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# Temperature- and Supply Voltage-Independent Time References for Wireless Sensor Networks

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*The only reason for time is so that  
everything doesn't happen at once.*

—A. Einstein

# Preface

“Most of what makes a book ‘good’ is that we are reading it at the right moment for us” is a quote from the Swiss-British philosopher Alain De Botton. The quote is right on top for a book where timing is the central given. Read this book about timing if at this moment your design is in need of an accurate yet efficient timing reference!

The book you are holding is the result of several years of Ph.D. research that started with the ambition to set important steps toward the implementation of batteryless wireless sensor nodes. Relatively soon it became clear that accurate timing is one of the key points in realizing this. Therefore the focus shifted to the construction of a low power, supply, and/or temperature independent timing reference. This topic in its turn surpasses the world of wireless sensor networks as it is much wider applicable.

We are convinced that the results we present here can help you in several ways. You can have a look at the circuits and concepts and adapt them for your system. More importantly the book can act as a source of inspiration for all those that are involved in sensor network design and the hardware for the Internet of Things. We dare to hope that the book brings you as much research inspiration as it brought us research joy while creating it.

Leuven, September 2014

Valentijn De Smedt  
Georges Gielen  
Wim Dehaene

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# Abstract

Back in 2000, the Nobel prize committee recognized the invention of the integrated circuit in 1958 by Jack Kilby as one of the most far-reaching steps forward in modern technology. Today, almost 60 years after this invention, electronics are found everywhere in our society. This is mainly caused by the characteristic exponential growth factors in the electronics industry (Moore's law), which result in an exponential miniaturization and cost decrease. This evolution goes hand in hand with a similar growth of wireless communication technology: devices become smaller, frequencies and data rates higher. As a result, the wearability and functionality of wireless electronic devices drastically increase.

This ongoing technological progress is a direct cause of the appearance of Wireless Sensor Networks (WSN). An increasing number of autonomously operating devices is wirelessly connected to a network and/or to the Internet, an evolution which will eventually result in the so-called Internet of Things. Since both the miniaturization as well as the cost decrease of these wireless sensor nodes is necessary to become economically feasible, a growing need for fully-integrated, single-chip wireless devices is observed. The use of modern deep-submicron CMOS technologies in analog electronics, however, has several drawbacks in terms of temperature sensitivity and linearity.

This work elaborately investigates the possible circuit techniques to overcome the temperature and supply voltage sensitivity of fully integrated time references for ultra-low-power wireless communication in WSN. In a first step, the basic needs to build a frequency reference are studied. Furthermore, a closer look at the short-term as well as the long-term frequency stability of integrated oscillators is taken. This results in a design strategy, which is applied to six different oscillator design cases. All six implementations are subject to a study of phase noise and long-term frequency stability.

The first two implementations are respectively a temperature- and a supply voltage-independent Wien bridge oscillator. The temperature independence is obtained by using a novel feedback amplifier topology of which the output resistance only depends on a temperature stable resistor. This requires advanced circuit techniques and a highly-stable amplitude regulation circuit. The second Wien

bridge implementation makes use of two nested regulators, resulting in an ultra-high supply voltage stability over a wide voltage range.

The third design case makes use of a high-quality bondwire LC tank. A novel pulsed driving technique is developed to decrease the power consumption of the high-frequency oscillator circuit. This driving technique reduces the impact of the oscillator circuitry on the output frequency and therefore also on the temperature- and supply voltage stability of the oscillator. To better understand the application field of the pulsed oscillator topology, the noise performance is analyzed as well. The processed implementation is a unique combination of power consumption and long-term frequency stability.

Next, two injection-locked oscillator implementations are discussed. Apart from a stable output frequency, a high absolute accuracy is also obtained due to the locking to a wirelessly received RF signal. The first design uses the received 2.4 GHz carrier frequency as a time reference. Despite its simple system topology, this approach has several drawbacks in terms of selectivity and power consumption. The second implementation locks to the envelope of the received RF signal. Therefore, the oscillator can run at a low frequency, drastically diminishing the power consumption. A second improvement is the addition of a network coordination receiver. For this purpose, a novel ultra-low-power receiver topology and demodulation technique are developed. As a result of the addressability, the overall power consumption in the network is reduced.

The last design case is a temperature- and supply voltage-independent oscillator-based sensor interface. Since the challenge in this design is rather the stability of the output value than the frequency stability, a different design strategy is used. It is shown that the matching of different oscillator delay stages can be applied to obtain a stable and highly-linear digitalization of a sensor input.

The wirelessly injection-locked oscillator, the coordination receiver, the sensor interface, and a transmitter are combined into one highly-flexible wireless tag. The content, the scrambling code, and the length of the transmitted data burst can be adapted freely, depending on the application. The developed tag can therefore be used in a wide range of applications, with different accuracy requirements and energy constraints.

Finally, an elaborate comparison between the developed oscillator designs and the state of the art is performed. It is shown that the free-running implementations as well as the injection-locked designs improve the state of the art. This discussion results in several suggestions for possible future work.

## About the Authors



**Valentijn De Smedt** (S'08) was born in Lubbeek, Belgium, in 1984. He received the M.Sc. degree in electrical engineering from the Katholieke Universiteit Leuven in 2007. The subject of his Master thesis was the design of an accurate integrated frequency reference. From 2007 to 2014 he was working as a research assistant at the MICAS laboratories of the Katholieke Universiteit Leuven towards a Ph.D. degree on the design of ultra-low-power time-based building blocks for wireless sensor networks, which he received in April 2014. At KU Leuven, he was involved in and has set up several extra-curricular educational projects, some of them in co-operation with the IEEE Student Branch of Leuven.

He has been vice-chair technical activities of the IEEE student branch of Leuven between 2009 and 2013 and chaired the IEEE Student Branch and GOLD congress 2010 (SBC 2010). Since 2011, he is IEEE Benelux GOLD (Young Professionals) chair and co-chair of the IEEE SSCS Benelux chapter. Since 2009 he is a guest lecturer at ACE Group-T on UWB standards and Zigbee.



**Georges G.E. Gielen** received the M.Sc. and Ph.D. degrees in Electrical Engineering from the Katholieke Universiteit Leuven (KU Leuven), Belgium, in 1986 and 1990, respectively. He is a full professor at the Department of Electrical Engineering (ESAT). From August 2013, Georges Gielen is also appointed as vice-rector for the Group Science, Engineering and Technology and Academic Personnel of the KU Leuven.

His research interests are in the design of analog and mixed-signal integrated circuits, and especially in analog and mixed-signal CAD tools and design automation. He is a coordinator or partner of several (industrial) research projects in

this area, including several European projects. He has authored or coauthored seven books and more than 450 papers in edited books, international journals, and conference proceedings. He is a Fellow of the IEEE since 2002.



**Wim Dehaene** was born in Nijmegen, The Netherlands, in 1967. He received the M.Sc. degree in electrical and mechanical engineering in 1991 from the Katholieke Universiteit Leuven. In November 1996, he received the Ph.D. degree at the Katholieke Universiteit Leuven. His thesis is entitled “CMOS integrated circuits for analog signal processing in hard disk systems.”

After receiving the M.Sc. Degree, Wim Dehaene was a research assistant at the ESAT-MICAS Laboratory of the Katholieke Universiteit Leuven. His research involved the design of novel CMOS building blocks for hard disk systems. The

research was first sponsored by the IWONL (Belgian Institute for Science and Research in Industry and Agriculture) and later by the IWT (the Flemish institute for Scientific Research in the Industry). In November 1996, Wim Dehaene joined Alcatel Microelectronics, Belgium. There he was a senior project leader for the feasibility, design and development of mixed mode systems on chip. The application domains were telephony, xDSL, and high-speed wireless LAN. In July 2002, Wim Dehaene joined the staff of the ESAT-MICAS Laboratory of the Katholieke Universiteit Leuven where he is now a full professor. His research domain is circuit level design of digital circuits. The current focus is on ultra low power signal processing and memories in advanced CMOS technologies. Part of this research is performed in cooperation with IMEC, Belgium where he is also a part-time principal scientist.

Wim Dehaene is teaching several classes on electrical engineering and digital circuit and system design. He is also very interested in the didactics of engineering. As such, he is guiding several projects aiming to bring engineering to youngsters and he is a teacher in the teacher education program of the KU Leuven.

Wim Dehaene is a senior member of the IEEE. Wim Dehaene is a member of the technical program committee of ESSCIRC and ISSCC.

# Abbreviations

A-ISF	Amplitude Impulse Sensitivity Function
AC	Alternating Current
AM	Amplitude Modulation
BBPLL	Bang–Bang Phase-Locked Loop
BER	Bit Error Rate
BJT	Bipolar Junction Transistor
CMB	Cosmic Microwave Background
DAC	Digital-to-Analog Converter
EEF	Energy Enhancement Factor
ELP	Extremely-Low Power
ENOB	Effective Number of Bits
ETF	Electro-Thermal Filter
FET	Field-Effect Transistor
FLL	Frequency-Locked Loop
FoM	Figure of Merit
FRS	Fellow of the Royal Society
FSM	Finite State Machine
IFF	Identification Friend or Foe
IoT	Internet of Things
IR-UWB	Impulse Radio Ultra-Wideband
ISF	Impulse Sensitivity Function
JFET	Junction Field-Effect Transistor
LDO	Low-DropOut
LFSR	Linear Feedback Shift Register
LTI	Linear Time-Invariant
LTV	Linear Time-Variant
MEMS	Microelectromechanical systems
MiM	Metal-insulator-Metal

MoI	Moment of Impact
MoM	Metal-oxide-Metal
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor
OCXO	Oven-Compensated Crystal Oscillator
P-UWB	Pulsed Ultra-Wideband
PC	Personal Computer
pcb	Printed Circuit Board
PDF	Probability Density Function
ppb	Parts per billion
ppm	Parts per million
PSD	Power Spectral Density
PVT	Process, Temperature and (Supply) Voltage
PW	Pulse Width
RF	Radio Frequency
RFID	Radio Frequency Identification
rms	Root mean square
SNDR	Signal to Noise and Distortion Ratio
SNR	Signal to Noise Ratio
TANSTAAFL	There Ain't No Such Thing As A Free Lunch
TCXO	Temperature-Compensated Crystal Oscillator
ULP	Ultra-Low Power
VCO	Voltage-Controlled Oscillator
VCXO	Voltage-Controlled Crystal Oscillator
WSN	Wireless Sensor Network
XO	Crystal Oscillator

# Symbols

$\alpha(\omega \cdot t)$	Deterministic, periodic function with period $2\pi$
$\alpha_k$	Phase shift of the $k$ -th harmonic of a voltage oscillator signal
$\alpha_{\mu_n}$	Temperature coefficient of the electron mobility
$\alpha_{V_{th}}$	Temperature coefficient of the transistor threshold voltage
$\beta$	Phase modulation index
$\beta_k$	Phase shift of the $k$ -th harmonic of a voltage oscillator signal
$\Delta\omega_{3dB}$	$-3$ dB width of a resonant peak in a transfer function
$\Delta f_{-3dB}$	Half-power width of an oscillator output spectrum
$\Delta T_n$	Delay error induced by noise
$\Delta V_n$	Output voltage error induced by noise
$\Delta V_{rel}$	Relative supply voltage span, as defined in Sect. 4.3.2
$\mathcal{A}(\Delta\omega)$	Amplitude noise density at frequency offset $\Delta\omega$ , relative to the carrier
$\Delta\omega_0$	Difference between the fundamental frequency $\omega_0$ and the natural frequency $\omega_n$ of the tuned network
$\Delta\omega_n$	Angular frequency difference between the natural oscillator frequency $\omega_n$ and the frequency of an injected signal $\omega_i$
$\Delta\omega_{inj}$	Frequency perturbation on the injected signal $i_{inj}$ , equal to $d\phi_{inj}(t)/dt$
$\Delta\omega_{nn}$	Frequency perturbation on the free-running oscillator signal (not the injection-locked oscillator signal), equal to $d\phi_{nn}(t)/dt$
$\Delta\phi_i(t)$	Perturbation of the phase shift between an oscillator current and an injected current (due to noise), equal to $\Delta\phi_{inj}(t) - \Delta\phi_{osc}$
$\Delta\phi_{inj}(t)$	Perturbation on the phase of the injected signal $i_{inj}$
$\Delta\phi_{osc}$	Perturbation on the phase of the (locked) oscillator signal

$\delta_k$	Relative amplitude of the $k$ -th harmonic of the voltage waveform, compared to the amplitude of the fundamental frequency
$\delta_{Xx}$	Skin-depth of a conducting material $Xx$ (for instance Silver, Ag)
$\varepsilon$	Parameter equal to $A_{inj}/A_{osc}$
$\varepsilon(t)$	Random amplitude variation of an oscillator signal
$\gamma$	Constant parameter used to determine the bandwidth of a Lorentzian spectrum, equal to $\pi \cdot f_0^2 \cdot c$
$\Gamma(x)$	The Impulse Sensitivity Function (ISF) with period $2 \cdot \pi$
$\Gamma_i(x)$	Impulse Sensitivity Function (ISF) on node $i$ with period $2 \cdot \pi$
$\gamma_k$	Relative amplitude of the $k$ -th harmonic of the voltage waveform, compared to the amplitude of the fundamental frequency
$\Gamma_{eff}(x)$	Effective Impulse Sensitivity Function (ISF), taking cyclo-stationary noise sources into account
$\Gamma_{rms}$	Root mean square (rms) value of the Impulse Sensitivity Function (ISF)
$\Lambda(x)$	Amplitude Impulse Sensitivity function (A-ISF), with period $2 \cdot \pi$
$\lambda_i$	Eigenvalue with corresponding eigenvector $\mathbf{u}_i$
$\Lambda_i(x)$	Amplitude Impulse Sensitivity function (A-ISF) on node $i$ , with period $2 \cdot \pi$
$\rho$	Generalized eigenvector with corresponding eigenvalue $\lambda$
$\mu$	Scalar nonlinearity parameter in the <i>van der Pol</i> equation
$\mu_{Xx}$	Magnetic permeability of a conducting material $Xx$ (for instance Silver, Ag)
$\Omega$	Unit of electrical resistance
$\omega$	Instantaneous angular frequency, pulsation, $2\pi/\tau$
$\omega(t)$	Instantaneous angular frequency over time
$\omega_0$	Constant mean angular frequency of an oscillator, often used as the angular frequency in standard conditions
$\omega_B$	Beating frequency of an injected oscillator
$\omega_i$	Angular frequency of an injected signal
$\omega_L$	One-sided lock range of an injection-locked oscillator
$\omega_m$	Angular frequency of a motor
$\omega_n$	Natural angular frequency of an electrical (tuned) network, natural angular frequency of an oscillator, $2\pi f_n$
$\Phi$	magnetic flux due to the magnetic poles in a motor
$\Phi(t)$	Instantaneous phase of an oscillator signal
$\phi(t)$	Random variations of noise in the phase function, also called excess phase function
$\phi_\infty$	Steady-state phase shift between the oscillator current and the injected current

$\phi_i(t)$	Phase shift between the oscillator current and the injected current
$\phi_{ETF}$	Phase shift of an Electro-Thermal Filter
$\phi_{i,0}$	Initial phase shift between the oscillator current and the injected current
$\phi_{inj}(s)$	Phase shift of the injected signal in the frequency domain
$\phi_{nn}(s)$	Phase shift of the free-running oscillator signal in the frequency domain
$\phi_{nn}(t)$	Phase shift of the free-running oscillator signal (not the injection-locked oscillator signal)
$\phi_{osc}(s)$	Phase shift of the locked oscillator signal in the frequency domain
$\psi$	Phase shift between the injected signal and the relaxation oscillator current
$\Psi(t)$	Systematic or deterministic variations in the phase function
$\rho_{Xx}$	Conductivity of a conducting material $Xx$ (for instance Silver, $Ag$ )
$\sigma(A, f, \tau)$	Allan variance of the output of a function $f$ with dead time $\tau$ between the subsequent samples
$\sigma(T_h, T_l)$	Covariance between two stochastic periods
$\sigma_c$	Cycle-to-cycle jitter of an oscillator output signal
$\sigma_{A,h,l}$	Allan covariance between two variables $T_h$ and $T_l$ (with dead time $\tau$ )
$\sigma_{abs}(t = N \cdot \tau_{avg})$	Standard deviation of the absolute jitter after $N$ oscillation cycles, sometimes also called the absolute jitter
$\sigma_{cc}$	Alternative definition of the cycle-to-cycle jitter, calculated using the difference between subsequent periods
$\sigma_{LC,Cte}$	Average cycle-to-cycle jitter of the output waveform of a resonant tank with constant output amplitude
$\sigma_{LC,Pulsed,N}$	Standard deviation of the length of the pulsed period of an LC tank, normalized to $T_{LC}/2$
$\sigma_{LC}$	Average cycle-to-cycle jitter of the output waveform of a resonant tank with decaying output amplitude
$\sigma_{T_n}$	Standard deviation (rms) of the time noise on a stage delay
$\sigma_{V_n}$	Standard deviation (rms) of the voltage noise
$\tau$	An arbitrary moment in time
$\tau_i$	Period of the $i$ -th oscillation cycle
$\tau_{avg}$	Average period of an oscillator output signal
$\theta(t)$	Phase shift of an oscillation over time (compared to $\omega_n \cdot t$ )
$\theta_0$	Initial phase shift of an oscillation
$\theta_i$	Phase shift induced by a small injected signal in the oscillator, angle between the oscillator signal and the resulting signal
$\theta_n$	Phase shift of a tuned network at $\omega \neq \omega_n$

$\Upsilon(t)$	Deterministic and systematic amplitude variations of an oscillator signal
$A$	Voltage gain of a generic amplifier
$A_0$	Ideal constant amplitude of an oscillator signal
$A_1$	Linear gain of a voltage dependent transconductance amplifier
$A$	Ampère, unit of electrical current
$A_D$	Area or size of a diode junction
$A_{ds}$	Amplitude of the drain-source current in an oscillator
$A_{env,DC}$	DC approximation of the complex envelope of an oscillator output signal
$A_{env}(t)$	Complex envelope of an oscillator output signal
$A_i$	$i$ -th-order gain of a voltage dependent transconductance amplifier
$A_{inj}$	Amplitude of a signal injected in an oscillator
$A_{osc}$	Amplitude of an oscillator signal current
$A_R$	Area or size of a resistor
$A_r$	Amplitude of the resulting oscillator signal, sum of the injected current and the oscillator current
$A(t)$	Instantaneous amplitude of an oscillator signal
$A_t$	Minimum attenuation of a feedback network
$A_v$	Voltage gain of an amplifier
$B$	Bandwidth of a modulating signal
$C$	Capacitance, a capacitor
$c$	Motor constant, speed of light
$c$	Constant parameter used to determine the bandwidth of a Lorentzian spectrum
$C_{ds}$	Drain-source capacitor of a MOS transistor
$C_{gd}$	Gate-drain capacitor of a MOS transistor
$C_{gs}$	Gate-source capacitor of a MOS transistor
$c_i$	Complex $i$ -th pole frequency
$C_{ox}$	Gate-oxide capacitance per unit area of a MOS transistor
$C_p$	Parallel capacitor in the equivalent quartz crystal model
$C_s$	Series capacitor in the equivalent quartz crystal model
$DC$	Duty cycle of a digital signal
$e$	Euler's constant
$E_{Decay}(N)$	Energy loss over $N$ oscillation cycles in a resonant tank with exponentially decaying output amplitude
$E_M$	Back electromotive force
$E_{Osc}(N)$	Energy loss over $N$ oscillation cycles in a resonant tank with constant output amplitude
$f(\Phi(t))$	Periodic function of $\Phi(t)$ with period $2 \cdot \pi$
$F$	Device excess noise factor
$F_{amp}$	Noise factor of an amplifier

$F$	Farad, unit of electrical capacitance
$f_i$	State variable of an oscillation on node $i$ , normalized to amplitude and frequency
$f_i(x_1, \dots, x_n)$	A smooth real-valued function of $x_1, \dots, x_n$
$FoM_{AD}$	Figure of Merit for Analog to Digital converters
$FoM_{JT}$	Jitter and Temperature-dependency Figure of Merit (FoM) of an oscillator
$FoM_{PN}$	Phase noise Figure of Merit (FoM) of an oscillator
$FoM_{PN,tuned}$	Phase noise Figure of Merit (FoM) of an oscillator, taking the tuning sensitivity into account
$FoM_{PNT}$	Phase noise and Temperature-dependency Figure of Merit (FoM) of an oscillator
$FoM_{PNTV}$	Phase noise, Temperature-dependency and Voltage-dependency Figure of Merit (FoM) of an oscillator
$FoM_{TV}$	Temperature- and Voltage-dependency Figure of Merit (FoM) of an oscillator
$FoM_V$	Voltage-dependency Figure of Merit (FoM) of an oscillator
$G$	Transconductance of a generic amplifier
$g_m$	Transconductance of a transistor
$g_{mb}$	Transconductance of the bulk of a transistor
$G_m(v)$	Gain of a nonlinear negative resistance or amplifier
$g_{m,wi}$	Transconductance of a MOS transistor in weak-inversion
$G(s)$	Transconductance of a generic amplifier, dependent on the Laplace variable (representing the frequency)
$G(v)$	Input-amplitude-dependent transconductance of a generic amplifier
$h_\phi(t, \tau)$	Unit impulse response for the excess phase for a charge injected at time $\tau$
$h_A(t, \tau)$	Unit amplitude impulse response for a charge injected at time $\tau$
$H$	Henry, unit of electrical inductance
$H_{inj}(j \cdot \Delta\omega)$	Transfer function of the phase perturbations on the injected signal $i_{inj}$ to the injection-locked oscillator output
$H_n(j \cdot \Delta\omega)$	Transfer function of the phase perturbations on the free-running oscillator signal $i_{osc}$ to the injection-locked oscillator output
$h_n(t)$	Normalized impulse response of a resonant network
$H(s)$	Transfer function of a linear system or network
$h(t)$	Impulse response of a linear system or network
$I$	Unity matrix, symbol of electrical current, mechanical moment of inertia
$I_A$	Current amplitude through an inductor
$I_b$	Biasing current of an oscillator or an oscillator stage

$I_{D0}$	Technology dependent parameter determining the weak-inversion current of a MOS transistor
$i_{d,n}(t)$	Small-signal differential noise current
$I_{ds}$	Large-signal drain-source current of a MOS transistor
$i_{ds}$	Small-signal drain-source current of a MOS transistor
$I_{dsf}$	Zero-temperature-coefficient DC biasing current of a transistor
$I_{ds,wi}$	Large-signal drain-source current of a MOS transistor in weak-inversion
$i_{inj}(t)$	Injected current in an oscillator
$I_{i,reg}$	Regulated output current of a current regulation circuit
$I_{i,reg,rep}$	Regulated output current through the oscillator replica when the regulator is loaded with a resistor
$I_k$	Fourier coefficient of a current oscillator signal
$\Im(x)$	The imaginary part of $x$
$I_n$	Amplitude of a noise current
$i_{n0}(t)$	Stationary current noise source
$i_n(t)$	Injected small-signal noise current
$i_{osc}$	Oscillator signal current
$i_r(t)$	Resulting current in an oscillator, sum of the injected current and the oscillator current
$I(s)$	Laplace transform of a current waveform
$i(t)$	Small-signal current
$j$	Imaginary unit, $\sqrt{-1}$
$j_{abs}(t = N \cdot \tau_{avg})$	Absolute jitter after $N$ oscillation cycles
$J(d)$	Current density in a conductor, depending on the distance $d$ from the surface
$J$	Joule, unit of energy
$J_s$	Current density in a conductor at the surface
$k$	The Boltzmann constant, approximately equal to $1.38 \text{ m}^2 \cdot \text{kg} \cdot \text{s}^{-1} \cdot \text{K}^{-1}$
$k_{100}$	Conversion gain of an envelope detector for an input signal with 100 % modulation depth
$k_{30}$	Conversion gain of an envelope detector for an input signal with 30 % modulation depth
$k_{DC}$	Conversion gain of an envelope detector for a DC (low-frequency) input signal
$K_{fD}$	Technology dependent $1/f$ noise fitting parameter of a diode
$K_{fR}$	Technology dependent $1/f$ noise fitting parameter of a resistor
$K_{fT}$	Technology dependent $1/f$ noise fitting parameter of a transistor
$K_{ij}$	Residue of a partial fraction corresponding to real pole $i$ with $j$ -th degree denominator

$K_{ILO}$	Injection-locked oscillator gain
$K_{mix}$	Mixing gain of a mixer circuit
$K_{VCO}$	Integration constant or sensitivity of a VCO to its control voltage $v_c$
$L$	Inductance, an inductor
$L$	Length of a MOS transistor
$\mathcal{L}(\Delta\omega)$	Phase noise density at frequency offset $\Delta\omega$ , relative to the carrier
$L_{ij}$	Residue of a partial fraction corresponding to complex pole $i$ with $j$ -th degree denominator
$\mathcal{L}_{total}(\Delta\omega)$	Total noise power density at frequency offset $\Delta\omega$ , relative to the carrier
$MoI$	Moment of Impact of a pulse applied to a resonant tank
$MoI_N$	Moment of Impact of a pulse applied to a resonant tank, normalized to $T_{LC}/2$
$N$	Number of free-running periods in a Pulsed-Harmonic oscillator
$NF_{amp}$	Noise figure of an amplifier
$N_{LF}$	Noise spectral density of the low-frequency noise at an amplifier output (expressed in $V^2/Hz$ )
$N_{o,ED}$	Noise spectral density of the noise at the output of an envelope detector (expressed in $V^2/Hz$ )
$N_{src}$	Noise spectral density of an input source (expressed in $V^2/Hz$ )
$p$	Frequency of a single pole
$p_i$	Real $i$ -th pole frequency
$P(s)$	Numerator of a transfer function $H(s)$
$P_s$	Average power dissipated in a resonant tank
$P_{sbc}(\Delta\omega)$	Single-sideband noise power at offset frequency $\Delta\omega$ , relative to the carrier
$\mathcal{P}_{side}(f_0 + \Delta f, 1 \text{ Hz})$	Single-sideband noise power in a 1 Hz interval at a frequency offset $\Delta f$ from the carrier
$PW$	Pulse width of a pulse applied to a resonant tank
$PW_N$	Pulse width of a pulse applied to a resonant tank, normalized to $T_{LC}/2$
$Q$	Quality factor of a resonant (second order) network
$q$	Unit charge of a single charge carrier (electron or hole)
$Q_C$	Quality factor of a capacitor $C$
$q_{Cte}$	Maximum charge displacement (compared to equilibrium) in a resonant network with constant output amplitude during one period
$q_{Decay}$	Maximum charge displacement (compared to equilibrium) in a resonant network with exponentially decaying output amplitude during one period

$Q_G$	Generalized Q factor
$Q_L$	Quality factor of an inductor $L$
$q_{max}$	Maximum charge displacement from the equilibrium state on the output node of a resonant tank during one oscillation cycle
$Q(s)$	Denominator of a transfer function $H(s)$
$R$	Resistance, a resistor
$r_0$	Small signal output (drain) impedance of a MOS transistor (also called $r_{ds}$ )
$R_B$	Bulk resistance of a transistor
$R_D$	Drain resistance of a transistor
$r_{ds}$	Small signal output (drain) impedance of a MOS transistor (also called $r_0$ )
$R_{eff}$	Effective noise resistance of a transistor
$\Re(x)$	The real part of $x$
$R_G$	Gate resistance of a transistor
$R_p$	Equivalent parallel resistance of an RLC network
$R_S$	Source resistance of a transistor
$R_s$	Series resistance of an inductor
$s$	Laplace variable, equal to $\sigma + j \cdot \omega$ , in steady-state equal to $j \cdot \omega$
$s$	Second, unit of time
$S_{\phi,f}(f)$	Power Spectral Density of the phase fluctuations as a result of the $1/f$ noise component
$S_a(f)$	Power Spectral Density of a waveform $a$
$S_{i_n}(f)$	Power Spectral Density of a current noise source $i_n$
$S_{inj}(\omega)$	Power spectral density of the injected signal $i_{inj}$
$S_{nn}(\omega)$	Power spectral density of the free-running oscillator signal
$S_{osc}(\omega)$	Power spectral density of the locked oscillator signal $i_{osc}$
$S_{RF}(f)$	Power Spectral Density of a general RF signal
$S_{V_n}(f)$	Power Spectral Density of the voltage noise on a capacitor
$S_{v_n}(f)$	Power Spectral Density of a voltage noise source $v_n$
$S_x(f)$	Power Spectral Density of phase time fluctuations
$S_y(f)$	Power Spectral Density of fractional frequency fluctuations
$S_\phi(f)$	Power Spectral Density of phase fluctuations
$S_{\Delta f}(f)$	Power Spectral Density of frequency fluctuations
$S_{\Delta\omega}(f)$	Power Spectral Density of angular frequency fluctuations
$T$	Absolute temperature, expressed in Kelvin [K] or (depending on the context), the oscillation period
$t$	Time variable
$T_0$	Period of an oscillator
$T_D$	Delay of an oscillator stage
$t_d$	Delay of an inverter

$T_h$	Time span that a digital oscillator output is high during one period
$T_{h,0}$	Time span that a digital oscillator output is high during one period with a sensor input signal equal to zero
$t_{hl}$	Input-output delay of an amplifier for a falling edge at the output
$T_{i,reg}$	Loopgain of a current regulation circuit
$T_l$	Time span that a digital oscillator output is low during one period
$t_L$	Time needed for an injection-locked oscillator to lock, lock time of an injection-locked oscillator
$T_{l,0}$	Time span that a digital oscillator output is low during one period with a sensor input signal equal to zero
$T_{LC}$	Period of a free-running LC tank
$t_{lh}$	Input-output delay of an amplifier for a rising edge at the output
$T_M$	Motor torque
$T_{Osc}$	Complete period of a multi-stage oscillator
$T(s)$	Frequency dependent loopgain of a feedback system
$T_{Sens}$	Sensitivity of the output frequency $f_0$ to temperature
$T_{Switch}$	Time interval needed for a differential oscillator stage to switch
$T(t)$	Phase time
$T_{v,reg}$	Loopgain of a voltage regulation circuit
$U$	Symbol of electrical tension
$\mathbf{u}_i$	Eigenvector with corresponding eigenvalue $\lambda_i$
$u(t)$	The Heaviside function
$V_A$	Voltage amplitude over a capacitor
$V_{bs}$	Large-signal bulk-source voltage of a MOS transistor
$v_{bs}$	Small-signal bulk-source voltage of a MOS transistor
$v_c(t)$	Control voltage of a VCO
$V_{Cte}$	Constant input voltage
$V_{ctrl}$	Control voltage to control the gain of an amplifier
$V_d$	Differential voltage applied to the input of a differential pair
$V_{dd}$	Supply voltage
$V_{ds}$	Large-signal drain-source voltage of a MOS transistor
$v_{ds}$	Small-signal drain-source voltage of a MOS transistor
$V_E$	Early-voltage of a MOS transistor
$V_{gs}$	Large-signal gate-source voltage of a MOS transistor
$v_{gs}$	Small-signal gate-source voltage of a MOS transistor
$V_{gs,f}$	Zero-temperature-coefficient DC biasing voltage of a transistor
$V_{gt}$	Overdrive voltage of a MOS transistor, equal to $V_{gs} - V_{th}$
$V_i$	Output voltage of the $i$ -th oscillator stage

$V_{i,reg}$	Regulated output voltage of a current regulation circuit
$V_k$	Fourier coefficient of a voltage oscillator signal
$V_{max}$	Maximum supply voltage at which a circuit can properly operate
$v_{max}$	Maximum output voltage (compared to equilibrium) of a resonant tank during one oscillation cycle
$V_{min}$	Minimum supply voltage at which a circuit can properly operate
$v_{n,diff}$	Differential noise voltage
$v_n(t)$	Small-signal noise voltage
$V_{ref}$	Output voltage of a voltage reference
$V_{reg}$	Output voltage of a voltage regulator
$V_{rep}$	Output biasing voltage coming from a replica circuit
$V(s)$	Laplace transform of a voltage waveform
$V_S$	Output voltage of a sensor, input voltage of a sensor interface
$V_{S,DC}$	DC value of the sensor output voltage
$V_{Sens}$	Sensitivity of the output frequency $f_0$ to the supply voltage
$V_{S-}$	Negative voltage output of a differential sensor
$V_{S+}$	Positive voltage output of a differential sensor
$v(t)$	Small-signal voltage
$v(t - \tau)$	Decay function of the excess amplitude
$V_T$	Threshold voltage of a relaxation oscillator
$V_t$	Thermal voltage, equal to $k \cdot T/q = 26$ mV at room temperature
$V_{th}$	Threshold voltage of a MOS transistor
$V$	Volt, unit of electrical tension
$V_{v,reg}$	Regulated output voltage of a voltage regulation circuit
$V_{v,reg,rep}$	Regulated output voltage over the oscillator replica when the regulator is loaded with a resistor
$V_{v,reg,res}$	Regulated output voltage of a voltage regulation circuit loaded with a resistor
$W$	Width of a MOS transistor
$x$	Normalized variable of the ISF $\Gamma(x)$ , typically equal to $\omega_n \cdot t$
$\dot{x}_i$	Time-derivative of $x_i$
$x_i(t)$	A real-valued function of $t$
$x(t)$	Random instantaneous phase time variation, $\phi(t)/\omega_0$
$y(t)$	Instantaneous fractional frequency variation, $dx(t)/dt$
$Z$	Complex impedance
$z$	Scalar for which $z \in \mathbb{Z}$
$Z_k$	Impedance of a resonant network, seen by the $k$ -th harmonic