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Radio Systems Engineering

A Tutorial Approach

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Este libro lo dedico a mis queridos padres
y a mis queridos Violeta, Mara, Hectorcito,
y Joseph.

Héctor J. De Los Santos

Para Yoyis y Suki

Juan Pontes

"Y sabemos que a los que aman a Dios todas las cosas les ayudan a bien, esto es, a los que conforme a su propósito son llamados."

Romanos 8:28

Preface

This book is the outgrowth of the course "Modern Radio Systems Engineering," developed and taught by the Dr. H.J. De Los Santos during the 2010/2011 Winter Semester, while he held a German Research Foundation *Mercator Visiting Professorship* at the Institute for High-Frequency Engineering and Electronics (IHE), Karlsruhe Institute of Technology/University of Karlsruhe (TH), Germany. In the German system, the courses consist of lectures and exercises; there is no homework! Both lectures and exercises are held once a week for one and one-half hours. The teaching process entails the literal transfer of information to the student, meaning that all the material required to be learned in a given course must be explicitly given to the student in lecture notes handouts. The American concept of urging/demanding that students learn material by themselves, e.g., "read this chapter on your own," "convince yourself by deriving a result by yourself," etc., is nonexistent. This book attempts to recreate this learning experience. In particular, the same team who carried out the lecturing and tutorial exercises in the above course, namely, Dr. H.J. De Los Santos, and former Research Associates (now Drs.) Dr.-Ing. C. Sturm and Dr. Ing. J. Pontes, respectively, has reunited once more to collaborate in producing this book.

The course is aimed at upper-level undergraduates/first-year graduate students, who already have knowledge of devices and circuits for radio-frequency (RF) and microwave communications and are ready to study the systems engineering-level aspects of modern radio communications systems. In particular, the course gives a general overview of radio systems, together with their components. In this context, the focus is on the analog parts of the system, with their non-idealities. Based on the physical functionality of the various building blocks of a modern radio system, block parameters are derived, which allows the examination of their influence on the overall system performance.

The chapters of the book are complemented by tutorial exercises, based on the Keysight SystemVue electronic system-level (ESL) design software. In these tutorials, the readers gain practical experience with slightly simplified real-world design examples of radio systems, both in the area of communications as well as radar sensing. The tutorials cover state-of-the-art system standards and applications and consider the characteristics of typical radio-frequency hardware components. For all tutorials, a comprehensive description of the tasks, including some hints to the solutions, is provided. The readers are then intended to perform these tasks independently. Then, complete simulation models and solutions to the tutorial exercises is given.

Radio Systems Engineering: *A Tutorial Approach* fills a niche not addressed by previous books; in fact, it would make an excellent prerequisite for them. For instance, previous books are typically aimed at advanced graduate students and practicing engineers. As a result, they tend to be specialized by providing an in-depth focus on specific applications and technologies, so as to be a resource to individuals developing these applications. This book, on the other hand, facilitates the integration of the knowledge gathered by undergraduate students during their pre-Senior years, which primarily focuses on devices and circuits, and gives them their first exposure to their exploitation in engineering an overall system to fulfill real-life performance requirements.

Radio Systems Engineering: *A Tutorial Approach* contains nine chapters. Chapter 1 starts by providing an overview of wireless communication systems, defining the fundamental wireless communications problem and motivating approaches to its solution. Chapter 2 introduces system-level block diagrams of Amplitude Modulation (AM) and Frequency Modulation(FM)/Phase Modulation (PM) modulators and demodulators, and explains their respective principles of operation. Chapter 3 addresses a number of topics surrounding system performance parameters as well as a system-level description of component models for systems analysis. Chapter 4 discusses the fundamentals of the radio channel and a discussion of antenna parameters. Chapter 5 deals with the topics of Noise, Nonlinearity and Time Variance as it pertains to relating the performance of building blocks to that of the overall system built as a cascade of them. Chapter 6 focuses on the topics of sensitivity and dynamic range for a receiver, including how the performance of the individual building blocks impacts the overall system. Chapter 7, addresses the topics of transmitter and receiver architectures, and practical aspects impacting the performance of oscillators. Chap. 8 integrates the knowledge presented in Chaps. 1–7 by engaging into a "case study" tutorial discussion that exposes the engineering considerations and thought processes behind the design of two real-life receivers, implemented after the WCDMA and LTE standards, respectively. Chapter 9, includes five tutorial exercises based on the Keysight SystemVue software tool, a free limited- time license of which may be downloaded from http://www.keysight.com/find/eesof-radio-systems-engineering.

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April, 2014

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Chapter 1
Introduction to Radio Systems

Abstract In this chapter, an overview of wireless communication systems is presented. We begin with a definition of the fundamental wireless communications problem and motivate approaches to its solution. Following definitions of carrier, baseband, and modulation, the simplified block diagram of a transmitter, together with a description of its typical building block elements, is presented. We then focus on the mathematical description and corresponding spectral properties of amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM). This is followed by a study of differences among modulation schemes, their bandwidth, and their vectorial representation. In particular, the separate cases of narrow- and wideband FM, and phase modulation (PM) are addressed.

1.1 Overview of Wireless Communication Systems

The fundamental problem of wireless communications consists in transferring information between a *source* and a *destination*, Fig. 1.1 [1, 2].

If the signal representing the information to be transmitted is in *analog* form, i.e., exists at all times, a straightforward way to accomplish this information transfer would be to feed this signal to an antenna (at the source), which will convert it into an electromagnetic (EM) wave and radiate it into space. If the signal at the source represents, for example, *music or voice*, which have a maximum frequency content of about $f = 20\ kHz$, this would mean that, for reasonable efficiency, the transmitting antenna required would have dimensions of half-wavelength given by,

$$\frac{\lambda}{2} = \frac{c}{2f} = \frac{3\times 10^8\, m/s}{2(20\times 10^3/s)} = 7000m = 4.35mi \tag{1.1}$$

where λ is the wavelength and c is the speed of light. Obviously, the required antenna length of this example, namely, 4.35 miles, would be impractical. This state of affairs is circumvented in practice by the process of *modulation*.

Modulation is the *process* by which the information whose transmission is desired is impressed upon a *high-frequency signal*. The set of frequencies that comprise the

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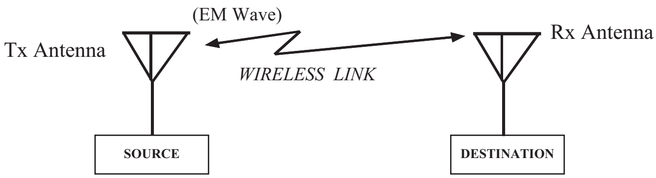


Fig. 1.1 Depiction of the wireless communications problem. *Tx* is transmitter; *EM* is electromagnetic; *Rx* is receiver; *Source* is the place of transmission; *Destination* is the place of reception

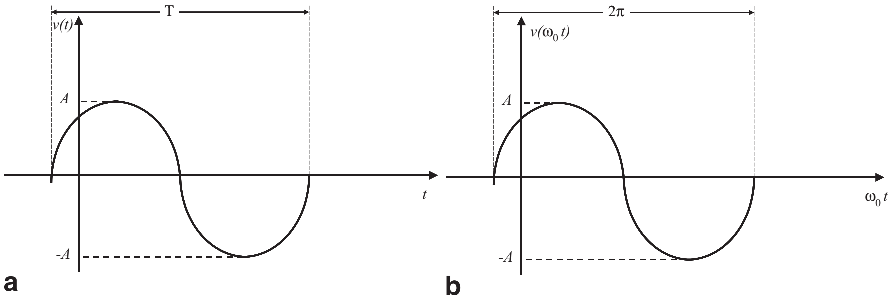


Fig. 1.2 Depiction of carrier signal. **a** As function of time. **b** As function of phase

desired information to be transmitted, $m(t)$, in its original form, is referred to as the *baseband* signal. On the other hand, the continuous high-frequency wave which carries the baseband signal is referred to as the *carrier*. The modulation of the carrier is effected when the amplitude, the frequency, or the phase of the carrier is made to vary in a manner dictated by the baseband signal.

The general equation for a continuous single-frequency time function is given by,

$$v(t) = A\cos(\omega_0 t + \varphi_0) = A\cos\left[\varphi(t)\right] \tag{1.2}$$

which may be represented as the function of time or phase $v(t)$ as depicted in Fig. 1.2.

The parameters in (1.2) embody the following. The factor A quantifies the peak amplitude of $v(t)$, the parameter ω_0 is called the radial frequency of the time function described by (1.2), and the time, T, shown in Fig. 1.2a, is called the *period* of the cosine function; it is the time required to change the phase of the time function by 2π radians. Differentiation of (1.2) with respect to time yields the fundamental relationship,

$$\omega_0 = \frac{d\varphi}{dt} \tag{1.3}$$

where, $\varphi = \omega_0 t + \varphi_0$. From Fig. 1.2 and (1.2) it is seen that a time change of T changes the phase by 2π, i.e.,

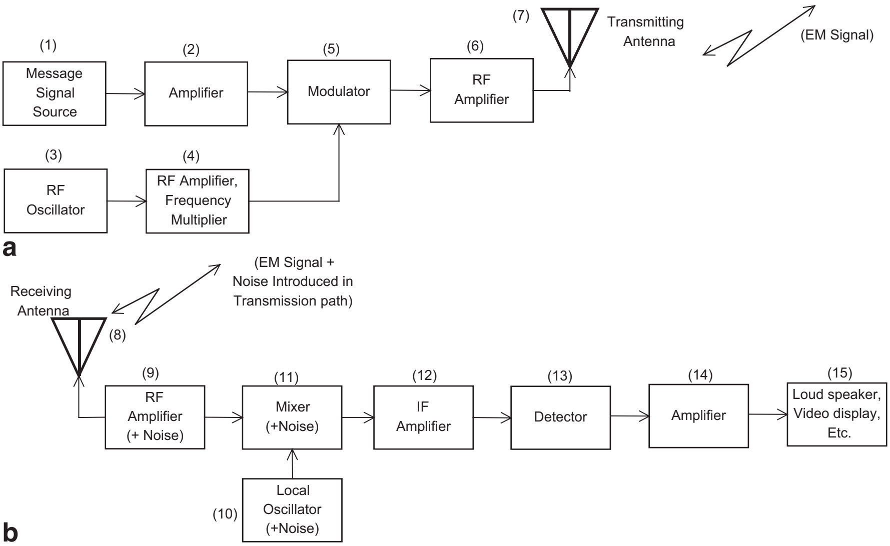


Fig. 1.3 Simplified block diagram of **a** transmitter, and **b** of a receiver

$$\omega_0(t+T)+\varphi_0-(\omega_0 t+\varphi_0)=2\pi \tag{1.4}$$

from where one obtains the relationship,

$$\omega_0 T = 2\pi \tag{1.5}$$

or

$$T = \frac{2\pi}{\omega_0} = \frac{1}{f_0} \tag{1.6}$$

where f_0 is the frequency of the continuous wave (CW) measured in *Hertz*. The relation between radial and cyclical frequency is given by,

$$f_0 = \frac{\omega_0}{2\pi} \tag{1.7}$$

1.2 Simplified Block Diagram of Transmitter and Receiver

In this section we begin elaborating on the constituents of the fundamental wireless communications system of Fig. 1.1. In particular, Fig. 1.3 depicts a simplified transmitter and receiver system [1, 2].

The source of the information in Fig. 1.3 (block 1), may be a microphone, a video camera, a temperature sensor, an accelerometer, a seismic sensor, a fluid level sensor or any other device (*transducer*) that transforms the desired information into an electrical signal.

The electrical (baseband) signal, in turn, is amplified (block 2) and usually passed through a lowpass filter to *limit* its bandwidth. The carrier frequency, or a sub-multiple of it, is generated by an RF oscillator (block 3), which is then multiplied and amplified (block 4) to establish the desired frequency. Due to the need for operation at the precise assigned frequency, the oscillator stability is typically controlled by a highly stable resonator, such as a quartz crystal.

One or more amplifier stages may be employed to increase the power level of the signal from that produced by the oscillator to that needed for input to the modulator (block 5). A variety of power amplification topologies may be used for obtaining high efficiency, for example, class C [2].

The modulator takes in two inputs, namely, the information-bearing (baseband) signal and the carrier frequency, and produces the modulated output carrier. If a higher power level than the one produced by the modulator is desired, additional amplification (block 6) may be added so the desired power level to be transmitted by the antenna (block 7) is reached.

On the other hand, the transmitted information is captured by a receiver, Fig. 1.3b. The receiving antenna (block 8) utilized may be *omnidirectional* for general service, or *highly directional* for point-to-point communication. The received *wave induces* a small voltage in the receiving antenna, with amplitudes ranging from tens of millivolt to less than a microvolt, depending on a wide variety of conditions, in particular, the nature of the intervening transmitter-receiver space or channel. Notice that, en-route to the receiver, the transmitted signal picks up *noise* from the environment; this refers to random signals which alter the amplitude, phase, or frequency of the transmitted carrier. The received signal delivered by the antenna is *amplified by a low noise amplifier* (LNA) (block 9) to increase the signal power to a level appropriate for input to a mixer. The LNA also provides isolation between the local oscillator (LO) (block 10) and the antenna, as well as increasing the received signal amplitude to overcome the noise that is inevitably introduced in the mixer (block 11). The mixer is a nonlinear circuit which produces multiples of the sum and difference of the RF and local oscillator (LO) signal frequencies, thus frequency-translating the received carrier signal, f_{RF}, to the intermediate frequency, f_{IF}, where demodulation is to be effected. In the receiver architecture shown, the LO is tuned to produce a frequency that differs from the incoming signal frequency f_{RF} by the intermediate frequency f_{IF}; in other words, f_{LO} can be equal to either $f_{RF}+f_{IF}$ or $f_{RF}-f_{IF}$. The IF amplifier (block 12) increases the signal amplitude to a level appropriate for detection, and provides most of the frequency selectivity necessary to "pass" the desired signal and reject the undesired signals that are found in the mixer's output spectrum. The detector (block 13) extracts the original message from the modulated IF input. The extracted signal is amplified (block 14) to an amplitude that is appropriate for driving a loudspeaker, a television tube, or other output device. The output transducer (15) converts the signal information back to its original form, e.g., a sound wave, a picture, etc.

1.3 Basic Modulation Definitions in Mathematical Terms

In the process of modulation, a property of the carrier signal is varied under the influence of the baseband signal. In particular, given the mathematical representation of the carrier by,

$$v(t) = A\cos(\omega_0 t + \varphi_0) = A\cos\left[\varphi(t)\right] \tag{1.8}$$

three parameters in this equation may be made independently time-varying, namely, the amplitude A, giving rise to Amplitude Modulation (AM), the frequency, giving rise to Frequency Modulation (FM), or the phase, giving rise to Phase Modulation (PM). To make these possible variations explicit in (1.8), we begin by assuming that the radial frequency, rather than being a constant, is a function of time, i.e.,

$$\omega = \frac{d\varphi}{dt} \tag{1.9}$$

which, when integrated, gives as the phase of the cosine wave,

$$\varphi = \int_0^t \omega dt + \varphi_0 \tag{1.10}$$

Permitting also the amplitude A to be a time-dependent variable, one obtains, following substitution of (1.10) into (1.8),

$$v(t) = A(t)cos\left(\int \omega dt + \varphi_0\right) \tag{1.11}$$

We now address the mathematical representation of each of these modulation schemes, AM, FM and PM [1].

1.3.1 Amplitude Modulation (AM)

If, in (1.11), the radial frequency is held constant and the amplitude, $A(t)$, allowed to vary in the manner given by,

$$A(t) = a_0\left[1 + mg(t)\right] \tag{1.12}$$

then, the time function $v(t)$ is said to be *amplitude modulated*. In (1.12), a_0 is a constant, m is the *modulation index*, and $g(t)$ is the *modulation function*.

1.3.2 Frequency Modulation (FM)

In frequency modulation, as its name implies, one holds the amplitude A constant and allows the radial frequency to vary in a manner given by,

$$\omega(t) = \omega_0 \left[1 + mg(t)\right] \tag{1.13}$$

Here, ω_0 is a constant, m is the modulation index, and $g(t)$ is the modulation function.

1.3.3 Phase Modulation (PM)

In phase modulation, the *envelope A* is held constant and the phase is allowed to vary in the fashion dictated by,

$$\varphi(t) = \omega_0 \left[1 + \varphi_0 mg(t)\right] \tag{1.14}$$

1.4 Spectral Properties of the Basic Modulation Schemes

In addition to enabling the transmission of information by a relatively small antenna, the process of modulating a high-frequency carrier signal yields peculiar spectral characteristics; these are presented next.

1.4.1 Amplitude Modulation Spectrum

For simplicity, we first consider a baseband modulation function, $g(t)$, that consists of a pure cosinusoid, i.e.,

$$g(t) = cos\ p_1 t \tag{1.15}$$

where p_1 is the baseband radial frequency. Successive substitutions of (1.15) into (1.12) into (1.11) yields (with $\omega = \omega_0, \phi_0 = 0$),

$$v(t) = a_0 \left[1 + m\, cos\ p_1 t\right] cos\ \omega_0 t \tag{1.16}$$

If one now uses the trigonometric identity,

$$cos\, x\, cos\, y = \frac{1}{2}\left[cos(x+y) + cos(x-y)\right] \tag{1.17}$$

one has, in place of (1.17),

$$v(t) = a_0 \left[cos\ \omega_0 t + \frac{m}{2} cos(\omega_0 + p_1)t + \frac{m}{2} cos(\omega_0 - p_1)t\right] \tag{1.18}$$

From (1.15) and (1.18), we find that the process of amplitude modulation has *changed* the radian frequency from baseband, p_1, to two *sidebands*, $\omega_0 + p_1$, $\omega_0 - p_1$,

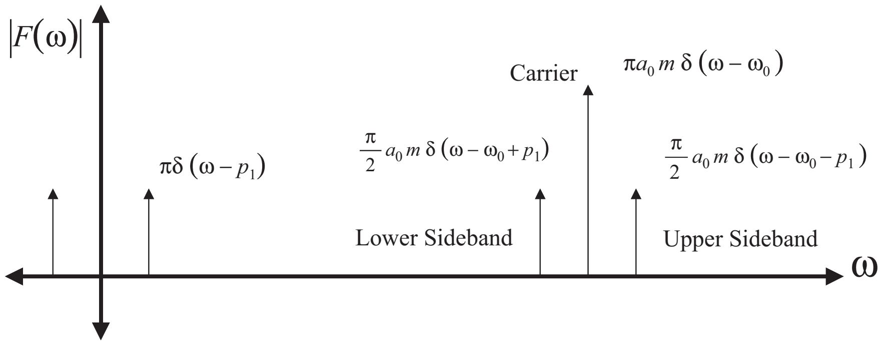


Fig. 1.4 Spectrum of amplitude modulated single-frequency signal. Whereas the single-frequency carrier has a spectrum represented by the delta function $\pi\delta(\omega - p_1)$, the spectrum of the modulated carrier has a spectrum represented by $\pi a_0 m\delta(\omega - \omega_0)$, the lower sideband $\frac{\pi}{2}a_0 m\delta(\omega - \omega_0 + p_1)$, and the upper sideband $\frac{\pi}{2}a_0 m\delta(\omega - \omega_0 - p_1)$

centered at the carrier radial frequency, ω_0. This results in the spectrum depicted in Fig. 1.4.

Rather than a single cosinusoid, let us next assume that the baseband modulation function is *general* in form. In this case, taking its Fourier transform results in,

$$F_B(\omega) = \int_{-\infty}^{\infty} mg(t)e^{-j\omega t}\,dt \tag{1.19}$$

On the other hand, for the Fourier transform of the modulated time function we obtain, similarly,

$$F_M(\omega) = a_0 \int_{-\infty}^{\infty} [1 + mg(t)]\cos\,\omega_0 t e^{-j\omega t}\,dt \tag{1.20}$$

Upon expansion, this yields,

$$F_M(\omega) = \frac{a_0}{2}\left[2\pi\delta(\omega - \omega_0) + 2\pi\delta(\omega + \omega_0) + m\int_{-\infty}^{\infty} g(t)\left\{e^{-j(\omega-\omega_0)t} + e^{-j(\omega+\omega_0)}\right\}\right]dt. \tag{1.21}$$

$$F_M(\omega) = \frac{a_0}{2}\left[2\pi\delta(\omega - \omega_0) + 2\pi\delta(\omega + \omega_0) + F(\omega - \omega_0) + F(\omega + \omega_0)\right] \tag{1.22}$$

where,

$$F(\omega \mp \omega_0) = m\int_{-\infty}^{\infty} g(t)e^{-j(\omega\mp\omega_0)t}\,dt \tag{1.23}$$

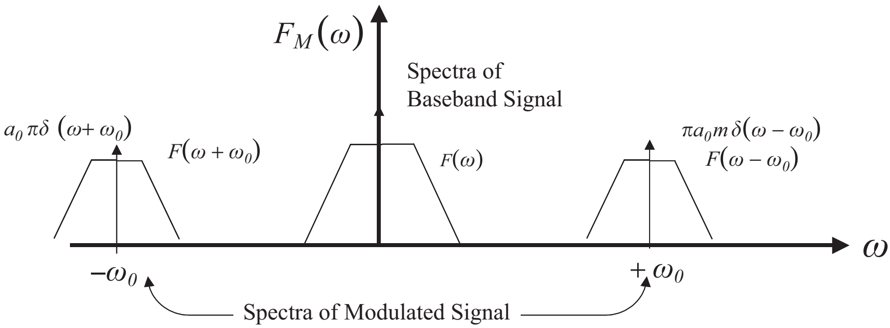
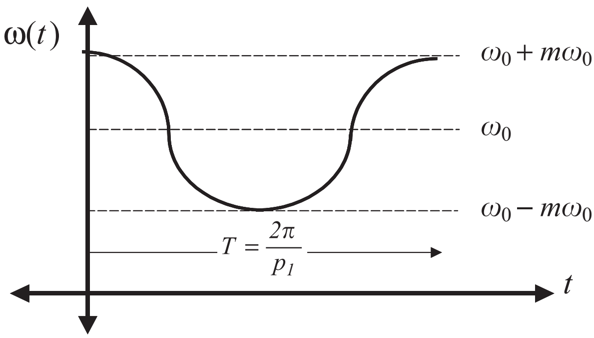


Fig. 1.5 Spectrum of amplitude modulated baseband signal

Fig. 1.6 Instantaneous frequency of FM signal



represents a shift in frequency from baseband to a frequency near the carrier. This results in the spectrum depicted in Fig. 1.5.

1.4.2 Frequency Modulation Spectrum

We begin our discussion of the mathematical properties of FM by writing the FM time function in its general form,

$$v(t) = A\cos\left[\int \omega_0(1+mg(t))dt + \varphi_0\right] \tag{1.24}$$

and then, restricting the modulation function to a single cosinusoid, with $\varphi_0 = 0$, (1.24) becomes,

$$v(t) = A\cos\left[\omega_0 t + \frac{m\omega_0}{p_1}\sin p_1 t\right] \tag{1.25}$$

For this special case, a plot of *instantaneous* frequency, $\omega = \dfrac{d\varphi}{dt}$, yields Fig. 1.6.

The ratio of the maximum deviation in instantaneous frequency, $\Delta\omega_{MAX}$, to the input modulating frequency, p_1, is called the *modulation index*, β or M_p. From Fig. 1.6 and (1.25), we have,

$$\beta \equiv \frac{\Delta\omega_{MAX}}{p_1} = \frac{m\omega_0}{p_1} = M_p \tag{1.26}$$

The substitution of (1.26) into (1.25) gives,

$$v(t) = A cos\left[\omega_0 t + M_p \ sin\ p_1 t\right] \tag{1.27}$$

If we now expand (1.27), making use of the trigonometric identity,

$$cos(x+y) = cos\,x\ cos\,y - sin\,x\ sin\,y \tag{1.28}$$

we obtain, in place of (1.27),

$$v(t) = A\,cos\ \omega_0 t\,cos(M_p\ sin\ p_1 t) - A\,sin\ \omega_0 t\,sin(M_p\ sin\ p_1 t) \tag{1.29}$$

A generic function for ordinary *Bessel functions* of the first kind is given by,

$$e^{\pm jM_p\,sin\,p_1 t} = J_0(M_p) + 2\sum_{k=1}^{\infty} J_{2k}(M_p)\,cos\,2kp_1 t \pm 2j\sum_{k=0}^{\infty} J_{2k+1}(M_p)\,sin\,(2k+1)p_1 t \tag{1.30}$$

This, in conjunction with the exponential equations for the sine and cosine functions, $sin\,x = \dfrac{e^{jx} - e^{-jx}}{2j}$ and $cos\,x = \dfrac{e^{jx} + e^{-jx}}{2}$, allows us to write, with, $x = M_p sin\ p_1 t$,

$$sin(M_p\ sin\ p_1 t) = 2\sum_{k=0}^{\infty} J_{2k+1}(M_p)\,sin(2k+1)p_1 t \tag{1.31}$$

and

$$cos(M_p\ sin\ p_1 t) = J_0(M_p) + 2\sum_{k=1}^{\infty} J_{2k}(M_p)\,cos(2k)p_1 t \tag{1.32}$$

Substitution of (1.31) and (1.32) into (1.29) yields,

$$v(t) = A cos\ \omega_0 t\left[J_0(M_p) + 2\sum_{k=1}^{\infty} J_{2k}(M_p) cos 2kp_1 t\right] - A sin\ \omega_0 t\left[2\sum_{k=0}^{\infty} J_{2k+1}(M_p) sin(2k+1)p_1 t\right] \tag{1.33}$$

If (1.33) is divided by the constant A, we obtain as the first few terms of the series,

$$\begin{aligned}\frac{v(t)}{A} &= J_0(M_p)\cos\,\omega_0 t - 2J_1(M_p)\sin p_1 t\sin\omega_0 t \\ &+ 2J_2(M_p)\cos\,\omega_0 t\cos 2p_1 t - 2J_3(M_p)\sin 3p_1 t\sin\omega_0 t \\ &+ 2J_4(M_p)\cos\,\omega_0 t\cos 4p_1 t - 2J_5(M_p)\sin 5p_1 t\sin\omega_0 t\end{aligned} \tag{1.34}$$

If the product terms are now changed to sum and difference terms, we have,

$$\begin{aligned}\frac{v(t)}{A} &= J_0(M_p)\cos\,\omega_0 t - 2J_1(M_p)\left[\cos(\omega_0+p_1)t - \cos(\omega_0-p_1)t\right] \\ &+ J_2(M_p)\left[\cos(\omega_0+2p_1)t + \cos(\omega_0-2p_1)t\right] + \\ &+ J_3(M_p)\left[\cos(\omega_0+3p_1)t - \cos(\omega_0-3p_1)t\right] + \\ &+ J_4(M_p)\left[\cos(\omega_0+4p_1)t + \cos(\omega_0-4p_1)t\right] + .\end{aligned} \tag{1.35}$$

The comparison of (1.35) with (1.18) reveals certain differences, which we now explore.

1.4.3 Differences Between AM and FM

It has been shown, see (1.18), that amplitude modulation (AM) produces a *single* set of sidebands. An inspection of (1.35), however, reveals that FM, on the other hand, produces an *infinite* number of sidebands. Each sideband is *separated* from the carrier by a *frequency* kp_1, where k is an integer and p_1 is the modulating frequency. Another more subtle difference between these two forms of modulation relates to the partitioning of power between the carrier and the sidebands; this subtlety is now shown. The total average power contained in any time function is given by,

$$\bar{P}_T = \frac{1}{T}\int_{T\to\infty} |v(t)|^2\, dt \tag{1.36}$$

For the cosinusoidal AM wave given by,

$$v(t) = a_0\left[\cos\,\omega_0 t + \frac{a_0 m}{2}\cos(\omega_0+p_1)t + \cos(\omega_0-p_1)t\right] \tag{1.37}$$

since the cosine function is orthogonal, the integral of cross-product terms obtained when (1.37) is squared, are zero. Therefore,

$$\bar{P}_T = \frac{a_0^2}{2} + \frac{a_0^2 m^2}{8} + \frac{a_0^2 m^2}{8} \tag{1.38}$$

Table 1.1 Amplitudes of FM wave components

$\Delta\omega_M$	p_1	M_p	$J_0(M_p)$	$J_1(M_p)$	$J_2(M_p)$	$J_3(M_p)$	$J_4(M_p)$	$J_5(M_p)$	$J_6(M_p)$	$J_7(M_p)$
1000	3000	1/3	0.9725	0.1644	0.03					
1000	1500	1/2	0.8930	0.3138	0.06					
1000	1000	1	0.7652	0.4401	0.1149	0.0196				
1000	500	2	0.2339	0.5767	0.3528	0.1289	0.034			
1000	333	3	−0.2501	0.3391	0.4861	0.3091	0.1320	0.043		
1000	250	4	−0.3971	−0.06604	0.3641	0.4302	0.4302	0.1321	0.0491	
1000	100	5	−0.1776	−0.3276	0.1697	0.2404	0.3912	0.2611	0.1311	0.0533

where the first term represents the power in the carrier, and the second and third terms that in the lower and upper sidebands, respectively. From (1.38) we find that the power contained in the carrier is *independent* of the properties of the modulating function (captured by *m*). The situation is somewhat altered in the case of FM waveforms. For FM waves, since the envelope is constant, the total power becomes,

$$\bar{P}_T = \frac{A^2}{2} \tag{1.39}$$

It follows, from (1.39), that in an FM wave the total average power, carrier plus sidebands, is *constant*. If (1.35) is squared, and the orthogonality property used, we obtain the relationship,

$$J_0^2(M_p) + 2\sum_{k=1}^{\infty} J_k^2(M_p) = 1 \tag{1.40}$$

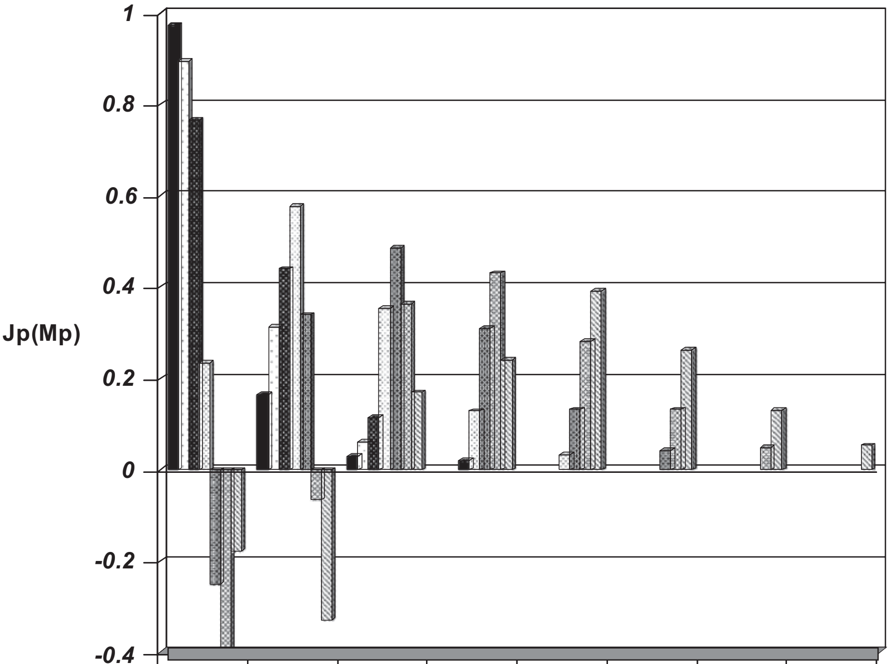
In (1.40), $J_0^2(M_p)$ is the *fraction* of total power in an FM wave that is contained in the carrier. Since $M_p = \frac{\Delta\omega_M}{p_1}$, the carrier power is clearly *dependent* on the properties of the modulating function. The following example illustrates some of the properties of a cosinusoidal FM wave. Let's take $\Delta\omega_M = 1000s^{-1}$, and allow p_1 to vary between 3000 and 200 s^{-1}. This produces the values shown in Table 1.1, and the histogram shown as Fig. 1.7.

It is observed that all significant sidebands are contained within a bandwidth, called the *FM bandwidth*, given by [1],

$$\omega_T \approx 2(\Delta\omega_M + p_1) \tag{1.41}$$

1.4.4 Vector Representations

It has been found that insight into the behavior and properties of the AM and FM modulation schemes may be attained by representing the pertinent waveforms by rotating phasors; these are introduced next.



	p=0	p=1	p=2	p=3	p=4	p=5	p=6	p=7
Mp=1/3	0.9725	0.1644	0.03					
Mp=1/2	0.893	0.3138	0.06					
Mp=1	0.7652	0.4401	0.1149	0.0196				
Mp=2	0.2339	0.5767	0.3528	0.1289	0.034			
Mp=3	-0.2501	0.3391	0.4861	0.3091	0.132	0.043		
Mp=4	-0.3971	-0.066	0.3641	0.4302	0.2811	0.1321	0.0491	
Mp=5	-0.1776	-0.3276	0.1697	0.2404	0.3912	0.2611	0.1311	0.0533
	J_0(Mp)	J_1(Mp)	J_2(Mp)	J_3(Mp)	J_4(Mp)	J_5(Mp)	J_6(Mp)	J_7(Mp)

Fig. 1.7 Amplitude of FM sidebands (Bessel coefficients) as function of modulation index, M_p. p is the order of the Bessel function (0 through 7)

1.4.4.1 AM Vector Representation

We begin our treatment of this subject by considering the complex quantity, $e^{j\omega_0 t}$. In particular, we can think of this as a two-dimensional vector of unit amplitude rotating at an angular speed ω_0, where projection of the vector on the real axis is the cosine function, and its projection on the imaginary axis the sine function. This is shown in Fig. 1.8.

Let us now write the cosinusoidal AM wave as,

$$v(t) = Re\left\{a_0(1 + m\cos p_1 t)e^{j\omega_0 t}\right\} \tag{1.42}$$

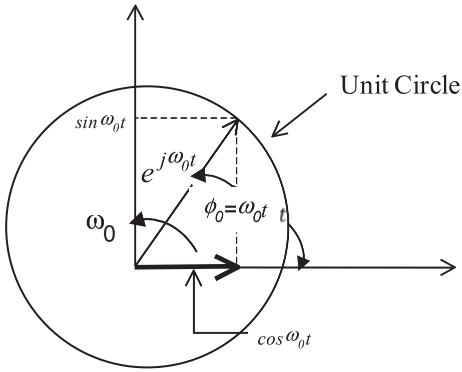


Fig. 1.8 Vector representation of complex quantity $e^{j\omega_0 t}$

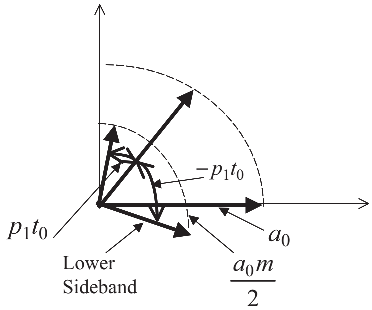


Fig. 1.9 Vector representation of (1.44) at time t_0

where *Re* signifies the "real part of." If the cosine term in (1.42) is written as,

$$cos\ p_1 t = \frac{e^{jp_1 t} + e^{-jp_1 t}}{2} \tag{1.43}$$

and substituted back into (1.42), we have,

$$v(t) = Re\left\{a_0 e^{j\omega_0 t} + \frac{a_0 m}{2} e^{j(\omega_0 + p_1)t} + \frac{a_0 m}{2} e^{j(\omega_0 - p_1)t}\right\} \tag{1.44}$$

From (1.44), we find that amplitude modulation with a cosinusoid produces two additional *vectors* with radial frequencies corresponding to upper and lower sidebands. Let us now freeze the motion of the vectors at some arbitrary time, t_0. Then, the lengths and directions would appear as shown in Fig. 1.9.

It is obvious, from (1.44) and Fig. 1.9, that the two vectors comprising the sidebands produce a composite vector that is in phase with the carrier. This is shown in Fig. 1.10.