

Marco Ceccarelli
Victor A. Glazunov *Editors*

Advances on Theory and Practice of Robots and Manipulators

Proceedings of ROMANSY 2014 XX
CISM-IFTToMM Symposium on Theory
and Practice of Robots and Manipulators

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Editors

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Proceedings of ROMANSY 2014 XX
CISM-IFTToMM Symposium on Theory
and Practice of Robots and Manipulators

 Springer

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Preface

ROMANSY 2014, the 20th CISM-IFTToMM Symposium on Theory and Practice of Robots and Manipulators has been the twentieth event of a series that was started in 1973 as a first conference activity in the world on Robotics. The first event was held at International Centre for Mechanical Science (CISM) in Udine, Italy on 5–8 September 1973. It was also the first topic conference of International Federation for the Promotion of Mechanism and Machine Science (IFTToMM) and it was directed not only at the IFTToMM community.

The ROMANSY aim was decided at the funding meeting as a conference event representing a forum of reference for discussing the latest advances in Robotics and for facilitating contacts among research people, scholars, students, and professionals from the Industry. From the beginning the acronym ROMANSY was used to name the symposium in short by using the first letter of the words: Robots, Manipulators, and Symposium.

The aim of the ROMANSY Symposium is still focused to bring together researchers, industry professionals, and students from broad ranges of disciplines referring to Robotics, in an intimate, collegial, and stimulating environment. In 2014, after 41 years the ROMANSY event still is very attractive since we have received increased attention toward the initiative, as can be seen by the fact that this Proceedings volume contains contributions by authors from all around the world.

The funding committee that also took the responsibility for the organization of the first ROMANSY event was composed of:

- Prof. A. E. Kobrinskii (USSR); Chair
- Prof. L. Sobrero (Italy), Vice-Chair and CISM Director
- Acad. I. I. Artoboleveskii (USSR); first IFTToMM President
- Prof. G. Bianchi (Italy)
- Prof. I. Kato (Japan)
- Prof. M. S. Konstantinov (Bulgaria)
- Prof. A. Morecki (Poland)
- Prof. A. Romiti (Italy)
- Prof. B. Roth (USA)

Prof. M. W. Thring (UK)
 Dr. M. Vukobratovic (Yugoslavia)
 Prof. H. J. Warnecke (Germany)
 Prof. D. E. Withney (USA)
 Mrs. A. Bertozzi (Italy) acting as secretary as from CISM

The funders were the first generation in ROMANSY and they have been active for several decades. Smoothly, new generations have contributed to the leadership of ROMANSY by preserving the original characters of the initiative but challenging to increase the influence and spread of the ROMANSY results within a growing world Robotics community. The current scientific committee is listed below with names of persons who are from a third generation (since they started their activity after 2000) and others, who are even pupils of the funders.

The ROMANSY series was established as cooperation between IFToMM and CISM with an initial plan to have conference events alternatively at the CSIM headquarters in Udine, Italy, and in Poland under the direct responsibility of IFToMM and CISMM leaders together with the scientific committee. Later, as it is still today, it was decided to have the conference events hosted in any world institution where the organizing chair is active. The following is the list of ROMANSY events over time:

- 1973: ROMANSY 1 in Udine, Italy with chairmanship of A. E. Kobrinskii
- 1976: ROMANSY 2 in Jadwisin, Poland with chairmanship of B. Roth
- 1978: ROMANSY 3 in Udine, Italy with chairmanship of L. Sobrero
- 1981: ROMANSY 4 in Zaborow, Poland with chairmanship of A. Morecki
- 1984: ROMANSY 5 in Udine, Italy with chairmanship of G. Bianchi
- 1986: ROMANSY 6 in Cracow, Poland with chairmanship of A. Morecki
- 1988: ROMANSY 7 in Udine, Italy with chairmanship of G. Bianchi and A. Morecki
- 1990: ROMANSY 8 in Cracow, Poland with chairmanship of A. Morecki and G. Bianchi
- 1992: ROMANSY 9 in Udine, Italy with chairmanship of G. Bianchi and A. Morecki
- 1994: ROMANSY 10 in Gdansk, Poland with chairmanship of A. Morecki and G. Bianchi
- 1996: ROMANSY 11 in Udine, Italy with chairmanship of G. Bianchi and A. Morecki
- 1998: ROMANSY 12 in Paris, France with chairmanship of A. Morecki and G. Bianchi and J. C. Guinot
- 2000: ROMANSY 13 in Zakopane, Poland with chairmanship of A. Morecki and G. Bianchi
- 2002: ROMANSY 14 in Udine, Italy with chairmanship of G. Bianchi and J. C. Guinot

- 2004: ROMANSY 15 in Montreal, Canada with chairmanship of J. Angeles and J. C. Piedboeuf
- 2006: ROMANSY 16 in Warsaw, Poland with chairmanship of T. Zielinska
- 2008: ROMANSY 17 in Tokyo, Japan with chairmanship of A. Takanishi and Y. Nakamura
- 2010: ROMANSY 18 in Udine, Italy with chairmanship of W. Schiehlen and V. Parenti-Castelli
- 2012: ROMANSY 19 in Paris, France with chairmanship of P. Bidaud and O. Khatib

Proceedings volumes have been always published to be available also after the symposium to large public of scholars and designers.

This Proceedings volume contains 62 papers that have been selected after review for oral presentation and one invited lecture that prepared by Prof. Bernard Roth to celebrate the 20th anniversary event with his vision and memories. These papers cover several aspects of the wide field of Robotics concerning Theory and Practice of Robots and Manipulators.

We would like to express grateful thanks to the members of the current International Scientific Committee for ROMANSY Symposium for cooperating enthusiastically for the success of the 2014 event:

Philippe Bidaud (France)
Marco Ceccarelli (Italy)
I-Ming Chen (Singapore), as Chair of the IFToMM Technical Committee for Robotics and Mechatronics
Victor Glazunov (Russia)
Qian Huang (China)
Oussama Khatib (USA)
Vincenzo Parenti-Castelli (Italy), CISM representative
Werner Schiehlen (Germany)
Atsuo Takanishi (Japan)
Teresa Zielińska (Poland)

We thank the authors who have contributed with very interesting papers on several subjects, covering many fields of Robotics as Theory and Practice of Robots and Manipulators and additionally for their cooperation in revising papers in a short time in agreement with reviewers' comments. We are grateful to the reviewers for the time and efforts they spent in evaluating the papers with a tight schedule that has permitted the publication of this proceedings volume in time for the symposium.

We thank the Blagonravov Institute of Machines Science (known also as IMASH) of Russian Academy of Science (RAS) in Moscow for having hosted the

ROMANSY 2014 event. We express our special thanks to academician Rivner Ganiev, director of IMASH, for supporting the hosting of ROMANSY 2014 in IMASH.

We would like to thank the members of the Organizing Committee: Academician, Prof. Vasilij Fomin; Mem. RAS., Prof. Alexandr Shipliyuk; Mem. RAS., Prof. Vacheslav Prihodko; Mem. RAS., Prof. Nikolay Bolotnik; Prof. Veniamin Goldfarb; Prof. Irina Demianushko; Prof. Vigen Arakelian; Prof. Alexandr Golovin; Prof. Sergey Yatsun; Prof. Sergey Gavryushin; Prof. Sergey Misyurin; Dr. Raphael Sukhorukov; Prof. Saygid Uvaisov; Prof. Alexey Borisov; Prof. Anrey Korabelnikov; Dr. Oleg Muguin, Dr. Constantin Salamandra; Dr. Nikolay Tatus for their help in the plans for ROMANSY 2014 in Moscow.

We also thank the support of International Federation for the Promotion of Mechanism and Machine Science (IFTToMM) and the auspices of Centre for Mechanical Science (CISM). The long cooperation between IFTToMM and CISM has ensured and will ensure the continuous success of ROMANSY as a unique conference event in the broad area of Robotics with tracking reached achievements and future challenges. Special thanks are expressed to IFTToMM Russia that very enthusiastically supported the plan to have ROMANSY in Moscow and promoted a significant participation of Russian colleagues.

We thank the publisher and Editorial staff of Springer and particularly Dr. Nathalie Jacobs, managing Editor, for accepting and helping in the publication of this volume within the book series on Mechanism and Machine Science (MMS).

We are grateful to our families since without their patience and understanding it would not have been possible for us to organize ROMANSY-2014, the 20th CISM-IFTToMM Symposium on Theory and Practice of Robots and Manipulators.

Moscow, March 2014

Marco Ceccarelli
Victor A. Glazunov

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Ro. Man. Sy.: Its Beginnings and Its Founders

B. Roth

Abstract This paper describes the origins of the ROMANSY series of symposia. It recounts the author's experience as one of the original founders. The emphasis is on the events leading to the first organizational meetings and the people involved in organizing the early symposia.

Keywords ROMANSY · CISM · IFToMM · Robot · Symposia

1 Introduction

My purpose in writing this paper is to record my personal recollections of the events that led to the founding of the Romansy symposia. I undertook this task at the kind invitation of Professors V. Glazunov and M. Ceccarelli. As organizers of the 20th symposium in the Romansy series, they felt it would be appropriate to mark this anniversary by adding an historical perspective. The task fell to me since, sadly, I am the last living member of the original group that conceived the project and issued the invitations to form the first Organizing Committee.

I do want to point out that Moscow is an especially appropriate venue for this 20th anniversary event since three prominent academics from Moscow, Academician I. I. Artobolevskii, Professor A. P. Bessonov and Professor A. E. Kobrinskii, played pivotal roles in Romansy's establishment and early implementations.

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2 My Voyage

Every event has many antecedents. Depending on the narrator, different founding stories start at different points in time. This paper is my version of what happened. I am fully aware that the other founders would have their own versions. Their stories would certainly overlap mine, yet this version could only be seen through my perspective. For me the essential part of the founding of Romansy began when my wife and I visited the Soviet Union for 30 days, starting April 7, 1969, under a USA-USSR Academy of Science exchange program. We were met at the airport by Professor Arcady Bessonov.

During that visit I met Professor Aaron Kobrinskii for the first time, and learned about his work in computer-aided manufacturing, vibrations, mechanisms and robotics. Most importantly, I met Academician Ivan Ivanovich Artobolevskii. I did not fully realize what an important figure Artobolevskii was until I was invited to lunch at his apartment and saw that he lived in a very prestigious location close to the Kremlin. I later found out that in addition to his academic achievements he had been elected to one of the top government bodies: the Supreme Soviet.

The next important connection for the role I was to play in the founding of Romansy occurred when we stopped for a two-week visit in Bulgaria on our way back from the USSR. Our host there was Professor Michael Konstantinov. He had been educated in a German school in Sofia, and his German was much better than his English. So, German was the language we mainly used. As an aside, his German was so beautifully and clearly spoken, that being with him was for me like attending an intensive and highly productive German refresher course.

The final steps in my personal voyage toward the formation of Romansy occurred four months later at the Second IFToMM International Congress on the Theory of Machines and Mechanisms that was held in Zakopane, Poland in September of 1969. I gave a paper on the kinematics of computer controlled manipulators that described some of the work I had been doing with my PhD student Donald Pieper. The presentation included a short film showing a computer controlled manipulator constructing a small tower of blocks while moving through an environment with obstacles that had to be avoided. It was the first autonomous manipulation demonstration that most people in the audience had ever seen. Such films are now commonplace. At that time however, it seemed excitingly new and forward looking.

At the IFToMM Congress Artobolevskii was somewhat of an aloof and imperial figure. He was housed separately and more luxuriously than the other participants. Generally he did not mix with participants during the off hours between sessions. However, when he attended technical sessions he always sat erectly in the front row and listened intently to the presentation.

When I gave my talk and showed my film he was in his usual front row seat. After my talk he came up to me, greeted me warmly and expressed, what seemed to me, very genuine admiration for my work. I could sense there was a strong interest on his part in further study on the topic of autonomous manipulation.

There were two other seemingly casual connections at this IFToMM Congress that later proved pivotal for the development of Romansy. They involved Professors Giovanni Bianchi and Adam Morecki. I had never met either of them before the Congress. Morecki was a professor at the Technical University in Warsaw who worked mainly in biomechanics and its application to medical rehabilitation. Bianchi's main interest was classical mechanics and its application to machine design. He was a professor at the Polytechnic in Milan.

The Romansy symposia were originated under the joint sponsorship of CISM and IFToMM. The founding of CISM and IFToMM both came to fruition after years of earlier discussion and organizational work. Both of these organizations were officially born at about the same time in the late 1960s. To understand the founding of Romansy it is useful to briefly review the connection of Romansy's founders to CISM and IFToMM.

3 CISM and IFToMM

CISM: The acronym is for the Italian rendering of International Center for Mechanical Sciences. Professor Luigi Sobrero was the longtime director of the Institute of Mechanics at the University of Trieste. In 1968, toward the end of a distinguished career, he succeeded in gathering enough support of eminent colleagues in Europe to establish CISM as an international institute for promoting post graduate study in the field of mechanics.

In order to foster economic growth in the region, the city of Udine gave CISM use of a palace that had been donated to it by Count Alessandro del Torso. His expressed wish was that it should be devoted to cultural activities. To this day, this palace, located in the heart of the city, provides office and meeting space for most of CISM's activities. Once established, CISM was governed by a board that consisted of representatives of member nations. One of these board members was appointed rector of CISM. It happened to be Professor Waclaw Olszak who had strong ties to his native Poland and the Polish scientific community. Especially important for Romansy was his close relationship with Adam Morecki. The other direct connection between CISM and Romansy was that Giovanni Bianchi was also deeply involved in CISM's governance and had close ties to its founder Luigi Sobrero.

IFToMM: Although it has now been modified, this acronym originally stood for the International Federation for the Theory of Mechanisms and Machines. Among the most active founders of IFToMM were Artobolevskii, Bessonov, and Konstantinov. In fact Artobolevskii was the founding president of IFToMM; Bessonov held various executive positions and Konstantinov was to become IFToMM's secretary general.

The financial arrangements were a bit unusual. IFToMM was funded by dues from member countries. It had a relatively small budget and could only make token contributions to Romansy. CISM received a land subsidy from the city of

Udine, but in the main it was also funded by dues from member countries. Some of the Eastern European countries paid their CISM dues in local non-dollar convertible currency. In particular CISM had resources it could make available for meetings that were held in Poland. That combined with Adam Morecki's organizational abilities created the early pattern of alternating the venue for Romansy between Udine and Poland. It also biased the venue of many of the organizing committee meetings to Poland.

4 Founders

I have outlined the circumstances that brought me into contact with the main originators of what became the Romansy symposia. The idea to organize an international symposium on robot research was brought to me by Kobrinskii in September 1971 during the Third IFToMM World Congress held in Kupari, Yugoslavia. Kobrinskii invited me to a meeting with Professor Sobrero, the founding Secretary General of CISM, and his secretary Mrs. A. Bertozzi. In addition to the four of us there were three other participants: Artobolevskii, Konstantinov and Bianchi. The seven of us held a meeting that what would in retrospect be the founding moment of Romansy. At this meeting we agreed to organize an international symposium. It was also decided that Kobrinskii would be the chairman of the organizing committee, and Sobrero would be the vice-chairman.

Artobolevskii assured us of IFToMM's support. Sobrero was anxious to grow activities at CISM and he wanted the symposium to take place at their headquarters in Udine.

We then had to decide what to call the proposed symposium. Kobrinskii who had a quick mind and enjoyed mental puzzles suggested Robot and Manipulator Symposium with the acronym ROMANSY. Bianchi objected. He felt the acronym was too frivolous and conveyed the wrong message. At the time, I did not know Bianchi well. I remember thinking, "there goes my stereotype of Italian lovers." We discussed this for a long time. Finally Bianchi acquiesced when we agreed to write the acronym as Ro. man. sy. Later, I became very close friends with Bianchi and his family. I realized that he had been brought up in a banking family with conservative social manners. This background caused Bianchi to have a visceral rejection of Kobrinskii's brilliant idea. Slowly over the years the acronym morphed into the original suggestion. First the spaces after the periods disappeared, and we had Ro.man.sy and also RO.MAN.SY. Next the periods disappeared and we got to RoManSy. Eventually the cover of the proceeding boldly proclaimed Kobrinskii's original ROMANSY.

The original meeting in Kupari was followed by a more extensive meeting in Nieborow, Poland in May 1972. The host for this meeting was Prof. Adam Morecki. He arranged for us to meet and live in the old Radzivil Palace, which now belonged to the Polish government. This venue fostered a very relaxed country atmosphere which provided opportunities for long strolls and off-the-record conversations.

In addition to Artobolevskii, Kobrinskii, Konstantinov, the meeting was attended by Prof. Medford Thring from University College, London, Adam Morecki from the Polytechnic of Warsaw, Miomir Vukobratović from Mihajlo Pupin Institute in Belgrade, Ikiro Khato from Wasada University, Tokyo. Giovanni Bianchi represented IFToMM and Luigi Sobrero and Mrs. Anna Bertozzi represented CISM. At this meeting we prepared the call for papers for the first symposium, which was set for September 1973 at CISM in Udine.

Figure 1 Shows a photo I took at that meeting. In the front row from left to right we have Kato, Artobolevskii, Sobrero, Konstantinov, and Thring. The second row has Bianchi and Bertozzi. The third row shows Morecki flanked by two staff members. The last row has two of Morecki's staff and Vukobratović on the right.

There was a final organizing meeting that took place in Split, Yugoslavia in April of 1973. This meeting was hosted by Professors Bazjanac and Jelovac from Zagreb. This meeting was attended by the same group as at Nieborow and, in addition, Professor Hans Wanecke from the University of Stuttgart and Professor Romiti from the Polytechnic in Turin.

At the Split meeting the submitted papers were reviewed. It was decided to accept 45 papers and publish them in a volume of conference preprints. I wanted the final proceedings to be as readable as possible. So, I went through all the papers written by non English speaking authors and, where necessary, annotated them with suggested language modifications. I still recall the gratitude I received when I passed out the edited manuscripts to the authors at the symposium.

5 First Symposium

The symposium was held on September 5–8, 1973 in the beautifully frescoed main hall of the Palazzo del Torso. All the papers were presented on a single track, so everyone was in the same sessions through the symposium.

After receiving the revisions, Mrs. Bertozzi arranged to have the final proceeding published as two-volume set by Springer-Verlag. The final proceedings contain the text of two Opening Lectures. The first one runs for a little over 7 pages. It is titled "The State of the Art in the field of Robots and Manipulators." It is signed A. E. Kobrinskii, Academy of Sciences of the USSR. The second one runs for two pages and is titled "Robots and Manipulators" it is signed by M. W. Thring, University of London. Whenever I look at this part of the proceedings I am reminded of what I consider the saddest event in the history of Romansy.

If there was a single person to be credited with the idea for Romansy it would be Kobrinskii. In recognition of this he was the chairman of the Organizing Committee for the first symposium. He attended all of the organizing committee meetings and was given the honor of presenting the opening lecture. It was unimaginable that he would not attend the symposium. Yet, when the scientists from the Soviet Union arrived, he was not among them. I was told that he had to cancel his trip at the last minute because his brother was stricken with an illness. It



Fig. 1 Organizing and program committee and staff, Nieborow, Poland in May 1972

was true that his trip had been cancelled at the last minute. However it had nothing to do with family illness. It was because he was not given an exit visa. I do not pretend to know the reason for this. I do know that Kobrinskii felt it was due to his Jewish origins and the Soviet Government's discriminative policies towards its Jewish citizens at that time.

These are not pleasant things to recall, especially in a paper meant to celebrate the cooperation between researchers from throughout the world. Kobrinskii is dead and so are most of the colleagues who attempted to hide the truth. In a cosmic sense the incident is meaningless. I mention it here mainly to emphasize the importance of Aaron Kobrinskii's role in Romansy's birth and to pay homage to an unusually creative colleague by acknowledging my empathy for what must have been an incredibly painful personal episode.

The rest of the symposium was uneventful and was full of good feelings and comradeship. We had achieved our goal of opening a multinational scientific exchange in the area of robotics. In addition to the two invited opening lectures, we had accepted 45 papers. The distribution according to country of these 45 papers was: USA 13, USSR 13, Japan 4, England 3, Yugoslavia 3, West Germany 3, Italy 2, Poland 2, Bulgaria 1, and France 1.

On the last day of the symposium the organizing committee was invited by Professor Sobrero to a lunch meeting in the restaurant of the upscale Astoria Hotel around the corner from CISM. I recall we were seated at a long rectangular table. Artobolevskii sat at the head and I was far away toward other end. We agreed that the symposium had been a success and that a second Romansy should be held in

Poland in 1976. The organizing committee was to have two meetings before the symposium, the first of which was to be in Warsaw in June of 1974.

The lunch went on a bit long and toward the end my mind was wandering. I was barely paying attention when Artobolevskii rose with wine glass in his hand and said, "I propose that Professor Roth should be the new chairman of the organizing committee". This was greeted with a round of applause. That was it, it was an imperial order and everyone obeyed. I was flabbergasted at Artobolevskii's action on two counts: I was both an American and the youngest person at the table. I suppose Artobolevskii must have discussed it with Sobrero and Bianchi beforehand, yet it always remained in my mind a personally generous gesture of great magnanimity on his part. I took it as both strengthening our personal connection and as a signal that Romansy had achieved a measure of Soviet-American cooperation that at the time was very much lacking in the world.

6 An Ongoing Series

The success of the first symposium led CISM and IFToMM to jointly form a permanent Technical Committee on Robots and Manipulators. This technical committee was to organize the second and all subsequent Romansy symposia. I had the honor to be the chair of this technical committee for its first years. For the second symposium the committee was:

Chairman: Prof. B. Roth (Stanford University), *Vice-Chairmen:* Prof. L. Sobrero (CISM), Prof. A. Morecki (Technical University of Warsaw). *Members:* Acad. I. I. Artobolevskii (Institute for the Study of Machines, Moscow), Prof. G. Bianchi (Technical University of Milan), Prof. I. Kato (Wasada University), Prof. A. E. Kobrinskii (Institute for the Study of Machines, Moscow), Prof. M. S. Konstantinov (High Mechanical and Electromechanical Institute, Sofia), Prof. R. B. McGhee (The Ohio State University), Prof. M. W. Thring (University of London), Mr. J. Vertut (Atomic Energy Commission, France), Prof. M. Vukobratović (Mihajlo Pupin Institute), Prof. H. J. Warnecke (University of Stuttgart). *Scientific Secretary:* Dr. K. Kędzior (Technical University of Warsaw). *Secretary:* Mrs. A. Bertozzi (CISM).

The Technical Davison of the Polish Academy of Sciences joined CISM and IFToMM's financial sponsorship, and two preparatory committee meetings for the second Romansy were held in Poland. The Second International CISM-IFToMM Symposium took place in the small village of Jadwisin near Warsaw, Poland on September 14–17, 1976. There was a volume of preprints available at the symposium.

The second Romansy was attended by 117 participants and 15 accompanying guests. They came from 4 continents and 20 countries (which doubled the 10 countries for the first symposium). The social programs included a banquet with groups of participants singing in various languages and a half-day excursion. This became the model for subsequent Romansy symposia.

For the second symposium we adopted the policy that a paper could only be published in the proceedings if it was actually presented at the symposium. I have always felt regret that this policy led to the exclusion of some good papers.

Artobolevskii passed away at the age of 71 on September 21, 1977, a year before the third Romansy. I happen to have been in the USSR at the time. I was attending a conference in Alma Ata. As soon as his death was announced everything seemed to shut down. Many participants hurried back to Moscow to attend his funeral. To me the most impressive reminder of what an important figure he was happened when the TV throughout the Soviet Union stopped its normal programs and simply played dirge music in his memory.

His colleague Prof. A. P. Bessonov had been acting as an aid-de-camp to Artobolevskii since the founding of Romansy. So, he was very knowledgeable about Romansy and was able to seamlessly take up Artobolevskii's role as a representative of the USSR on the Organizing Committee.

The third Romansy was held in Udine at CISM on September 12–15, 1978. About 8 months after the third symposium our great benefactor Prof. Luigi Sobrero passed away at the age of 69. CISM had been the crowning passion of his life, and he left it in very good shape as far as Romansy was concerned. Prof. Giovanni Bianchi had served as an aid-de-camp to Sobrero from the beginning of the Romansy discussions. After Sobrero died, he was appointed Secretary General of CISM and became the main contact between Romansy and CISM. Also, the incredibly efficient Mrs. Bertozzi was also still available. After the symposium, Professors McGhee, Thring and Warnecke left the committee, and Prof. Morecki became the chairman of the CISM-IFTOMM Technical Committee on Robots and Manipulators. Prof. Bianchi became the vice chairman. The fifth Romansy was back at CISM and, accordingly, Bianchi and Morecki had switched chairman and vice chairman roles.

The sixth was back in Poland on September 9–12, 1986. This time the venue was in the city of Krakow. The symposium opened with a memorial session dedicated to the work of Jean Vertut, who had died of a heart attack at age 56. After the sixth Kato, Kobrinskii, Kędzior and I left the Organizing and Program Committee. For me, it was time to let the next generation pilot the Romansy voyage.

7 Conclusions

When Romansy was started it was the first research based conferences dealing with robot design, mechanics and control. Furthermore, before Romansy there was little or no interchange between Eastern Europe and the West on these subjects. Romansy was created on the premise of free scientific communication between all areas of the world. The fact that it achieved its original goals and continues to thrive is of great personal satisfaction to me. Even greater satisfaction for me is the great gift of the enduring and cherished friendships formed through Romansy.

Parametric Method for Motion Analysis of Manipulators with Uncertainty in Kinematic Parameters

Vahid Nazari and Leila Notash

Abstract In this paper, the motion performance of manipulators considering the uncertainty in the kinematic parameters is investigated. Interval analysis is employed to deal with the uncertainty in the kinematic parameters in the form of small uncertainty boxes. For a given range of uncertainties in the kinematic parameters, the interval linear equations are formulated to relate the velocity of joints to the end effector velocity with the Jacobian matrix. A novel approach for calculating the exact size and shape of the solution for the system of interval linear equations is presented. A 2 degrees of freedom planar serial manipulator is used as a case study to analyze the motion performance of the manipulator in the presence of uncertainties.

Keywords Interval analysis · Robot manipulators · Uncertainty · Parametric method · Parameter solution set

1 Introduction

Robot manipulators are typical of systems that are intrinsically subjected to uncertainties. The nominal relationship between the end effector pose and joints displacement is known but this relationship is not necessarily accurate due to changes in the robot hardware and uncertainties in the kinematic parameters [1]. A real robot analysis should be performed in the presence of uncertainties in the

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modeling of the manipulator and measurements of the kinematic parameters. The sources of uncertainties include the manufacturing tolerances of the mechanical parts, measurement error, control error, and round-off error. All types of uncertainties can be accommodated as bounded variations in the kinematic parameters.

Several methods are known for calculating the lower and upper bounds for each component of the solution set in the interval linear systems. One of the first contributions on determining the bounds of the solution set was given in [2]. It was shown that the solution set for this system is a polyhedron. More general algorithms for determining the bounds containing the exact solution were presented in [3–5]. These bounds were not necessarily identical to the exact solution. The exact solution was determined in [6] as the union of finitely many convex polytopes whose vertices were denoted by matrices with entries equal to the lower or upper bounds of the interval coefficient matrix. The shape of the solution set, in general, was a non-convex polyhedron.

The exact solution of the interval linear systems is generally complicated and not easily described. Therefore, calculation of this solution is computationally expensive and, hence, is not convenient to use for the real time application. Accordingly, the researchers are drawn to find the fastest methods to enclose the exact solution. One of the first publications on parametric interval systems for special coefficient matrices, such as symmetric and skew-symmetric matrices, was presented in [7, 8]. The characterization of the boundary of the solution set of the parametric system based on a set of inequalities was done by [9]. This approach was designed particularly for visualizing the boundary of the parametric solution set.

In this paper, the motion performance of manipulators with uncertainty in the kinematic parameters is investigated using parametric interval method. The organization of paper is as follows. The basic principles of the interval analysis and the parametric interval systems are given in Sect. 2. The proposed methodology for formulating the exact solution, which is based on parameterizing the interval linear systems, is presented in Sect. 3. The simulation results are reported in Sect. 4 and the paper is concluded in Sect. 5.

2 Parametric System of Interval Linear Equations

Interval analysis is a numerical method of representing the uncertainty in values by replacing a number with a finite range of values. An interval denoted by $[X] = [\underline{X}, \overline{X}]$ is the set of real numbers X verifying $\underline{X} \leq X \leq \overline{X}$ where \underline{X} and \overline{X} are the lower and upper bounds of the interval, respectively. The interval is also represented by the midpoint, X_c , and the radius, ΔX , as $[X] = [X_c - \Delta X, X_c + \Delta X]$ or $[X] = X_c + \Delta X[-1, 1]$. A real number is a special case of an interval in which $\underline{X} = \overline{X}$. The width of the interval $[X]$ is defined as $w(X) = \overline{X} - \underline{X}$. The midpoint of $[X]$ is given by $m(X) = \frac{1}{2}(\overline{X} + \underline{X})$. A matrix whose entries are interval is called an interval matrix and denoted by $[A]$, A_c , is the midpoint of $[A]$ whose entries are the

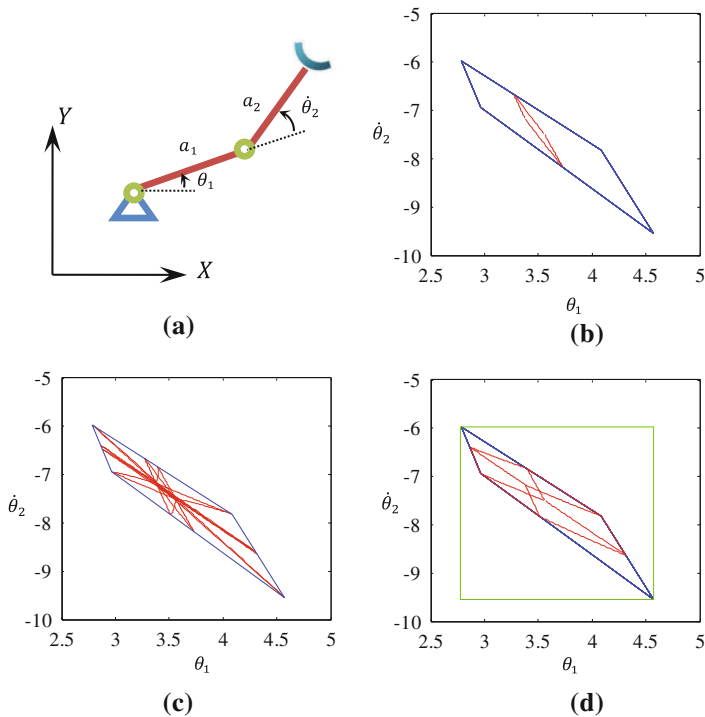


Fig. 1 **a** 2 DOF planar serial manipulator, **b** one of two-parameter solution sets in *red*, **c** all two-parameter solution sets in *red* and the exact solution in *blue*, **d** a three-parameter solution set in *red* and the smallest box containing the exact solution in *green*

midpoints of the corresponding entries of $[\mathbf{A}]$, the radius of the interval matrix, Δ , is defined as $\frac{1}{2}(\overline{\mathbf{A}} - \underline{\mathbf{A}})$.

In manipulators, the Jacobian matrix relates the joint velocity vector to the end effector velocity vector. Due to the uncertainty in the kinematic parameters, the relationship between the joint velocity vector and the end effector velocity vector takes the form of the interval linear system. This interval system is parameterized as $[\mathbf{J}([\mathbf{p}]])\dot{\mathbf{q}} = [\mathbf{V}([\mathbf{p}])]$ in which the entries of the Jacobian matrix and the end effector velocity vector linearly depend on parameters $[\mathbf{p}] = ([p_1], [p_2], \dots, [p_K])$ even though in general, the entries of the Jacobian matrix and the velocity vector could be nonlinear functions of the interval parameters $[\mathbf{p}]$. The exact values of these parameters are unknown but bounded within given intervals. Considering the serial manipulator in Fig. 1a and using a linear parametric model for each entry of $[\mathbf{J}([\mathbf{p}])]$ and $[\mathbf{V}([\mathbf{p}])]$, the entries of the Jacobian matrix and the velocity vector could be defined as

$$[J_{jk}(\mathbf{p})] = J_{jk,0} + \sum_{\mu=1}^K J_{jk,\mu} [p_{\mu}], [V_j(\mathbf{p})] = V_{j,0} + \sum_{\mu=1}^K V_{j,\mu} [p_{\mu}] \quad (1)$$

where $J_{jk,\mu}, V_{j,\mu} \in \mathbb{R}; \mu = 1, \dots, k; j = 1, \dots, m; k = 1, \dots, n$; m is the task space dimension and n is the number of joints. The value of parameter K depends on the number of the interval parameters used to parameterize the interval system. The maximum number of the interval entries of $[\mathbf{J}]$ and $[\mathbf{V}]$ is mn and m , respectively. Depending on the uncertainty of the kinematic parameters in the manipulator, some entries of the Jacobian matrix and the end effector velocity vector may not be interval.

3 Parametric Method for Exact Solution

In this section, the exact solution of the interval system is calculated using solution sets obtained from parameter groups of interval systems. Depending on the number of interval parameters involved in the Jacobian matrix and the velocity vector, the exact solution will be characterized. The parameter assignment of the entries of $[\mathbf{J}]$ and $[\mathbf{V}]$ in the manipulator is performed by selecting some interval entries of either $[\mathbf{J}]$ or $[\mathbf{V}]$ as parameters $[p_{\mu}]$ and formulating other entries as functions of interval parameters $[p_{\mu}]$. All parameter assignments of the entries of $[\mathbf{J}]$ and $[\mathbf{V}]$ which lead to the same solution set are collected as one parameter group. That is, a parameter group may consist of one or several different parameter assignments.

The number of parameter groups in each interval system depends on the total number of interval entries of $[\mathbf{J}(\mathbf{p})]$ and $[\mathbf{V}(\mathbf{p})]$, denoted as η , and the number of interval parameters in the interval system, K . The solution sets of all parameter assignments of the interval system are checked and the parameter assignments which result in the same solution set are categorized as one parameter group. Considering a general spatial serial manipulator, to form $[\mathbf{J}] \dot{\mathbf{q}} = [\mathbf{V}]$ with η interval entries in $[\mathbf{J}]$ and $[\mathbf{V}]$ and 2 interval parameters, $K = 2$, there exist $\frac{1}{2} \sum_{i=K-1}^{\eta-1} \binom{\eta}{i} = \frac{1}{2} \sum_{i=1}^{\eta-1} \frac{\eta!}{(\eta-i)!i!}$ different parameter groups. When the number of interval parameters is $K = 3$, all the number of possible parameter groups is

$$\text{calculated as } \sum_{i=K-1}^{\eta-2} \frac{\binom{\eta}{i} \binom{\eta-i}{2}}{(\eta-i-2)!} = \sum_{i=2}^{\eta-2} \frac{(\eta-i-1)(\eta-i)\dots(\eta)}{2!i!(\eta-i-2)!}.$$

In this paper, once the numerical interval matrix $[\mathbf{J}]$ is calculated, the entries of $[\mathbf{J}(\mathbf{p})]$ and $[\mathbf{V}(\mathbf{p})]$ are expressed as linear functions of the interval parameters $[p_{\mu}]$, $1 \leq \mu \leq K$. Considering entry $[J_{jk}]$ and $[V_j]$ as linear function of $[p_{\mu}]$, then $[J_{jk}([p_{\mu}])] = J_{jk,0} + J_{jk,\mu}[p_{\mu}]$ and $[V_j(p_{\mu})] = V_{j,0} + V_{j,\mu}[p_{\mu}]$. The lower and upper bounds of any interval entry $[J_{jk}] = [\underline{J}_{jk}, \overline{J}_{jk}]$ are related to those of interval parameter $p_{\mu} \in [p_{\mu}, \overline{p}_{\mu}]$, $p_{\mu} \neq \overline{p}_{\mu}$ through the following system of linear equations

$$\begin{cases} \overline{J_{jk}} = J_{jk,0}\overline{p}_\mu + J_{jk,\mu} \\ \underline{J_{jk}} = J_{jk,0}\underline{p}_\mu + J_{jk,\mu} \end{cases} \Rightarrow \begin{bmatrix} \overline{p}_\mu & 1 \\ \underline{p}_\mu & 1 \end{bmatrix} \begin{bmatrix} J_{jk,0} \\ J_{jk,\mu} \end{bmatrix} = \begin{bmatrix} \overline{J_{jk}} \\ \underline{J_{jk}} \end{bmatrix} \quad (2)$$

The coefficients ${}^i J_{jk,0}$ and ${}^i J_{jk,\mu}$ are calculated by taking inverse of Eq. (2) as

$$\begin{bmatrix} J_{jk,0} \\ J_{jk,\mu} \end{bmatrix} = \begin{bmatrix} \overline{p}_\mu & 1 \\ \underline{p}_\mu & 1 \end{bmatrix}^{-1} \begin{bmatrix} \overline{J_{jk}} \\ \underline{J_{jk}} \end{bmatrix} \quad (3)$$

The same procedure is performed to formulate the entry of $[V_j]$ as a function of $[p_\mu]$. It should be noted that the entry of $[\mathbf{J}([\mathbf{p}]])$ or $[\mathbf{V}([\mathbf{p}]])$ nominated for the interval parameter must be interval. Otherwise, the matrix in Eq. (2) would be singular and the entry $[J_{jk}]$ cannot be formulated in terms of parameter $[p_\mu]$. If $[\mathbf{J}([\mathbf{p}]])$ is an $n \times n$ square matrix and non-singular for each $p_\mu \in [\underline{p}_\mu, \overline{p}_\mu]$, $\mu = 1, \dots, K$, $[\mathbf{J}^{-1}([\mathbf{p}]])$ exists and $[\dot{\mathbf{q}}([\mathbf{p}]]) = [\mathbf{J}^{-1}([\mathbf{p}]])[\mathbf{V}([\mathbf{p}]])$ is a function of K interval parameters which is continuous [9]. This parametric joint velocity vector provides the solution set for each parameter group.

When the parametric Jacobian matrix is of full-rank, the solution which minimizes the 2-norm of the joint velocity vector is selected. If the square parametric matrix $[\mathbf{J}([\mathbf{p}]])[\mathbf{J}^T([\mathbf{p}]])$ is regular for every $p_\mu \in [\underline{p}_\mu, \overline{p}_\mu]$, the minimum 2-norm solution set to the parametric system exists and is formulated as a function of interval parameters $[\dot{\mathbf{q}}([\mathbf{p}]]) = [\mathbf{J}^T([\mathbf{p}]])([\mathbf{J}([\mathbf{p}]])[\mathbf{J}^T([\mathbf{p}]])^{-1}[\mathbf{V}([\mathbf{p}]])]$. If the manipulator has a combination of revolute and prismatic joints, the joint velocity vector is not physically consistent. If the interval entries with the same dimension are parameterized, a weighting matrix would be required to calculate the generalized (Moore-Penrose) inverse of $[\mathbf{J}([\mathbf{p}]])$ as $\mathbf{J}^\# = \mathbf{W}[\mathbf{J}^T([\mathbf{p}]])([\mathbf{J}([\mathbf{p}]])\mathbf{W}[\mathbf{J}^T([\mathbf{p}]])^{-1}]$.

Similarly, when parametric Jacobian matrix $[\mathbf{J}([\mathbf{p}]])$ is of full column-rank and $[\mathbf{J}^T([\mathbf{p}]])[\mathbf{J}([\mathbf{p}]])$ is regular for every $p_\mu \in [\underline{p}_\mu, \overline{p}_\mu]$, the least square solution set is calculated. The weighted left generalized inverse of $[\mathbf{J}([\mathbf{p}]])$ is calculated as $\mathbf{J}^\# = [\mathbf{J}^T([\mathbf{p}])\mathbf{W}[\mathbf{J}([\mathbf{p}]])^{-1}\mathbf{J}([\mathbf{p}])\mathbf{W}]$ if the interval entries of the Jacobian matrix are parameterized using the interval parameters with the same dimension.

4 Case Study

In this section, the 2 DOF planar serial manipulator in Fig. 1a with two revolute joints is used as a case study for the interval analysis to visualize the solution set. The manipulator has uncertainty in two joint variables θ_1 and θ_2 and the link lengths a_1 and a_2 .

For the joint variables $\theta_1 = \frac{\pi}{6}$ rad and $\theta_2 = \frac{\pi}{4}$ rad, the link lengths $a_1 = a_2 = 0.5$ m, the radius of uncertainty $\frac{\pi}{180}$ rad in θ_1 and θ_2 and the radius of

uncertainty 0.010 m in link lengths, the interval Jacobian matrix is $\mathbf{J} = \begin{pmatrix} [-0.760, -0.706] & [-0.497, -0.469] \\ [0.530, 0.595] & [0.110, 0.149] \end{pmatrix}$. The desired end effector velocity is $\mathbf{V} = [v_x v_y]^T = [1 \ 1]^T$ (m/s).

If the Jacobian matrix and the end effector velocity vector are functions of two parameters $[p_1]$ and $[p_2]$, i.e., $\mu = 1, 2$, the parametric linear system will be

$$\mathbf{J}([\mathbf{p}]) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{pmatrix} [v_x([\mathbf{p})]] \\ [v_y([\mathbf{p})]] \end{pmatrix} \quad (4)$$

The parameter solution set is derived using the inverse of $\mathbf{J}([\mathbf{p}])$ as

$$\dot{\theta}([\mathbf{p}]) = \begin{pmatrix} [\dot{\theta}_1([\mathbf{p})]] \\ [\dot{\theta}_2([\mathbf{p})]] \end{pmatrix} = \mathbf{J}([\mathbf{p}])^{-1} \begin{pmatrix} [v_x([\mathbf{p})]] \\ [v_y([\mathbf{p})]] \end{pmatrix} \quad (5)$$

Generally, the entries of the Jacobian matrix and the end effector velocity vector can be parameterized such that the entries with the consistent dimension are categorized in the same groups. In this example, the Jacobian matrix has physically consistent entries. Therefore, the parameter assignment can be performed to any entries of the Jacobian matrix. If the entries of the end effector velocity vector are interval and have the same dimension, e.g., m/s, these entries could be parameterized using an interval parameter with the same dimension, e.g., m/s. In the case study, the entries of the end effector velocity vector are not interval. Therefore, they are not functions of an interval parameter, i.e., $[V_j] = V_{j,0} = 1$, $j = 1, 2$.

Entries $[J_{12}]$ and $[J_{11}]$ are selected as the interval parameters $p_1 \in [-0.497, -0.469]$ and $p_2 \in [-0.760, -0.706]$, entries $[J_{21}]$ and $[J_{22}]$ are assigned as functions of $[p_1]$ and entries v_x and v_y are constant values 1. The interval entries of Eq. (3) are substituted into Eq. (5) and the two-parameter solution set for this parameter group is formulated as

$$\dot{\theta}(p_1, p_2) = \begin{pmatrix} \frac{-0.372[p_1] - 0.792}{1.669[p_1] - 0.792[p_2] - 1.372[p_1][p_2] + 2.291[p_1]^2} \\ \frac{2.291[p_1] - [p_2] + 1.669}{1.669[p_1] - 0.792[p_2] - 1.372[p_1][p_2] + 2.291[p_1]^2} \end{pmatrix} \quad (6)$$

Similar to the procedure in calculating the two-parameter solution set in Eq. (6), the two-parameter solution set for each parameter group is formulated. Other parameter groups are obtained by new parameter assignment of the interval entries of $\mathbf{J}([\mathbf{p}])$ as either $[p_1]$ or $[p_2]$ and the rest of entries as functions of $[p_1]$ and $[p_2]$. The new parameter solution set for each parameter assignment forms a parameter group. The boundary curves of the solution set for each group of parametric linear system are specified by 4 curves; two curves $\dot{\theta}(p_1, \underline{p}_2)$ and $\dot{\theta}(p_1, \bar{p}_2)$ in 2-dimensional space when p_1 varies from \underline{p}_1 to \bar{p}_1 and p_2 is set once to

the lower bound and then to the upper bound. Similarly, the other two curves $\dot{\theta}(p_2, p_1)$ and $\dot{\theta}(p_2, \bar{p}_1)$ are formulated when p_2 varies from \underline{p}_2 to \bar{p}_2 and p_1 is set to the lower bound and the upper bound, respectively. In the resulting solution set enclosed by four curves, each curve is connected to the other two curves in two points and the two attached curves share a point. Therefore, four points $\dot{\theta}(\underline{p}_1, \underline{p}_2)$, $\dot{\theta}(\bar{p}_1, \bar{p}_2)$, $\dot{\theta}(\underline{p}_1, \bar{p}_2)$ and $\dot{\theta}(\bar{p}_1, \underline{p}_2)$ form vertices of the solution set for each parameter group. This two-parameter solution set (in red color) is illustrated in Fig. 1b and completely lies inside the exact solution (in blue color).

To characterize the exact solution, first all parameter groups which result in the same solution sets are determined and then plotted in $\dot{\theta}_1$ - $\dot{\theta}_2$ plane. In this example, since there are four interval entries in the Jacobian matrix, $\eta = 4$, there will exist $\frac{1}{2} \sum_{i=1}^{\eta-1} \frac{\eta!}{(\eta-i)!i!} = \frac{1}{2} \sum_{i=1}^3 \frac{4!}{(4-i)!i!} = \frac{1}{2} (4 + 6 + 4) = 7$ different parameter groups among all possible solution sets, i.e., $2^4 = 16$. These 16 solution sets are illustrated in red color in Fig. 1c. The outer vertices of the different groups of the two-parameter solution sets are connected to form the boundary of the exact solution (in blue color). Generally speaking, when the exact solution is non-convex, the two-parameter solution sets might not be able to distinguish the indented vertices.

In the three-parameter case, each parameter group includes interval parameters $[p_1]$, $[p_2]$ and $[p_3]$, i.e., $\mu = 1, 2, 3$. The procedure to calculate the solution set for each parameter group is similar to that of the two-parameter case. The parameter groups for three interval parameters are $\sum_{i=2}^{\eta-2} \frac{(4-i-1)(4-i)\dots(4)}{2!i!(4-i-2)!} = 6$. The solution set corresponding to each parameter group consists of 12 curves; the two parameters p_1, p_2 are set to either lower or upper bounds and the resulting 4 curves, which are functions of parameter p_3 , are plotted when p_3 varies within the lower and upper bounds. The formulation of the solution set of the interval system including three parameters is applicable to the Jacobian matrices of the manipulators with more than 2 joints such as planar 3 DOF manipulators. The process is repeated when $[p_1], [p_3]$ are set to either the lower or upper bounds and the next 4 curves are functions of $[p_2]$. The last 4 curves are formulated as functions of $[p_1]$ when $[p_2], [p_3]$ are set to either the lower or upper bounds. The resulting 12 curves form a hypersurface which may have surfaces on the boundary surface of the exact solution.

To show the solution set for a group of parametric linear system with three interval parameters, the same example as the two-parameter case is considered. For entries $[J_{11}]$ and $[J_{12}]$ and $[J_{21}]$ as interval parameters $p_1 \in [-0.760, -0.706]$, $p_2 \in [-0.497, -0.469]$ and $p_3 \in [0.530, 0.595]$, respectively, $[J_{22}]$ as a function of $[p_1]$, and $[v_x]$ and $[v_y]$ as constant values, the three-parameter solution set is plotted in Fig. 1d. As illustrated, some edges of this solution set lie on the boundary of the exact solution. The commonly calculated smallest box containing the exact solution is depicted in Fig. 1d in green color. As shown, this solution is much larger than the exact solution.