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Margarita N. Favorskaya
Lakhmi C. Jain *Editors*

Computer Vision in Control Systems-1

Mathematical Theory

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Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

Lakhmi C. Jain, University of Canberra, Canberra, Australia, and
University of South Australia, Adelaide, Australia
e-mail: Lakhmi.Jain@unisa.edu.au

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Margarita N. Favorskaya · Lakhmi C. Jain
Editors

Computer Vision in Control Systems-1

Mathematical Theory

 Springer

Editors

Margarita N. Favorskaya
Department of Informatics and Computer
Techniques
Siberian State Aerospace University
Krasnoyarsk
Russia

Lakhmi C. Jain
Faculty of Education, Science, Technology
and Mathematics
University of Canberra
Canberra
Australia

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Foreword

The mathematics underlying computer vision can be traced back to the work of A. Rosenfeld on the digital topology,¹ G. Matheron² on decidability through regionals, and J. Serra³ on mathematical morphology. Digital topology begins with the separation of the pixels in a digital image into subsets called segments and studying the basic properties of the subsets such as adjacency and connectedness. It was G. Matheron who suggested regionals as a basis for decideability and called attention to the importance of a hit-or-miss topology in the study of closed compact subsets in a bounded region in \mathbb{R}^2 . And it was J. Serra who suggested using hit-or-miss topology⁴ as a basis for mathematical morphology. In practice, we start with a small set called a structuring element A_h , used to probe the parts of a binary image. A probe is used to check whether A_h hits a subset B (the intersection $A_h \cap B$ is not empty) in an image viewed as a hyperspace (collection of closed subsets of a topological space) or whether A_h misses B ($A_h \cap B$ is empty).

Digital topology has its roots in the work by Archimedes and Apollonius in defining the locations of points in a plane by their distances from two straight lines and later by R. Descartes in defining nonnegative coordinates in the plane. Negative coordinates were introduced by I. Newton. The set theoretic view of digital images can be traced back to the work by H. Poincaré on the similarities between point-sets in a physical continuum⁵ and by F. Hausdorff on topological spaces,⁶ focusing on open and closed sets and on metrizable spaces (spaces that are homeomorphic to metric spaces). It was Hausdorff who pointed out that every point has a least one neighborhood.⁷

¹ see [1].

² see, e.g., [2].

³ see, e.g., [3].

⁴ For a detailed view of hit-or-miss topology, see [4].

⁵ see, e.g., [5].

⁶ see, e.g., [6].

⁷ For a detailed view of various types of neighborhoods in digital images, see [7].

Appropriately, this book begins with an introduction to morphological image analysis for computer vision applications by Yu.V. Vizilter, Yu.P. Pyt'ev, A.I. Chulichkov, and L.M. Mestetskiy. These authors focus on skeleton-based continuous binary morphology, a morphological pattern spectrum, and what is known as Pyt'ev morphology. The notion of a skeleton (middle set of points) of a closed region in the Euclidean plane is a locus of centers of maximum empty circles. This leads to a very interesting presentation of what is known as a discrete morphological pattern spectrum.⁸ The Pyt'ev morphology is based on vector algebra and functional analysis, whereas the Serra morphology is based on nonlinear set-theoretic (complete lattice) models.

The central motifs in this book are threefold.

1. **Mathematical morphology**

Morphological spectral patterns and Pyt'ev morphology (Yu.V. Vizilter, Yu.P. Pyt'ev, A.I. Chulichkov and L.M. Mestetskiy, Chap. 2), fuzzy morphological contour basis for image segmentation (V.L. Fox, M. Milanova, S. Al-Ali, Chap. 8).

2. **Image correspondence**

Structural image similarity based on spectral criteria (Y.S. Radchenko, A.V. Bulygin, Chap. 3), digital image correlation (R. Kountchev, R. Kountcheva, Chap. 4), recognition of digital images with geometric transforms (V. Lutsiv, Chap. 5).

3. **Image-Based Signal analysis**

Energy and phase-energy spectra in analysing interframe differences in video signals (A. Bogoslovsky, I. Zhigulina, Chap. 6), cooperative measurement using multiple visual motion sensors (S. Gepshtein, I. Tyukin, Chap. 7), digital video stabilization via motion vector separation using fuzzy set theory (M. Favorskaya, L.C. Jain, V. Buryachenko, Chap. 9), Strip-method of image transformation that entails cutting a 1-dimensional signal into n strips, forming an n -dimensional vector, mixing and superimposing image fragments on each other using a Hadamard matrices and variations of such matrices (L. Mironovsky, V. Slaev, Chap. 10), and criteria useful in estimating the efficiency of telecommunication systems (A.A. Borisenko, V.V. Kalashnikov, A.E. Goryachev, N.I. Kalashnykova, Chap. 11).

This book ably demonstrates the utility of the basic mathematical framework provided by morphology (and its underlying attention to aspects of set theoretic topology and the basic geometric structures found in digital images) as well as the utility of a variety of approaches in image correspondence detection and image-based signal analysis. The interplay of these concepts is cogently demonstrated by the contributors to this volume.

⁸ In this book, see, e.g., Fig. 2.14, page 30.

I strongly recommend this book as a concise and very original introduction to the mathematical foundations of image analysis and the practical application of the mathematics across a broad spectrum in the study of digital images.

June 2014

James F. Peters
Department of Electrical and Computer Engineering
University of Manitoba
Winnipeg, MB, Canada

and

Faculty of Arts and Sciences
Department of Mathematics
Adıyaman University
Adıyaman, Turkey

References

1. Rosenfeld A, Pfaltz JL (1968) Distance functions on digital pictures. *Pattern Recognit* 1:33–61; Rosenfeld A (1979) Digital topology. *The Amer Math Monthly* 86(8):621–630
2. Matheron G (1975) *Random sets and integral geometry*. Wiley, New York; Matheron G (1989) *Estimating and choosing. An Essay on Probability in Practice* (trans: Hasofer AM). Springer-Verlag, Berlin, pp ix + 141
3. Serra J (1976) *Lectures on image analysis by mathematical morphology*, Tokyo Noko University; Serra J (1982) *Image analysis and mathematical morphology I*, Academic Press, London-New York, pp xiv + 610
4. Beer G (1993) *Topologies on closed and closed convex sets*, Kluwer, Dordrecht
5. Peters JF, Wasilewski P (2012) Tolerance spaces: origins, theoretical aspects and applications, *Inform Sci* 185:211–225
6. Hausdorff F (1957) *Set theory* (trans: Aumann JR) AMS Chelsea Pub, 1957
7. Peters JF (2014) *Topology of digital images. Visual Pattern Discovery in Proximity Spaces*, Springer, Ch. 1

Preface

The research book is focused on the recent advances in computer vision methodologies and technical solutions using conventional and intelligent paradigms. The contemporary solutions based on advanced mathematical achievements emphasize more information and visual monitoring in natural and human environment. The real challenge of designing such observation models are to make them close to realistic visualization and interpretation of events in our world.

The book presents some of the research results from some of the most respectable researchers in the field of computer vision stressing on mathematical theory. Of the 11 chapters, the first chapter presents a brief introduction of the chapters presented in the book. Chapter 2 is on the Morphological Image Analysis for Computer Vision Applications. Chapter 3 presents techniques for Detecting the Structural Changes in Computer Vision. Chapter 4 is on Hierarchical Adaptive KL-based Transform: Algorithms and Applications. Chapter 5 is on Automatic Estimation for Parameters of Image Projective Transforms Based on Object-invariant Cores. Chapter 6 is on the Analysis of Energy for Image and Video Sequence Processing. Chapter 7 is on Optimal Measurement of Visual Motion Across Spatial and Temporal Scales. Chapter 8 presents the Analysis of Scene Using Morphological Mathematics and Fuzzy Logic. Chapter 9 is on Digital Video Stabilization in Static and Dynamic Scenes. Chapter 10 presents the Implementation of Hadamard Matrices for Image Processing. The final chapter is on A Generalized Criterion of Efficiency for Telecommunication Systems.

The book is directed to the Ph.D. students, professors, researchers and software developers working in the areas of digital video processing and computer vision technologies.

We wish to express our gratitude to the authors and reviewers for their contribution. The assistance given by the Springer-Verlag and team is acknowledged.

Russia
Australia

Margarita N. Favorskaya
Lakhmi C. Jain

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About the Editors



Margarita N. Favorskaya received her engineering diploma from Rybinsk State Aviation Technological University, Russia, in 1980 and was awarded a Ph.D. by S.-Petersburg State University of Aerospace Instrumentation, S.-Petersburg, in 1985. Since 1986 she worked as Associate Professor of Siberian State Aerospace University, Krasnoyarsk. Margarita Favorskaya defended her doctoral dissertation in Siberian Federal University in 2011. Since 2011 she is a Professor and a Head of Department of Informatics and Computer Techniques at Siberian State Aerospace University.

Her main research interests are digital image and videos processing, pattern recognition, fractal image processing, artificial intelligence, information technologies, remote sensing. She is the author or the co-author of nearly 130 scientific publications and 20 educational manuals in these fields. Margarita Favorskaya is a member of KES organization, IPC member of International Conferences, and Co-Chair of Invited Sessions. She serves as the Reviewer, Guest Editor, and Associate Editor in International Journals.



Lakhmi C. Jain is with the Faculty of Education, Science, Technology and Mathematics at the University of Canberra, Australia and University of South Australia, Australia. He is a Fellow of the Institution of Engineers Australia.

Dr. Jain founded the KES International for providing a professional community the opportunities for publications, knowledge exchange, cooperation and teaming. Involving around 5,000 researchers drawn from universities and companies world-wide, KES facilitates international cooperation and generate synergy in teaching and research. KES regularly provides networking opportunities for professional community through one of the largest conferences of its kind in the area of KES. www.kesinternational.org.

His interests focus on the artificial intelligence paradigms and their applications in complex systems, security, e-education, e-healthcare, unmanned air vehicles and intelligent agents.

Chapter 1

Development of Mathematical Theory in Computer Vision

Margarita N. Favorskaya and Lakhmi C. Jain

Abstract This chapter presents a brief description of chapters devoted to the theoretical development of computer vision. Original investigations in mathematical morphology, estimations of structural changes, the hierarchical adaptive Karhunen-Loeve and projective transforms, among others, provide the great contribution in mathematical foundations of computer vision. Each theoretical chapter involves practical implementations, which demonstrate the merit of proposed methods in practice.

Keywords Computer vision · Image processing · Videos processing

1.1 Introduction

In the past decades, computer vision techniques have progressed significantly and are widely used in many implementations of control systems. Great advances have been made in image filtering, segmentation, pattern recognition, and events understanding. However, the excellent mathematical models and methods cannot be directly applied in many practical situations. The majority of efforts focus on designing the efficient and real-time methods to analyze images and video data on various levels of processing. The contemporary solutions based on advanced mathematical achievements emphasize on more information and visual monitoring

M.N. Favorskaya (✉)

Institute of Informatics and Telecommunications, Siberian State Aerospace University,
31 Krasnoyarsky Rabochy, Krasnoyarsk 660014, Russian Federation
e-mail: favorskaya@sibsau.ru

L.C. Jain

Faculty of Education, Science, Technology and Mathematics, University of Canberra,
Canberra, ACT 2601, Australia
e-mail: lakhmi.jain@unisa.edu.au

in natural and human environment. The goal of current investigations is designing such observation models, which are close to realistic visualization and interpretation of events in our world.

1.2 Chapters Included in the Book

The main purpose of this research book is to present a sample of research results on recent advances in computer vision. This book includes eleven chapters on the “Mathematical Theory” aspect of the computer vision.

Chapter 2 introduces the morphological framework as a very wide theoretical platform for creation of mid-level image analysis tools for specialized computer vision applications. It utilizes the structural image modeling and decides some image filtering, segmentation, and comparison problems. Mathematical Morphology (MM) by Serra [1] and Matheron [2] is still the most well-known version of MM until these days. Another morphological approach proposed by Pyt’ev is based on geometrical and algebraic reasoning. In the framework of Pyt’ev morphology, images are considered as piecewise-constant 2D functions [3]. The tessellation of image frame by a set of non-intersected connected regions with constant intensities determines the “shape” of the image. The main idea of this approach is the projection of one image onto the shape of other image. The detection of morphological changing is performed by comparison of image and its projection to the reference image. Such morphological tools are invariant relative to image intensity transforms and stable relative to noise. The idempotent operators such as morphological filters or projectors are introduced using a concept of figure filling by structured elements. In other version, morphological filters are based on merging of grayscale image connected regions (“flat” zones). The continuous binary morphology is based on computational geometry and provides very fast tool for computation of continuous figure skeletons using approximation of 2D binary image by region border polygons and calculation of Voronoi diagram for segments these polygons [4]. A skeletal representation of the figure is formed as its skeleton and the radial function determined in skeleton points. The projective morphology is a generalized framework based on Serra’s MM and Pyt’ev’s Morphological Analysis. It combines the ideas of both morphological approaches and allows construction of some new morphological systems and operators based on different image decompositions and transforms and/or criteria (energy functions). Criterion-based projective morphological filters are implemented using numeric optimization techniques (linear programming, dynamic programming, graph cutting, and so on) [5]. The morphological spectrum as a multi-scale morphological shape analysis tool based on “granulometry” also contains in this chapter.

Chapter 3 discusses the criteria of Mean Structural Similarity Index Measure (MSSIM) and the developed Mean Nonparametric Structural Similarity Index Measure (MNSSIM), as well as the spectral algorithm for detecting structural changes in a frame, which have been used to good effect in video codec analysis [6].

These criteria provide the estimations for structural (texture) variations of images. The growing popularity of these criteria is proved by their quite appropriate compliance with the human vision system [7]. The detection of variations in the image segment structure is based on spectral and correlation analysis of space-time fields. At present, the quasi-optimum heuristic algorithms applying variations of field correlation features, non-invariance of spectrum in various bases (in relation to a segment movement and change of their texture features) exist. The different estimation methods and algorithms for images presented by numbered blocks as well as the criteria and metrics being a basis to detect these differences are investigated. In this chapter, the reader can find practical examples using of pixel and spectral algorithms in image analysis.

Chapter 4 investigates a novel approach to process a single image or sequences of frames through the Hierarchical Adaptive Karhunen-Loeve (KL)-based Transform (HA-KLT). This approach is suitable for image block coding and for inter-frame processing of correlated frames in groups [8]. The basic aim of a new transform is to achieve a decorrelation of the image blocks, respectively of all frames in the processed group. This is realized by a multiple applying of the HA-KLT. After each level of the hierarchical transform, all sub-blocks (respectively groups) are rearranged so, that the components with highest correlation, which are obtained in the preceding level, would be placed in a new sub-blocks of the current level. The kernel of the multi-level transform is the Adaptive KL Transform (AKLT). The AKLT with a transform matrix of size 2×2 and 3×3 is used for the processing of the image sub-blocks and the pixels with same position in the sub-groups of frames respectively. The algebraic method for the calculation of the elements of the AKLT transform matrix of size 2×2 and 3×3 is presented in this chapter. The 2D and 3D HA-KLT algorithms for the blocks of a single image and for inter-frame processing of sequences (groups) of frames are also developed [9]. The computational complexity of these algorithms is compared with the “classic” KLT. On the one hand, the proposed approach ensures a higher accuracy of color segmentation in all cases, when a distribution of color vectors is not Gaussian. This is achieved by using a polynomial kernel for the color space expansion, after which the HA-KLT is applied to the expanded color vectors. In result, a decorrelation of the transformed vectors and an information concentration in their first components are achieved. On the other hand, this permits to reduce a number of components of the transformed vectors, retaining the first two only. In a new 2D space, the color vectors clusterization in respect to RGB space is enhanced, and they can be classified with high accuracy by using the support vector machine algorithm or other similar methods [10]. The HA-KLT method is a basis for the creation of novel efficient algorithms for a fusion of 3D images in face recognition task, an objects tracing in videos, a compression with movement compensation and without visual quality loss of TV and multi-view visual information, medical and multispectral images, etc [11].

Chapter 5 provides the design of object-invariant cores, which correspond to all types of spatially compact object images (previously segmented from a background), under the affine and projective transformations caused by an image

projection through the spherical (or almost spherical) lenses being the traditional parts of photo- and video-cameras [12]. The object-invariant core is synthesized by means of truncating the high-frequency harmonics in a spatial image spectrum. These rejected high-frequency harmonics present the object peculiarities, while the rest (extremely low-frequency) harmonics contain the information about spatial image transformations. It is shown that such object-independent core is mathematically described by elliptic paraboloid (quadratic parabola in 1D image projection). All parameters of affine geometric transformation (except a rotation and a mirror-like reflection) are measured analytically from this object invariant core. The parameters of rotation and mirror-like reflection are calculated from the cyclic narrow-band harmonic cores of image projection on the angular coordinate in a polar system. While the 6-parametric affine transformation is entirely linear, the full projective transformation contains additionally a nonlinear part described by two additional parameters. Due to this nonlinearity, the specific parameters of projective transformation cannot be measured analytically. A novel iterative optimization procedure is proposed to measure all parameters of projective transformation [13]. It is proposed to measure the missing parameters of projective transformation by a displacement of object-invariant core under the test transformations. The convergence of iterative measurement procedure is rigorously proven. At the end of the chapter, the examples of practical applications for automatic measurement of all projective transformation parameters are presented.

Chapter 6 presents a way of energy analysis for image and video sequence processing as a preliminary processing in vision systems [14]. Usually the object movements are determined by the analysis of an Inter-Frame Difference (IFD) in video signals. It is the simplest universal method. However, it doesn't exhaust opportunities for intelligent processing, especially in extremely low luminance. The IFD of energy spectrums and phase-energy spectrums are considered as an alternative analysis. The phase-energy spectrum is a product of partial derivatives in spatial phase-frequency spectrum over their spatial frequencies. It provides the detailed information about motion in finite frames [15]. The modeling of the IFD of frequency responses shows the necessity of analysis for pixels located near the moving boundaries. A processing of such pixels intensities increases a probability of movement's detection. Also distortions of moving object's shape, movement's characteristics, and a quantity of moving objects are possible to define based on the analysis of the IFD types. The phase-energy spectrums are used for edges analysis, if any movement is detected in a scene. The analysis of the energy spectrums is applied to design the effective 2D filters. The changes of the energetic indexes in static images determine the efficiency function on a whole set of impulse responses of the filter. The function of efficiency has a positively certain quadratic form with the coefficients of energy spectrum decomposition into the 2D Fourier series over the cosines. The analysis of stationary points by using this function of efficiency allows to synthesize the optimum and the quasi-optimum 2D filters. The proposed way of energy analysis provides some novel possibilities, for example, the detection of objects with extremely small contrast image.

Chapter 7 studies on how visual motion can be estimated at the lowest overall uncertainty of measurement across the entire range of useful sensor sizes (in artificial systems) [16] or the entire range of receptive fields (in biological systems). In other words, the following is an attempt to develop an economic normative theory of motion-sensitive systems. Such norms are derived for efficient design of systems, and then the norms are compared with facts of biological vision. This approach from the first principles of measurement and parsimony helps to understand the forces that shape the characteristics of biological vision. These characteristics include the spatiotemporal contrast sensitivity function, the adaptive transformations of this function caused by stimulus change, and also some characteristics of the higher-level perceptual processes such as perceptual organization [17]. In the following, the minimax strategy is implemented by assuming the maximal (worst-case) uncertainty of measurement on the sensors that span the entire range of the useful spatial and temporal scales. This strategy is used in two ways. First, the consequences of Gabor's uncertainty relation are investigated by assuming that the uncertainty of measurement is as high as possible. Second, the outcomes of measurement on different sensors are anticipated by adding their component uncertainties.

Chapter 8 presents the segmentation of natural images as a challenging task in image processing. Many methods have been proposed in the literature regarding algorithms for segmentation of such images [18]. Many of algorithms are complex in nature and inefficient in practice with unaltered images. In order to efficiently use the algorithms it is beneficial to pre-process the natural images. However, natural images often involve subjects and background that are not easily quantified with crisp pre-processing parameters. A partial solution to the problem of segmenting complex images is to use features that discriminate in the active contour algorithms [19]. These feature descriptions range from curvature to the orientation of level sets and usually result in better segmentation. An unfortunate side effect of using feature discriminates is that the complexity of the algorithm greatly increases resulting in even higher computational cost and difficulty in implementing the method. The goal is to develop a morphological level set active contour segmentation method that can robustly and efficiently segment multiphase textural images of high complexity [20]. To do this the usage of region statistics inside and outside the contour, membership functions from fuzzy logic methodology, and a Gaussian kernel function are required. In this chapter, a number of existing methods for shape feature extraction and representation are presented. At the end, application examples for using object shape representation in application for object recognition and human activity recognition are show.

Chapter 9 is devoted to digital video stabilization oriented on removal of intentional motions from video sequences caused by camera vibrations under strong wind in static scenes, by motion of robots unstable platforms in dynamic scenes, or jitters during a human hand-held shooting [21]. The analysis of dynamic scenes is required in advanced intelligent methods and directly depends from a problem statement. Several sequential stages connect with the choice of anchor frame, local and global motion estimations, and the jitters compensation algorithm. The choice of anchor frame into static scenes may be random with duration 1 s or 24 frames. In

the case of dynamic scene, the additional problem of scenes' separation should be solved for receiving a 'good' anchor frame. Most existing methods and algorithms do not work in real time. For investigation purposes, a non-real time approach is developed, however practical applications need in fast and reliable solutions. Several strategies are used for Local Motion Vectors (LMVs) building based on the keypoint detectors and block-matching algorithm [22]. The application of fuzzy logic operators improves the separation results between the unwanted motion and the real motion of rigid objects. For dynamic scenes, the kurtosis estimations are calculated and tracking curves are built in the case of small vibrations, and frame interpolation is applied, if vibrations have large values. The fuzzy model based on triangular, trapezoidal, and *S*-shape memberships partitions the LMVs concerning them to an unwanted camera motion and objects motion into a scene. The output of fuzzy logic model indicates a final reliability of matching quality by using the Takagi-Sugeno-Kang model. Such zero-order fuzzy model generates the quality index (a value in the range [0, 1]). The quality of the points matching is classified into four categories: excellent, good, medium, and bad. Therefore, fuzzy logic is used for improvement of local and global motion estimations and determines the novelty of approach. The similar procedure is applied for estimation of Global Motion Vectors (GMVs). The corrective algorithm compensates the unwanted motion into frames. Thereby, the scene is aligned. For restoration of current frame, pixels are shifted on a value of Accumulated Motion Vector (AMV) of unwanted motion. However, the sizes of stabilized frames became less relatively the original video sequence and the restoration of "missing" frame edges is required.

Chapter 10 examines the problems of transforming information and studying data connected with processing and transmitting images. The strip-method for storage and noise-immune transmission of images is studied [23]. Before transmission, the matrix transformations of an original image are executed, during which the image fragments are mixed and superimposed on each other. The transformed image is transmitted over a communication channel, where it is distorted with a pulse noise, the latter being for example a possible reason for a complete loss of separate image fragments. In the process of receiving a signal at the receiving end, an inverse transformation is performed. At the end of this transformation, the reconstruction of the image takes place. If it is possible to provide a uniform distribution of the pulse noise over the whole area the image occupies (without any changes of its energy), then a noticeable decrease of noise amplitude will take place and an acceptable quality of all fragments of the image reconstructed. In this chapter, many tasks are considered such as versions of the two-sided strip-transformation of images, choice of optimal transformation matrices, investigation of root images of the strip-transformation, and illustration of capabilities of the method suggested using particular examples. In order to get the maximum decrease of the pulse noise amplitude, it is necessary to achieve a uniform distribution of the noise over the image by applying the inverse transformation at the receiving end of the communication channel. This will allow information about distorted or "lost" fragments to be reconstructed. Now a problem of determining the type of the transformation matrices A and B arises. The solution of this problem will provide

the possibility to minimize the noise amplitude in the reconstructed image. A well-known solution of this problem is related to the cases of n , which can be divided by four, i.e. the so-called normalized Hadamard matrices [24]. The less-known solution for even n , not divisible by four, consists in so-called C -matrices (Conference-matrices). Such matrices have a zero diagonal and their remaining elements are equal to ± 1 .

Chapter 11 provides a discussion about the generalized criterion of efficiency for telecommunication systems [25]. Besides the partial criteria, there exists also a need in developing generalized ones allowing to compare various telecommunication systems and to choose the most efficient ones among them. To this end, the generalized criteria should consider and incorporate the partial ones, establish certain relationships between them, and hence possess the highest possible objectivity. Such criteria should be rather simple, easily computable, and provide the way to compare the telecommunication systems within a definite numerical scale, that is, they should be normalized [26]. The chapter develops a generalized criterion to estimate the efficiency of telecommunication systems that can be applied to economics information systems, too. The criterion combines evaluation of such special properties as the information quantity, noise immunity, the data transmission speed, and the transmission cost. In contrast to other criteria, the proposed one is non-dimensional and normalized, thus estimating a telecommunication system by means of real number between 0 and 1. The design of the developed criterion based upon the concept of conditional entropy is rather simple. It allows one to calculate the system's characteristic value with sufficient accuracy for practice, thus comparing various telecommunication systems to transfer the economic information. The generalized criterion is composed as a product of some partial criteria, which permits one to estimate the telecommunication systems not only as a whole, but also with respect to their partial characteristics, such as their productivity, reliability, and transmission cost.

1.3 Conclusion

The chapter has provided a briefly description of ten chapters with original mathematical investigations in computer vision techniques applied in advanced control systems. All included chapters involve the recent achievements in mathematical morphological theory, advanced criteria for structural similarity and the efficiency for telecommunication systems, the analysis of energy spectrums, complicated image transforms such as hierarchical adaptive Karhunen-Loeve transform and projective transform, optimal measurement of visual motion based on perception theory, intelligent methods for digital video stabilization, approaches for transmitting images based on Hadamard matrices Each chapter of the book explores experimental results, illustrating its use and applicability.

References

1. Serra J (1988) Image analysis and mathematical morphology. Theoretical advances. Academic Press, London
2. Matheron G (1975) Random sets and integral geometry. Wiley, New York
3. Pyt'ev Y (1998) Methods for morphological analysis of color images. *Pattern Recognit Image Anal* 8(4):517–531
4. Mestetskiy L (2010) Skeleton representation based on compound Bezier curves. In: 5th international conference on computer vision theory and applications (VISAPP'2010), vol 1. INSTICC Press, pp 44–51
5. Vizilter YV (2009) Design of data segmentation and data compression operators based on projective morphological decompositions. *J Comput Syst Sci Int* 48(3):415–429
6. Sheikh HR, Bovik FC (2011) Image information and visual quality. *IEEE Trans Image Process* 20(1):88–98
7. Pratt WK (2001) Digital image processing. PIKS inside, 3rd edn. Wiley, New York
8. Dony R (2001) Karhunen-Loeve transform. In: Rao K, Yip P (eds) *The transform and data compression handbook*. CRC Press, Boca Raton, London, New York, Washington, DC
9. Kountchev R, Kountcheva R (2013) Decorrelation of multispectral images, based on hierarchical adaptive PCA. *Int J WSEAS Trans Sign Proc* 3(9):120–137
10. Ivanov P, Kountchev R (2013) Hierarchical principal component analysis-based transformation of multispectral images. *Int J Reasoning-Based Intell Syst (IJRIS)* 5(4):260–273
11. Rao K, Kim D, Hwang J (2010) *Fast Fourier transforms: algorithms and applications*. Springer, Dordrecht, Heidelberg, London, New York
12. Shapiro LG, Stockman GC (2001) *Computer vision*. Prentice Hall, Upper Saddle River, New Jersey
13. Lutsiv V (2009) Method of iteratively compensating projective image distortions. *J Opt Technol* 76(7):417–422
14. Jähne B (2005) *Digital image processing*, 6th edn. Springer, Berlin
15. Xiao Z, Hou Z (2004) Phase based feature detector consistent with human visual system characteristics. *Pattern Recogn Lett* 25(10):1115–1121
16. Marr D (1982) *Vision: a computational investigation into the human representation and processing of visual information*. W. H. Freeman, San Francisco
17. Gepshtein S, Tyukin I, Kubovy M (2011) A failure of the proximity principle in the perception of motion. *Humana Mente* 17:21–34
18. Ilea DE, Whelan PF (2011) Image segmentation based on the integration of colour-texture descriptors—a review. *Pattern Recogn* 44(10):2479–2501
19. Marquez-Neila P, Baumela L, Alvarez L (2014) A morphological approach to curvature-based evolution of curves and surfaces. *IEEE Trans Pattern Anal Mach Intell* 36(1):2–17
20. Fox V, Milanova M, Al-Ali S (2013) A morphological multiphase active contour for vascular segmentation. *Int J Bioinform Biosci* 3(3):1–12
21. Battiato S, Lukac R (2008) *Video stabilization techniques*. Encyclopedia of multimedia. Springer, New York, pp 941–945
22. Rawat P, Singhai J (2011) Review of motion estimation and video stabilization techniques for hand held mobile video. *Int J Signal Image Process* 2(2):159–168
23. Mironowsky LA, Slaev VA (2011) Strip-method for image and signal transformation. De Gruyter, Berlin
24. Hadamard J (1893) Resolution d'une Question Relative aux Determinants. *Bull Sci Math ser 2* 17(1):240–246
25. Yüksel S, Başar T (2013) *Stochastic networked control systems*. Springer, New York, Heidelberg
26. Govindan M, Tang CM (2010) Information system evaluation: an ongoing measure. *Int J Business Inform Syst* 6(3):336–353

Chapter 2

Morphological Image Analysis for Computer Vision Applications

Y.V. Vizilter, Y.P. Pyt'ev, A.I. Chulichkov and L.M. Mestetskiy

Abstract Some original and novel morphological concepts and tools are presented in this chapter as well as required amount of mathematical morphological basics. The continuous binary morphology based on a computational geometry is presented as a very fast approach to shape representation via real-time computation of figures' skeletons. A skeletal representation of the figure is formed as a skeleton graph, and the radial function is determined in skeleton points. The proposed morphological spectrum is the multi-scale morphological shape description and analysis tools based on granulometry. It is shown how the tasks of change detection and shape matching in images can be solved using a morphological image analysis. The projective morphology as a generalized framework based on the mathematical morphology and the morphological image analysis provides fast and efficient solutions of morphological segmentation problem in complex images.

Keywords Mathematical morphology · Shape representation · Continuous skeleton · Morphological spectrum · Image analysis · Change detection · Image matching · Image segmentation

Y.V. Vizilter (✉)

State Research Institute of Aviation Systems, 7, Viktorenko str., Moscow 125319,
Russian Federation
e-mail: viz@gosniias.ru

Y.P. Pyt'ev · A.I. Chulichkov

M.V. Lomonosov Moscow State University, 1, Leninskie Gory, Moscow 119991,
Russian Federation
e-mail: achulichkov@gmail.ru

L.M. Mestetskiy

Moscow International University, 17, Leningradsky av., Moscow 125040,
Russian Federation
e-mail: mestlm@mail.ru

2.1 Introduction

The morphological framework is very wide theoretical platform for creation of mid-level image analysis tools for specialized computer vision applications. It utilizes the structural image modeling and is useful for some image filtering, segmentation and comparison problems. The foundation of Mathematical Morphology (MM) by Serra and Matheron was in 1960 [1], and this original version of MM with structuring elements, erosion/dilation operators, and monotonous opening/closing filters is still the most well-known version of MM until nowadays. However, the current morphological framework contains more ideas and tools than the initial MM. Some of them are just unknown for computer vision developers and engineers. The purpose of this chapter is to provide a brief sketch of some novel morphological techniques useful for different practical applications. The chapter contains the following issues:

- Basics of mathematical morphology.
- Skeleton-based continuous binary morphology.
- Morphological pattern spectrum: concepts and computation.
- Morphological image analysis (Pyt'ev morphology).
- Projective morphologies, morphological segmentation, and complexity analysis.

The MM is the most well-known morphological technique based on a set theory and (later) a lattice theory. The idempotent operators (morphological filters or projectors) are introduced using concept of figure filling by structuring elements. In other version, the morphological filters are based on merging of grayscale image connected regions (flat zones). A brief description of basic MM notions and concepts is required for understanding of following techniques.

The continuous binary morphology is a skeleton-based approach for description and analysis of figure shapes proposed and developed by Mestetskiy. It is based on a computational geometry and provides very fast tool for real-time computation of continuous figure skeletons using approximation of 2D binary image by region border polygons and calculation of Voronoi diagram to segment these polygons. A skeletal representation of the figure is formed as a skeleton graph, and the radial function is determined in skeleton points. The computational efficiency of such approach is based on the fact that skeleton-based continuous binary morphology uses the finite and relatively small number of analytical structuring elements for representation of binary image shape. Each analytical structuring element is connected with one edge of continuous skeleton.

A morphological spectrum is a multi-scale morphological shape description and analysis tool based on granulometry—a set of filters with different grades. Each of filters provides the details of certain size and shape to pass. The original “Pattern spectrum” proposed by Maragos is based on the Serra MM filters and describes the distribution of local figure thickness. Many modifications and generalizations of this idea are known and utilized now. In this chapter, the fast algorithm for pattern

spectrum calculation using the continuous binary skeletons is described. Such implementation allows to apply the morphological spectra in the real-time machine vision systems.

The Morphological Image Analysis (MIA) proposed by Pyt'ev is well-known in Russia since 1970. It is based on geometrical and algebraic reasoning. In the framework of Pyt'ev morphology, images are considered as piecewise-constant 2D functions. The tessellation of image frame by a set of the non-intersected connected regions with constant intensities determines the "shape" of the image. From mathematical point of view, any shape is a hyperplane in a linear space of images. The crucial idea of this approach is the projection of one image onto the shape of other image. Here a morphological image comparison is performed using the normalized morphological correlation coefficients. The morphological change detection is performed by a comparison of an image and its projection to the reference image. Such morphological tools are invariant relative to transforms of image intensity and stable relative to noise. In this chapter, a morphological shape matching technique is described that generalizes a morphological approach to shape-to-shape comparison.

A projective morphology is a generalized framework based on the Serra mathematical morphology, the Pavel shape theory, and the Pyt'ev morphological analysis. It combines ideas of these morphological approaches and allows to construct some new morphological systems and operators based on different image decompositions and transforms and/or criteria (energy functions). The criterion-based projective morphological filters are implemented using numeric optimization techniques (linear programming, dynamic programming, graph cutting, and so on). The use of morphological shape complexity as a criterion for shape regularization provides tools for shape complexity analysis those are more general than tools based on the MM granulometry concept. In particular, the definitions of the morphological filters and the morphological spectra by complexity are given.

Thus, some original and modern morphological concepts and tools are presented in this chapter as well as required amount of morphological basics. From one hand, this material allows to learn of modern morphology techniques without any previous background in the MM. From the other hand, some tools and techniques those are applicable for real-time technical vision systems, especially for vision systems of moving vehicles and other controlled real-time technical devices with video cameras or several imaging sensors, are selected and presented.

The chapter is organized as follows. The basics of mathematical morphology are discussed in Sect. 2.2. The skeleton-based continuous binary morphology is described in Sect. 2.3. The concept and computation of morphological spectrum are represented in Sect. 2.4. The morphological image analysis (Pyt'ev morphology) is given in Sect. 2.5. Section 2.6 describes the projective morphologies, a morphological segmentation and a complexity analysis. Conclusion is situated in Sect. 2.7.

2.2 Basics of Mathematical Morphology

The MM is a well-known theoretical framework for image processing and shape analysis. It was originally developed for binary image processing, and classically stated in the set-theoretical terms. Then the MM was extended to grayscale images, color images, graphs, among others. At present, the description of the MM in terms of complete lattices is the widest MM theoretical formalism.

The MM was originally developed by Serra [1] in 1964 and Matheron [2] in 1975. In 1960–1970, the set of popular MM operators was proposed including Hit-or-miss transform, dilation, erosion, opening, closing, granulometry, thinning, skeletonization, ultimate erosion, etc. In 1970–1980, some novel MM operators like morphological gradients, top-hat transform, and the watershed were proposed. In 1986, the MM generalization based on complete lattices was proposed by Serra. In 1990–2000, some further theoretical advancement was developed including the concepts of connection and leveling.

The basic MM concepts and operations in order to explore the interconnections between well-known morphological tools and some novel morphological concepts and ideas are briefly introduced in Sect. 2.2.1. A binary morphology and a grayscale mathematical morphology based on structuring elements are introduced in Sects. 2.2.2 and 2.2.3, respectively. The mathematical morphology as a lattice-theoretic scheme is discussed in Sect. 2.2.4. The novel morphological concept based on connected filters is given in Sect. 2.2.5. A building of morphological skeleton is presented in Sect. 2.2.6.

2.2.1 Mathematical Morphology as a Set-Theoretic Scheme

In a set-theoretic terms [1] the MM operations are defined for any Euclidean space E^N equipped by the set-theoretic inclusion (\subset), union (\cup), and intersection (\cap). Any operator (transform) of this space $\Psi: E^N \rightarrow E^N$ is called:

- The increasing, if it preserves the inclusion $(X \subset Y) \Rightarrow (\Psi(X) \subset \Psi(Y))$, $X, Y \subset E^N$.
- The dilation, if it preserves the union $\Psi(\cup X_i) = \cup \Psi(X_i)$, $\forall X_i \subset E^N$.
- The erosion, if it preserves the intersection $\Psi(\cap X_i) = \cap (\Psi(X_i))$, $\forall X_i \subset E^N$.
- The extensive, if $\Psi(X) \supseteq X$, and anti-extensive, if $\Psi(X) \subseteq X$.
- The idempotent (or algebraic projector), if $(\Psi(\Psi(X))) = \Psi(X)$.

All inclusion-preserving operators are called the morphological operators. Well-known Matheron theorem states that any morphological operator can be represented as a union of erosions or as an intersection of dilations. Due to this theorem, the erosion and the dilation are called the basic morphological operators.

The idempotent morphological operators are called the morphological filters:

- The anti-extensive morphological filter is called the opening.
- The extensive morphological filter is called the closing.

Let us consider the original implementation of these terms and notions.