of elastically connected beams and geometrically nonlinear vibrations of damaged beams, which together may represent a group of complex beam systems.

The co-author of the book (the first author's supervisor) Dr. Predrag Kozić, full professor of the Faculty of Mechanical Engineering, University of Niš, provided meaningful assistance in the research within the field of linear vibrations. During the author's specialization at the Faculty of Mechanical Engineering, University of Engenharia in Porto, the author conducted research in the field of geometrically nonlinear vibrations with Dr. Pedro Ribeiro, and as a result the chapter which describes the vibrations of damaged beams is presented here.

The authors would like to express their sincere acknowledgements to Dr. Ratko Pavlović, Dr. Goran Janevski, Dr. Zoran Golubović, Dr. Stanislav Stoykov and Dr. Marko D. Petković for cooperation during the theoretical investigation. This research was supported by the research grant of the Serbian Ministry of Science and Environmental Protection under the number ON 174011.

The presented work consists of seven parts which are separately formed by chapters. The first chapter relates to the introductory discussion and review of previous research in the theory of elastic and related damaged structures. It is one of the ways to perform partial differential equations of motion of mechanical systems and provides a basic overview of the methods used. Chapters 2-6 are devoted to the analysis of linear elastic oscillations. The seventh chapter is devoted to geometric nonlinear oscillations of damaged beams using the new finite element method.

Free oscillations and static stability of two elastically connected beams are considered in Chapter 2. Through various examples analytically obtained results are shown and impacts of some mechanical parameters of the system on the natural frequency and amplitudes are presented. The verification of the obtained analytical results is shown by comparison with the results of the existing classical models. A new scientific contribution in this chapter is the formulation of the new double-beam model described with new derived equations of motion with rotational inertia effects and with inertia of rotation with transverse shear (Timoshenko's model, Reddy-Bickford's model). The static stability conditions of two elastically connected beams of different types are formulated with analytical expressions for the values of critical forces. Numerical experiments confirmed the validity of the analytical results obtained by comparing the results of the models existing in the literature. From chapter 2 it can be concluded that the effects of rotational inertia and transverse shear must be taken into account in the model of thick beams because errors that occur by ignoring them increase with the mode of vibration.

Chapter 3 presents the solution for forced vibrations of two elastically connected beams of Rayleigh, Timoshenko and Reddy-Bickford type under the influence of axial forces. The scientific contribution lies in the presented analytical solutions for the forms of three types of forced vibration: harmonic arbitrarily continuous excitation, continuous uniform harmonic excitation, and harmonic concentrated excitation. Analytical solutions were obtained by using the modal analysis method. The chapter also presents the analytical solutions of forced vibration for the case when harmonic excitation effects are concentrated on one of the beams under the effect of compressive axial forces. Based on the results derived in this chapter, it can be concluded that the differences in the approximations of the solutions depending on the used model provided good solutions only in the case of Timoshenko and Reddy-Bickford theory for thick beams in higher modes. Classical theories did not yield good results.

Chapter 4 considers the static and stochastic stability of two and three elastically connected beams and a single beam on elastic foundation. A new set of partial differential equations is derived for static analysis of deflections and critical buckling force of the complex mechanical systems. The critical buckling force is analytically determined for each system individually. It is concluded that the system is most stable in the case of one beam on elastic foundation.

Chapters 5 and 6 analyze free vibrations of more elastically connected beams of Timoshenko and Reddy-Bickford type on elastic foundation under the influence of axial forces. Analytical solutions for the natural frequencies and the critical force are determined by the trigonometric method and verified numerically.

Chapter 7 presents geometrically nonlinear forced vibrations of damaged Timoshenko beams. The study develops a new p-version of the finite element method for damaged beams. The advantage of the new method is compared with the traditional method, showing that it provides better approximations of solutions with a small number of degrees of freedom used in numerical analysis. The scientific contribution can be found in two topics-computational mechanics and non-linear vibrations of beams. It is concluded that the traditional method cannot provide good approximations of solutions in the case of a very small width of damage. This benefit is also shown in the comparison with the obtained results in the commercial software Ansys. A new p-version finite element is suggested to deal with geometrically non-linear vibrations of damaged Timoshenko beams. The novelty of the p-element comes from the use of new displacement shape functions, which are the functions of the damage location, therefore, providing more efficient models, where accuracy is improved at lower computational cost. In numerical tests in the linear regime, coupling between cross-sectional rotation and longitudinal vibrations is discovered, with longitudinal displacements suddenly changing direction at the damage location and with a peculiar change in the cross-section rotation at the same place. Geometrically nonlinear, forced vibrations are then investigated in the time domain using Newmark's method and further couplings between displacement components are found.

Dr. Vladimir Stojanović

Contents

Chapter 1
Introductory Remarks

1.1 Introduction

A great number of mechanical systems are complex structures composed of two or more basic mechanical systems whose dynamic behavior is conditioned by their interaction. The systems connected by an elastic layer constitute one group of such mechanical structures which are commonly encountered in mechanical, construction and aeronautical industry. Mechanical systems formed by elastic connection of their members, due to the nature of the dynamic interaction conditioned by elastic connections are characterized by complex vibration and a higher number of natural frequencies. Since the number of natural frequencies depends on the number of basic elements joint together, such mechanical systems are exposed to an increased likelihood of creating resonance conditions which can cause breakage and damage. With the view to putting theoretical research into engineering practice, a great number of linear dynamic models describing the motion of a system were created. Such models are important as they give the initial approximation of the solution and a general insight into a dynamic behavior of the system at slight motion. If technical practice requires further investigation of system behavior, these dynamic models provide solid ground for the continuation of research with non-linearity effects.

The parts of mechanical structures performing a great number of cyclical operations exposed to loads often sustain damage. As soon as the damage occurs, the dynamic behavior of the system changes requiring good knowledge thereof in order to prevent resonance conditions and the resulting consequences. In already damaged structures, the displacement of deformable elements is greater; hence the effect of geometric non-linearity is more common. Such dynamic behaviors are different in non-damaged structures, and it is essential to determine the conditions leading to resonance as the consequence of interaction between vibration modes.

The effects of rotary inertia and transverse shear have been taken into consideration in creating mathematical models for both elastically connected beams and damaged beams. Vibrations and the stability of such systems have been the subject of scientific, academic and practical research by many authors for decades. Increased interest by the scientific public in the problem of vibrations

V. Stojanović and P. Kozić, *Vibrations and Stability of Complex Beam Systems*,
Springer Tracts in Mechanical Engineering, DOI: 10.1007/978-3-319-13767-4_1

and the stability of such systems is motivated by breakage prevention and the prevention of permanent damage to real mechanical, construction and other systems. This led to the development of analytical and numerical methods used for analyzing the vibrations of both linear and non-linear mechanical systems.

The problem concerning the vibrations of beams joint by a Winkler elastic layer has attracted the interest of a large group of scientists. The problem of two elastically connected beams joint by the Winkler elastic layer emerged in order to determine the conditions for the behavior of the system acting as a dynamic absorber in technical practice. A mathematical model was developed by Seelig and Hoppman [1]. They investigated the problem of an impulse load effect on a beam and produced a system of partial differential equations describing its vibration. The obtained theoretical and experimental results confirmed a sound approximation of an analytical solution obtained for slender beams at small transverse motions using the Euler-Bernoulli theory.

Oniszczuk [2, 3] analyzed the problem of free and forced vibration of two elastically connected Euler-Bernoulli beams. He determined analytical solutions for eigenfrequencies, amplitudinous functions and vibration modes. He discussed the effect of stiffness which the elastic interlayer had on the frequencies and amplitudes of the system. He determined the conditions for the occurrence of resonance and the behavior of the system as a dynamic absorber.

The analysis of the system composed of two connected beams was carried on by Zhang et al. [4, 5] in their investigation of free and forced vibrations by two elastically connected Euler-Bernoulli beams affected by axial compression forces. They presented analytical solutions for natural frequencies of the system in the function of axial compression force impact and their effect on the vibration amplitude. They determined the co-dependency between the system's critical force and the Euler critical load in the function of an axial force of the other beam.

Vu et al. [6] studied the problem of forced and damped vibrations of the system composed of two elastically connected Euler-Bernoulli beams. They provided analytical solutions to partial differential equations with a damping factor. They determined the influence of system damping on vibration and amplitudes under harmonic forcing. Their analytical results showed close correspondence to experimental results obtained by Seelig and Hoppmann [1] for dynamic absorbers.

Li and Hua [7], using the spectral finite element method arrived at numerical solutions for natural frequencies of the two elastically connected Timoshenko beams with different supports. They determined vibration modes and amplitude-frequency dependence for forced vibration of the system.

The investigation of system vibration in complex structures which are encountered in technical practice as physical models of reinforcement, coupled support structures in mechanical engineering and multistory buildings in civil engineering, has made its way into the study fields of many researchers. These structures made up of three or more elastically connected elementary member beams were the subject of recent investigations. The development of computer resources and the application of numerical mathematics on such models led to the increased interest in the matter by a group of scientists in the 20th century.

Li et al. [8] addressed the problem of three elastically connected Timoshenko beams in their paper. They numerically determined natural frequencies for different types of support, mode shapes and the effect the stiffness of elastic inter-layers had on vibration of the system. Kelly and Srinivas [9] discussed the problem of multiple elastically connected Euler beams. They determined natural frequencies and mode shapes for the beams with identical characteristics using the Rayleigh-Ritz numerical method. Ariaei et al. [10] studied the problem of a movable body according to the system of multiple elastically connected Timoshenko beams on an elastic foundation using the Transfer-matrix method. They concluded that the maximum deflection of the beam system decreases in case of a body moving on the beams closer to the surface. Mao [11] determined mode shapes for the first ten system modes of multiple elastically connected Euler beams on an elastic foundation using the Adomian decomposition method.

Stojanović et al. [12] analyzed free vibration and static stability of two elastically connected beams taking into account the effects of rotary inertia and transverse shear which leads to the rotation of the cross-section. They used an example to show the analytical solutions for natural frequencies and determined the critical force for the coupled beam system.

Stojanović and Kozić [13] discussed the case of forced vibration of two elastically connected beams and the effect of axial compression force on amplitude ratio of system vibration for three types of external forcing - arbitrarily continuous harmonic excitation, uniformly continuous harmonic excitation and concentrated harmonic excitation. They determined general conditions of resonance and dynamic vibration absorption. In the paper [14], Stojanović et al. discussed the analytic analysis of static stability of a system consisting of three elastically connected Timoshenko beams on an elastic foundation. They provided expressions for critical force of the system under the influence of elastic Winkler layers. Stojanović et al. [15] using the example of multiple elastically connected Timoshenko and Reddy-Bickford beams, determined the analytical forms of natural frequencies, their change under the effect of axial compression forces and the conditions for static stability for a different number of connected beams.

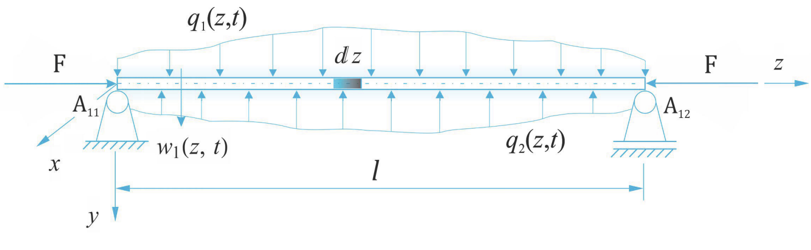
The issues of the vibration in damaged beams have a significant place in theoretical and experimental research into the behavior of dynamic systems. The occurrence of various types of damage in constructions leads to their permanent modification and increased risk of breakage. Considerable engineering practice in the field prompted a great number of researchers to examine the vibration of damaged dynamic system. Christides and Barr [16] experimentally determined the impact factor on natural frequencies of a simply supported beam with closed damage. The obtained results showed that the occurrence of damage decreases the natural frequencies of the system. Several studies in the field of damaged beam analysis have been carried out on a beam model with altered stiffness as a consequence of damage occurrence and its impact on natural frequencies, vibration modes and the damping factor in linear mode. Thus Sinha et al. [17] developed a model of the apparent change in the beam geometry at the place of damage. Using the experimentally determined impact factor on natural frequencies of damaged beams published by Christides and Barr [16] for a closed crack, they determined the required change in the beam geometry at the place of damage using the triangle model type of an open crack. They examined the vibration of a thus formulated mathematical model using the finite element method and confirmed the results by experimental verification thereof. Pandey et al. [18] investigated the effect of damage on natural mode shapes in their paper. A group of researchers focused dynamic analysis on damage detection and localization. They used the occurrence of greatest deviation from the mode shapes as a method of detecting the damage on the beam. Panteliou et al. [19] used the change in the damping factor to localize the damage. A great number of papers in non-linear vibration of damaged beams confirmed the interest of scholars in the field. Thus Bikri et al. [20] considered geometrically non-linear vibration of a doubly-clamped beam with a crack and showed the difference in mode shapes between the linear and non-linear model. Andreaus et al. [21] investigated geometrically non-linear vibration of a bracket under the influence of harmonic excitation. They formed the model as a bilinear oscillator which implied the opening and closing of the damage at vibration. Based on earlier research on non-linear vibration of damaged beams, Stojanović et al. [22, 23, 50] developed a new *p*-version of a finite element method for geometrically non-linear vibration of damaged Timoshenko beams. Unlike its previous versions, they formed the damage model so as to take into account the change in the weight of the system in addition to the change in the stiffness thereof. The advantage of the new method is the ability to determine natural frequencies of the system using a smaller number of functions of motion, i.e. lower vibration freedom degree. The paper shows the occurrence of longitudinal vibration of the doubly clamped beam which occurs solely as the consequence of damage.

Numerical experiments were performed on the Timoshenko beam model taking into account the effects of rotary inertia and transverse shear as in the case of elastically connected beams. It was shown that the coupling between transverse vibration and component vibration of cross-sections caused asymmetry in a non-linear vibration time mode. For the non-linear vibration region the Newmark method of direct integration was used to solve non-linear partial differential equations for forced vibration of the damaged Timoshenko beam. Continuation method was applied to the free non-linear vibration model to determine bifurcation points, Stojanović and Ribeiro [23]. The new *p*-version of the finite element method was developed within the mathematical model of Timoshenko beams, determining the effects of the cross-section rotation (rotary inertia and transverse shear) on geometrically non-linear vibration of beams with different thickness.

1.2 Vibrations of Euler, Rayleigh, Timoshenko and Reddy-Bickford Beams

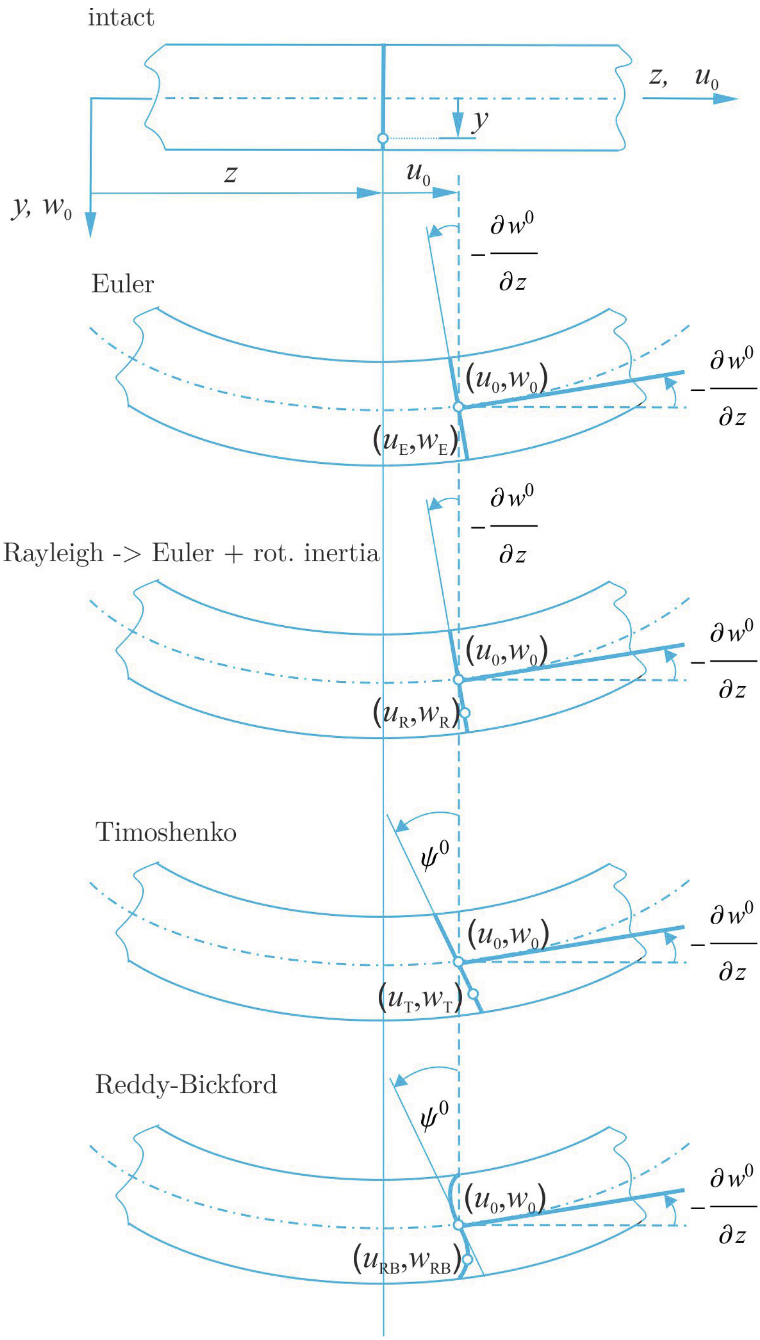
This part of the chapter relates to the basic review of the beam theories. It is shown the different aspect and perform of partial differential equations of motion which are used in further theoretical investigation of the complex beam systems. The Euler–Bernoulli theory for a beam originated in the 18th century. The effect of rotary inertia was introduced by Rayleigh in 1894. In 1921, Timoshenko proposed his theory where shear is also taken into account. Since shear and rotary inertia are ignored in the Euler–Bernoulli theory is reasonable to assume that the Timoshenko theory is an improvement. Reddy-Bickford theory ensures a more exact stress-deformation mathematical interpretation in the analysis and provide a approximations which are more accurate and important in vibrations of thick beams.

Euler-Bernoulli theory on homogeneous elastic slender prismatic beams (Figure 1.2.1a,b) implies that the vibration takes place so that the beams' cross-sections are always normal to the neutral axis at deformation ignoring the effects of rotary inertia and transverse shear, [2, 3]. The theory gives a good approximation of solutions at small transverse motions for slender beams.



(a)

Fig. 1.2.1 (a) A simply supported beam under the influence of axial compression forces F and continuous loads $q_1(z,t)$ and $q_2(z,t)$ (b) Deformation in various beam theories



(b)

Fig. 1.2.1 (*continued*)