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Chiara Esposito

Formality Theory

From Poisson Structures to Deformation Quantization

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Chiara Esposito
Department of Mathematics
University of Würzburg
Würzburg
Germany

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Preface

This book aims to be a survey of the theory of formal deformation quantization of Poisson manifolds, in the formalism developed by Kontsevich. It is intended to be a pedagogical introduction to mathematicians and mathematical physicists who are first approaching the subject. The main topics are the theory of Poisson manifolds, star products and their classification, deformations of associative algebras, and the formality theorem. Furthermore, the aim is to introduce the reader to the relevant physical motivations behind the purely mathematical constructions.

Despite the fact that deformation quantization is a broad research area, the pedagogical literature dealing with this topic, and in particular formal deformation quantization, is limited.

The construction of star products on symplectic manifolds can be found in [3], a monograph focused on Fedosov's approach. Recently, S. Waldmann [6] wrote a more extensive book on the argument, which is only available in German. Further interesting reviews on the subject can be found, for instance, in [1, 2, 4, 5]. In particular, [1] is a sketchy introduction to formal deformation quantization. A detailed, although brief, presentation of the Kontsevich theory from a purely mathematical point of view is included in [2, 4, 5]. It has to be noted that these reviews are oriented to researchers with a basic knowledge of Poisson geometry. There, the authors describe Kontsevich's formality theorem and its relation to the existence and classification of star products on a Poisson manifold. In [4] the author introduces the theory of star products and their classification on symplectic manifolds. The notes in [2, 5] are also focused on further developments of Kontsevich's theory. In particular, the author in [5] also introduces the Tamarkin approach.

The above list, although not exhaustive, represents the state of the art of the introductory literature on formal deformation quantization. At the present time, a book that covers both the basic mathematical tools and Kontsevich's theory, and one that is also designed for first-time readers, is missing. These notes aim to fill this gap writing a didactical exposition to the subject for nonexperts without assuming from the reader too many prerequisites. In particular, this book is addressed to mathematical physicists, who have basic knowledge of differential geometry, classical, and quantum mechanics.

We describe the Poisson structures and their role in classical mechanics, present the results of Kontsevich on the classification of star products on Poisson manifolds, and discuss the physical motivations underlying this theory.

Würzburg, May 2014 Chiara Esposito

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