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Andrea Puglisi

Transport and Fluctuations in Granular Fluids

From Boltzmann Equation
to Hydrodynamics,
Diffusion and Motor
Effects



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Transport and Fluctuations in Granular Fluids

From Boltzmann Equation
to Hydrodynamics, Diffusion
and Motor Effects

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A Fabiana e Luca

Preface

The purpose of the book is to discuss nonequilibrium phenomena in fluidized granular materials, with an accent on granular kinetic theory and some of its stochastic extensions. A few other books exist on this subject. The difference in this new one is to provide the reader with a *brief* introduction, which goes through a few salient points in the subject: models for collisions, Boltzmann equation, fundamental boundary conditions, transport equations and hydrodynamics, macroscopic ordered phenomena, the motion of tracer particles, and the breaking of time-reversal symmetry.

This book is *not* a topical review. Therefore reference sections are not meant to be exhaustive. My intent is to offer a selection of starting points, for instance by citing reviews or other books, where the reader will find more detailed bibliographies. This book merges material from two courses given for Ph.D. students at the Physics Department of Sapienza University (2010 and 2012), and a general reorganization of the results of GranularChaos project. GranularChaos is a 5-year-long project (2009–2014) funded by the Italian Ministry for University and Research, after winning the selection at the European Research Council (Starting Grant 2007). The focus of the project is fluctuations in granular media.

I am indebted to Angelo Vulpiani for most of what I learned about nonequilibrium statistical mechanics and granular materials: collaboration with Angelo has always been enjoyable and fruitful. The hint to write this Brief came from him: again an interesting and challenging incitement. Umberto Marini Bettolo Marconi, Alberto Petri and Vittorio Loreto are the other three friends who greatly improved my knowledge and understanding of the subject, since the beginning of my study of granular fluids. Many of the ideas and results contained in this book are due to collaboration and discussions with them, during the last 15 years and more. I wish to say thanks also to Andrea Baldassarri, who shared with me many progresses on granular kinetic models and, in the early years of my doctorate, was an intense stimulus to become a better c-programmer and a more careful researcher. My understanding of granular fluids in the wider context of nonequilibrium steady states has received a great impulse during a stay of 2 years in Orsay (Paris), where I collaborated with Alain Barrat, Emmanuel Trizac, and Frederic van-Wijland,

whom I wish to warmly thank. In the last years I had the exciting possibility, as a coordinator of the GranularChaos project, to interact with brilliant young collaborators, in particular with Giulio Costantini, Giacomo Gradenigo, Alessandro Sarracino, and Dario Villamaina, whom I acknowledge for a constant passion, curiosity, and their many intriguing interrogatives: they shaped my ability to explain and teach. My hope is that, as a consequence, this book will be clear and useful to students and young researchers. The GranularChaos project, thanks to the crucial help of Andrea Gnoli, has allowed me to enter in the fascinating world of real experiments with granular fluids, an experience which has deeply influenced my perspective on this subject. A special acknowledgment goes to Andrea Gnoli, Alessandro Sarracino, Camille Scalliet, and Angelo Vulpiani, who read the manuscript, found plenty of errors, and gave me many useful advices.

Last, but certainly not least, I wish to thank my beloved family, in particular Fabiana (well before that unstoppable and joyful creature which dwells in our house since a couple of years), for their patience, tolerance, and love.

Roma, July 2014

Andrea Puglisi

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Symbols

| | |
|--------------------------------|---|
| σ | Particle's diameter |
| \hat{n} | Unit vector joining particles' centers of mass in a collision |
| λ | Mean free path |
| τ_c | Mean free time between collisions |
| ω_c | Mean collision frequency |
| S | Total scattering cross section |
| v_T | Thermal velocity |
| r | Restitution coefficient |
| N | Number of particles |
| $P(\mathbf{r}, \mathbf{v}, t)$ | Single particle probability density function (normalized to 1) |
| $Q(P, P)$ | Collisional integral in the Boltzmann equation |
| V_{12} | Relative velocity between colliding particles 1 and 2 |
| \mathbf{g} | Modulus of relative velocity between colliding particles projected along \hat{n} |
| $g_2(\sigma)$ | Spatial pair correlation evaluated at contact |
| ϕ | Packing (or "volume") fraction |
| μ_p | p -th velocity moment of the collisional integral |
| ζ | Collisional cooling rate |
| a_2 | Coefficient of the second polynomial in the Sonine expansion |
| T | Granular temperature |
| ε | Small parameter for perturbative expansions, e.g. Knudsen number (Chap. 3) or square root of the mass ratio (Chap. 4) |
| γ | Shear rate |
| η | Viscosity |
| κ | Thermal conductivity |
| μ | Dufour-like thermal conductivity |

| | |
|---------------------------------------|--|
| $\xi_{\perp}, \xi_{\parallel}, \xi_T$ | Correlation lengths related to linearized hydrodynamics |
| ξ, ξ', ξ'' | Noises in fluctuating hydrodynamics |
| U_{\perp} | Component of the (Fourier-transformed) velocity field perpendicular to the wave vector |
| C_{\perp} | Time autocorrelation of U_{\perp} |
| \mathcal{E} | Noise in Langevin equations for tracer's dynamics |

Introduction and Motivation

A granular material is a substance made of grains, i.e., many macroscopic particles with a spatial extension (average diameter) that ranges from tenths of microns to millimeters. In line of principle the size of grains is not limited as far as their behavior can be described by classical mechanics. For example, the physics of planetary rings (made of objects with a diameter far larger than centimeters) is sometimes studied with models of granular media. More often the term “granular” applies to industrial powders: in chemical or pharmaceutical industries the problem of mixing or separating different kinds of powders is well known; the problem of the transport of pills, seeds, concretes, etc., is also widely studied by engineers; the prevention of avalanches or the study of formation and motion of desert dunes are the subject of important studies, often involving granular theories; silos containing granular products from agriculture sometimes undergo to dramatic breakages, or more often their content become irreversibly stuck in the inside, because of huge internal force chains; the problem of diffusion of fluids through densely packed granular materials is vital for the industry of natural combustibles; the study of ripples formations in the sand under shallow seawaters can solve important emergencies on many coasts of the world. Rough estimates of the losses suffered in the world economy due to ignorance of granular *laws* amount to billions of dollars a year.

The study of granular materials dates back to the nineteenth century, with the first studies by Coulomb in 1773, Faraday in 1831, Reynolds in 1885 and much more recently by Bagnold, who really opened the way to the systematic study of granular rheology. For decades granular systems have been a blessing and a curse for engineers. In the last 50 years they have become more and more present in physics laboratories. The rise of computer simulations has led to a huge increase of interest in the study of realistic granular models.

In parallel, a closer look at the fundamental properties of granular media (inelasticity of collisions and entropic constraints) motivated the introduction of new minimal models. These “granular cartoons” have the remarkable charm of displaying an intriguing behavior in spite of their simplicity. Granular gases represent a noteworthy example. As for spin glasses, some models of granular gas