

GeoPlanet: Earth and Planetary Sciences

Ewa Niemczura  
Barry Smalley  
Wojtek Pych *Editors*

# Determination of Atmospheric Parameters of B-, A-, F- and G-Type Stars

Lectures from the School of  
Spectroscopic Data Analyses

 Springer

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# Determination of Atmospheric Parameters of B-, A-, F- and G-Type Stars

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# Preface

During the past years, asteroseismology has greatly benefited from space missions such as *Kepler*, *CoRoT* and *MOST*. Data of unprecedented quality have challenged both observers and theorists, allowing us to improve our knowledge of stars significantly. However, during this process it became very clear that ground-based follow-up spectroscopy is crucial for an in-depth seismic study, as it provides information on different stellar parameters. Keeping this in mind, the ‘Spring School of Spectroscopic Data Analyses’ was organised by the Astronomical Institute of the University of Wrocław, Poland, from April 8 to 12, 2013.

The aim of this school was to provide researchers with an introduction to methods used to obtain the atmospheric parameters of B-, A-, F- and G-type stars. The lecture topics included the determination of atmospheric models and synthetic spectra, application of LTE and NLTE analysis and the analysis of high- and low-resolution data. The practical exercises undertaken during the workshop not only allowed the participants to learn how to compute atmospheric models and synthetic spectra, but also how to determine the atmospheric parameters (effective temperature, surface gravity, microturbulence, etc.), abundances of chemical elements and stellar rotation.

The school was an initiative of *Kepler* Asteroseismic Science Consortium (KASC) working group on main-sequence pulsators and was primarily intended for Ph.D. students and postdocs. The lectures presented here were given by experienced scientists who actively work on stellar atmospheres. We are confident that these lectures will provide an important tool for all students interested in stellar spectroscopy.

The school was sponsored by the Nicolaus Copernicus Astronomical Centre of the Polish Academy of Sciences, Polish Academy of Arts and Sciences, Astronomical Institute of the University of Wrocław and Copernicus Foundation for Polish Astronomy.

Ewa Niemczura  
Barry Smalley  
Wojtek Pych

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# Spring School of Spectroscopic Data Analyses: Determination of Atmospheric Parameters of B, A, F and G-type Stars—Introduction

Katrien Uytterhoeven

**Abstract** Several aspects of astrophysics require accurate atmospheric parameters and abundances. A powerful way of retrieving information on a star's effective temperature, surface gravity, rotation rate, and chemical composition is through the stellar spectrum. This spectroscopic school aims at training researchers in performing spectral analyses, with the motivation of recruiting manpower to join the project on a systematic characterisation of *Kepler* Main Sequence pulsators, for which a collection of low-, mid-, and high-resolution spectra is available for more than 1,000 asteroseismic targets of spectral types B, A, and F.

**Keywords** Stars: fundamental parameters · Stars: abundances · Stars: general

## 1 The Need for Accurate Atmospheric Parameters and Abundances from an Astrophysical Point of View

Why has it been important to develop methods on derivation of stellar effective temperature ( $T_{\text{eff}}$ ) and surface gravity ( $\log g$ )? Why are we interested in a refined picture of the chemical composition of a particular star? Because  $T_{\text{eff}}$ ,  $\log g$ , and chemical composition are stepping-stones towards the characterisation of the star.

Stars are described by mass ( $M$ ), luminosity ( $L$ ), radius ( $R$ ), age, chemical composition, angular momentum, and magnetic field, among other parameters. Generally, not many of these stellar parameters can be directly observed, and hence other ways must be found to infer them. Effective temperature and  $\log g$  are key parameters as they not only define the physical conditions of the stellar atmosphere, but are also directly related to  $M$ ,  $L$ , and  $R$  through the relations:

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$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad (1)$$

$$g = GM/R^2, \quad (2)$$

with  $\sigma$  the Stefan–Boltzmann constant and  $G$  the gravitational constant. Subsequently, the stellar age can be estimated through theoretical evolutionary tracks in the Hertzsprung–Russell (H–R) diagram.

Several aspects of astrophysics require accurate atmospheric parameters, chemical abundances, and/or values of the projected rotational velocity ( $v \sin i$ ). As an illustration we mention a few.

- **Stellar classification** is an important prerequisite to detailed spectral analyses. Stars are classified in spectral classes, which depend on their surface temperature and luminosity. For most stars, excluding (chemically) peculiar stars, an accurate spectral type is a good indicator of their  $T_{\text{eff}}$  and  $\log g$ .
- **Stellar atmosphere models** are governed and defined by processes in the stellar atmosphere. Effective temperature,  $\log g$ , element abundances, the stellar rotation rate (projected rotational velocity  $v \sin i$ ), and less well-understood parameters such as microturbulence ( $\xi_t$ ) play an important role in a good description of the models. To improve stellar atmosphere models, observed spectra of stars with well-known atmospheric parameters for stars of all ages and at different positions in the H–R diagram are needed.
- A precise determination of the stellar rotation rate in combination with element abundances allow the study of **abundance surface inhomogeneities** (stellar spots).
- What causes **chemical peculiarity**? Is there a relation between chemical peculiarity, rotation, and/or magnetic field?
- Models of **convection** can be tested through the mechanism of microturbulence  $\xi_t$ . In massive stars  $\xi_t$  finds its origin in sub-surface convection zones, while for less massive stars outer convection layers are the places of origin.
- In **asteroseismic studies**, accurate values of  $T_{\text{eff}}$  and  $\log g$  are crucial to define the best-fitting and unique asteroseismic model. Moreover, they are indispensable to test instability strips of specific pulsating classes along the Main Sequence (MS), such as  $\beta$  Cep, Slowly Pulsating B stars (SPBs),  $\delta$  Sct and  $\gamma$  Dor stars. A good knowledge of the stellar rotation rate, on the other hand, is needed to understand and recognise rotational splitting of stellar pulsation modes. Moreover, in case of rapidly rotating pulsating stars the rotation causes a deformation of the surface velocity fields. To be able to test and improve pulsational models for rapidly rotating stars, a careful determination of  $v \sin i$  is very important.
- Understanding **stellar structure and evolution of the Milky Way** requires a detailed knowledge of fundamental properties, such as mass, age, kinematics and chemical abundances of its stellar populations. An accurate characterisation of stars at various stages of their lives and at different positions in the H–R-diagram is needed.
- Analysis of the chemical composition of different populations of stars probes **galactic chemical evolution** and test models on element production by thermonu-



clear reactions in stellar interiors and by supernovae. Additionally, comparison of observations and models of chemical evolution test abundance gradients across the Milky Way disk, gas infall episodes, and star formation rates.

## 2 Spectra as a Powerful Tool

Several methods are in use to derive atmospheric parameters. We refer to Chap. 8 for an overview of methods involving photometric indices and spectrophotometry. Here we focus on spectra. High-resolution spectra are a powerful tool for the characterisation of stars, as information on  $T_{\text{eff}}$  as well as  $\log g$ , element abundance,  $v \sin i$  and  $\xi_t$  can be inferred.

### 2.1 How to Choose the Ideal Spectrograph?

Spectrographs come in different types and with different specifications. When selecting a spectrograph, one needs to keep in mind what information one would like to derive from the spectra and what scientific goals one wants to achieve. Some important spectrograph specifications are: the resolving power of the spectrograph ( $R$ ), available wavelength range, and instrumental efficiency, i.e. throughput of the starlight, which affects the signal-to-noise ratio (SNR).

#### 2.1.1 Spectral Resolution

Low-resolution spectra ( $R < 5,000$ ) are suited to perform a spectral classification, to distinguish between rapid and slow rotation, and to detect chemical peculiarities. For hotter stars, and depending on the available wavelength range, information on  $T_{\text{eff}}$  can mainly be derived from Balmer lines. Medium-resolution spectrographs ( $5,000 < R < 40,000$ ) are generally well-suited for the spectral analysis of many stars, provided that non-blended, well-resolved lines are selected. For fast rotating stars ( $v \sin i > 100 \text{ km s}^{-1}$ ), medium-resolution spectra are generally sufficient for a spectral analysis. In the case of slow rotators, one optimally would prefer high-resolution spectra ( $R > 40,000$ ). The higher the spectral resolution, the higher the confidence in the results from the chemical analysis, and in derived values of  $T_{\text{eff}}$ ,  $\log g$ ,  $v \sin i$ , and  $\xi_t$ .

#### 2.1.2 Wavelength Range

The available wavelength range varies from instrument to instrument. Slit spectra focus on a limited range, while échelle spectra can cover the entire optical spectrum,

often with gaps. If possible, adapt the order definition such that the most important spectral lines you are interested in are completely recorded. Make also sure that sufficient numbers of element lines are available for a reliable analysis. Slit spectra with a limited wavelength range limit the output of the spectral analysis. When planning on using Balmer lines, make sure that they are well defined in the spectrum. In case of échelle spectra, the broad wings of Balmer lines are often located at the edge of spectral orders or fall in interorder gaps, making them unusable for a reliable analysis. Using spectral lines that are ‘reconstructed’ by merging two different orders is not advisable.

### 2.1.3 Signal-to-Noise Ratio

The accuracy of the spectral analysis depends not only on the spectral resolution, but also on the SNR value of the spectra. For a good analysis, a SNR value of at least 100–150 is needed. The higher the SNR value, the better! However, keep in mind that for fainter targets ( $V > 11$ ) obtaining a spectrum with SNR  $> 100$  becomes very time consuming with 1–2 m-class telescopes, or is simply not feasible. In these cases, multiple spectra can be obtained and combined to one spectrum with a higher SNR value (e.g. two spectra of SNR  $\sim 80$  or three spectra of SNR  $\sim 60$  result in one combined spectrum with SNR  $\geq 100$ ).

## 3 KASC and the Characterisation of Asteroseismic *Kepler* Targets

Successful asteroseismic modelling depends strongly on the accuracy of the parameters  $T_{\text{eff}}$  and  $\log g$ , while information on  $v \sin i$  and chemical abundances is important for the understanding of the internal processes. Many of the thousands of stars observed continuously for several months to years with the NASA *Kepler* space mission (Borucki et al. 2010) are pulsating and of interest to the Kepler Asteroseismic Science Consortium (KASC) for asteroseismic studies. However, the *Kepler* one-filter photometric observations do not provide information on stellar atmospheric parameters. Although a characterisation of all *Kepler* targets based on multi-band photometry in Sloan filters and 2MASS HJK bands is available through the Kepler Input Catalogue (KIC), (Latham et al. 2005; Brown et al. 2011), an alternative characterisation of the asteroseismic targets is needed for stars of spectral types earlier than F-type as the KIC values are less reliable in these cases (Lehmann et al. 2011; Molenda-Żakowicz et al. 2011). In particular, KIC values of  $T_{\text{eff}}$  are not accurate for hot stars ( $T_{\text{eff}} > 7,000$  K) as the Sloan filters and 2MASS photometry do not include the Balmer jump, from which  $\log g$  is derived. Also, the general KIC uncertainty on  $\log g$  is 0.5 dex which is not good enough for asteroseismic modelling. Therefore, an independent derivation of atmospheric parameters is needed.

To this end, a systematic ground-based follow-up campaign has been set up to obtain multi-colour photometry and/or spectra for as many of the over 5,000 KASC targets as possible (Uytterhoeven et al. 2010a,b). This ambitious task has many challenges. First of all, it is a time consuming project knowing that at most 14 stars can be observed per night with a high-resolution spectrograph on a 2 m-class telescope. Also, a systematic spectral characterisation is only feasible for bright targets ( $V < 11$ ) even though the majority of *Kepler* targets are fainter than  $V = 12$ . This restriction follows from the requirement of  $\text{SNR} > 100$  to perform a reliable spectral analysis, the fact that with a high-resolution spectrograph on a 2.5 m telescope exposure times to obtain  $\text{SNR} = 100$  are of the order of 20, 45, and  $> 90$  min for a star of magnitude  $V = 9, 10, 11$ , respectively, and the sparse number of high-resolution spectrographs on large ( $> 4$  m-class) telescopes.

The KASC ground-based observations so far involved 40 different instruments at 35 telescopes at 23 observatories in 12 countries in the Northern Hemisphere, and resulted in over 1,500 observing nights.

## 4 Motivation of the Spectroscopic School

The aim of the spectroscopic school is to train researchers in performing spectral analyses, including the derivation of  $T_{\text{eff}}$ ,  $\log g$ ,  $v \sin i$ , and chemical abundances. The motivation of the school lies within the project on the systematic characterisation of pulsators of spectral types B, A, and F of the KASC Working Group on MS pulsators (KASC WG#3). The spectral characterisation is needed to describe the pulsational nature of these stars and to investigate possible connections between stellar oscillations, rotation, and chemical peculiarity. In particular, the instability strips of various classes of pulsating stars need to be tested. For instance, it needs to be verified that  $\delta$  Sct,  $\gamma$  Dor and hybrid  $\delta$  Sct/ $\gamma$  Dor stars are not confined to the current observational pulsation instability strips but are ubiquitous, as suggested from *Kepler* observations by Uytterhoeven et al. (2011), which would call for a new explanation for the observed pulsational behaviour.

A collection of low-, mid-, and high-resolution spectra is currently available for more than 1,000 asteroseismic targets of spectral types B, A, and F, that partly still need to be analysed, while more data is coming. As within KASC there is a lack of manpower with time and expertise in spectral analysis, new experts in spectral analysis need to be trained.

### 4.1 Available Spectra of Kepler Main Sequence Pulsators

So far and within KASC WG#3, spectra have been collected for 1,014 out of 3,119 (i.e. 33%) classified  $\delta$  Sct,  $\gamma$  Dor,  $\beta$  Cep, and SPB stars. Table 1 lists the instruments involved, sorted according to increasing spectral resolution. The different columns

**Table 1** Spectroscopic instruments used for the ground-based follow-up observations of B-, A-, and F-type *Kepler* pulsators

Spectrograph	Telescope	Observatory*	Resolution
IDS	2.5 m INT	ORM (E)	1,400
Multi-object	LAMOST	Xinglong (CN)	1,000/2,000
B&C	2.12 m	OAN-SPM (MX)	2,000
BFOSC	1.5 m Cassini	Loiano (I)	5,000
TWIN	3.5 m	CAHA (E)	10,000
FRESCO	0.91 m	Catania (I)	21,000
ARCES	3.5 m ARC	APO (USA)	33,000
HRS	9.2 m HET	McDonald (USA)	30,000/60,000
FIES	2.56 m NOT	ORM (E)	25,000 /46,000/67,000
Coudé échelle	2.0 m Alfred Jensch	TLS (D)	35,000/67,000
SOPHIE	1.92 m	OHP (F)	46,000
SARG	3.58 m TNG	ORM (E)	57,000
SES	2.1 m Otto Struve	McDonald (USA)	60,000
CS23	2.7 m Harlan J. Smith	McDonald (USA)	60,000
HES	3.0 m Shane	Lick (USA)	60,000–100,000
ESPADONS	3.6 m CFHT	Mauna Kea (USA)	81,000
NARVAL	2.0 m TBL	Pic du Midi (F)	81,000
HERMES	1.2 m Mercator	ORM (E)	90,000

\* *APO* Apache Point Observatory, New Mexico

*CAHA* Calar Alto Astronomical Observatory

*OAN-SPM* Observatorio Astronómico Nacional, San Pedro Mártir

*OHP* Observatoire de Haute Provence

*ORM* Observatorio Roque de Los Muchachos, La Palma

*TLS* Karl Schwarzschild Observatory, Tautenburg

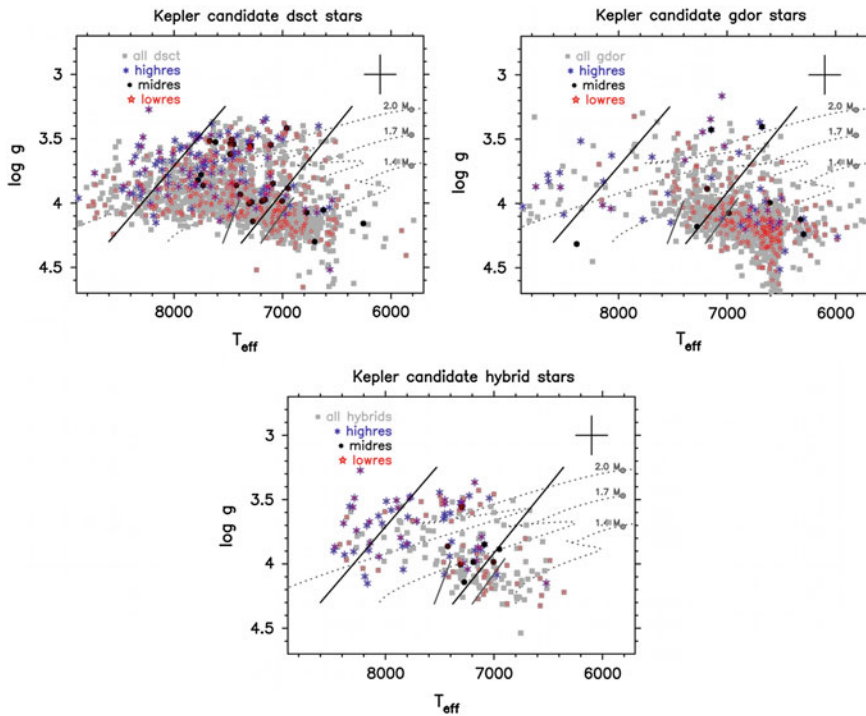
give the name of the instrument, the telescope, the observatory, and the spectral resolution. Note that some instruments have various settings of spectral resolution.

Table 2 gives an overview of the total number of the considered sample of candidate B-type ( $\beta$  Cep and SPB),  $\delta$  Sct,  $\gamma$  Dor, and hybrid  $\delta$  Sct/ $\gamma$  Dor pulsators that have been identified through analysis of *Kepler* light curves. The remaining columns subsequently indicate for each group the number of stars for which low-resolution ( $R < 5,000$ ), medium-resolution ( $5,000 < R < 40,000$ ), and high-resolution ( $R > 40,000$ ) spectra are available, and the total number of stars for which spectra have been obtained, together with the percentage with respect to the total sample. We note that there is an observational bias towards bright targets ( $V < 11$ ), as obtaining spectra with  $\text{SNR} > 100$  becomes very time consuming for fainter targets.

Figure 1 illustrates the location of the candidate  $\delta$  Sct (top, left panel),  $\gamma$  Dor (top, right panel), and hybrid  $\delta$  Sct/ $\gamma$  Dor (bottom panel) pulsators with respect to the observed instability strips of  $\delta$  Sct and  $\gamma$  Dor stars in the ( $T_{\text{eff}}, \log g$ )-diagram. The low-resolution, medium-resolution, and high-resolution spectra available for analysis are indicated by asterisks, bullets, and stars, respectively. Note that this concerns candidate  $\gamma$  Dor, hybrid,  $\delta$  Sct stars [see discussion in Uytterhoeven et al. (2011)].

**Table 2** Number of currently available low-resolution, medium-resolution, and high-resolution spectra for candidate B-type,  $\delta$  Sct,  $\gamma$  Dor, and hybrid  $\delta$  Sct/ $\gamma$  Dor pulsators

Type	# Stars	# Low-res	# Mid-res	# High-res	# Stars with spectra
B-type	48	13	15	39	45 (=94%)
$\delta$ Sct	1,607	389	37	151	513 (=32%)
$\gamma$ Dor	1,205	281	12	70	331 (=27%)
Hybrid	259	84	10	67	125 (=48%)



**Fig. 1** ( $T_{\text{eff}}$ ,  $\log g$ )-diagram for candidate  $\delta$  Sct (top, left panel),  $\gamma$  Dor (top, right panel), and hybrid  $\delta$  Sct/ $\gamma$  Dor (bottom panel) pulsators observed by *Kepler*. All values plotted are taken from the KIC. Light grey squared symbols represent the total sample of stars. Asterisks, bullets, and stars indicate the pulsators that have been observed with high-resolution, medium-resolution, and low-resolution spectrographs, respectively. The solid thick black and light grey lines mark the blue and red edge of the observed instability strips of  $\delta$  Sct and  $\gamma$  Dor stars. Evolutionary tracks for  $MS$  stars with masses  $1.4 M_{\odot}$ ,  $1.7 M_{\odot}$ , and  $2.0 M_{\odot}$  have been calculated with the code CLES (Scuflaire et al. 2008) using input physics similar as described in Dupret et al. (2005), and are plotted with grey dotted lines (for a full description of the evolutionary tracks see Uytterhoeven et al. (2011)). The cross at the top of each panel indicates the typical error bar on  $T_{\text{eff}}$  and  $\log g$  values.

## 4.2 Outlook

It is obvious that the collection of spectra for so many variable B-, A-, and F-type stars is very valuable. This unique sample together with the high-quality *Kepler* light curves promise progress and breakthroughs in the understanding of the pulsational mechanisms and other physical processes occurring inside MS pulsators. To observe also the fainter *Kepler* stars ( $V > 11$ ) in a systematic way and to complete the sample, a significant amount of observing time on high-resolution spectrographs at large telescopes ( $>4$  m) in the Northern Hemisphere is needed. Unfortunately, this is currently not feasible to achieve due to the limited number of specialised high-resolution instruments available and the high demand for observing time. The spectral analysis of over 1,000 stars requires a large effort. Additional manpower is needed. This spectroscopic school has trained new people for joining the project.

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# Stellar Atmospheres: Basic Processes and Equations

Giovanni Catanzaro

**Abstract** The content of this chapter is a very quick summary of key concepts that concern the interaction between photons created in the stellar interior and plasma, which is the basis of the physical processes occurring in stellar atmospheres. The dominant mechanism of energy transport through the surface layers of a typical star is radiation. This is the reason why radiative transfer is our main focus here. We start by setting up the differential equation describing the flow of radiation through an infinitesimal volume and all the related quantities. We conclude with a generic description of the equations used to compute an atmospheric model.

**Keywords** Stars: atmospheres · Stars: fundamental parameters

## 1 Introduction

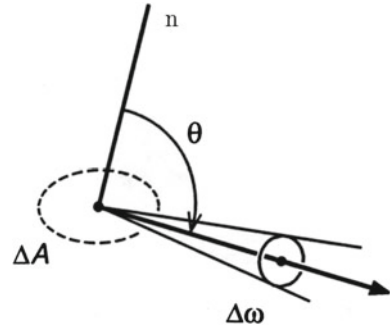
The main goal of this school was to provide students with the tools to analyse stellar spectra with particular reference to the determination of the atmospheric parameters of B, A, F, and G type stars. It is obvious that a careful spectral analysis is not possible without knowledge of the theory of stellar atmospheres. So the purpose of this introductory lesson on this important subject is to provide students with a refresher on the main equations describing the physical processes that occur when the radiation, generated in the interior of the star, interacts with the stellar matter which composes the atmosphere.

If we consider a star as a succession of layers of gas, we know that going deep in the atmosphere gas becomes opaque and our line-of-sight cannot penetrate into the interior layers. We call the stellar atmosphere the ensemble of the outer layers to which the energy, generated in the nucleus, is carried, either by radiation,

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**Fig. 1** The increment of area  $\Delta A$  of a radiating element of material, is seen under an increment of solid angle  $\Delta\omega$  and tilted by an angle  $\theta$  with respect the direction of the normal to the surface.



convection or conduction, before flowing away in the interstellar medium. Interacting with the matter present in the outer layers, this energy finally produces the observed electromagnetic spectrum.

In general, we can say that the theory of stellar atmospheres translates into the study of how the radiation produced in the stellar interior propagates and interacts with the external layers of the star. That is why, during the reading of this introductory lecture on stellar atmospheres we must have always clear in mind this schematic description:

- we call the stellar atmosphere the external layers of a star,
- these are the layers where radiation created in the stellar interior can escape freely into the interstellar medium,
- the atmosphere is the only part from which we receive photons.

Of course, this lecture does not claim to be exhaustive of the topic, but rather a quick recall of the main concepts and definitions. Please refer to specific texts, (i.e. Gray 2005; Hubeny 1996; Mihalas 1978), for a complete and rigorous discussion. In the next sections, before getting to the heart of our topic, we draw some important definitions useful to properly describe light and its interaction with the atmospheric material.

## 2 Basic Definitions

### 2.1 Specific Intensity

Looking at the situation represented in Fig. 1, the **specific intensity** is the quantity of energy  $\Delta E_\nu$  that flows through the element  $\Delta A$  toward the generic direction  $\theta$ , in the solid angle  $\Delta\omega$ , during the time  $\Delta t$ , in the interval of frequency  $\Delta\nu$ . When all these increments become smaller, we can take the limit toward zero:

$$I_\nu = \lim \frac{\Delta E_\nu}{\cos \theta \Delta A \Delta \omega \Delta t \Delta \nu} = \frac{dE_\nu}{\cos \theta dA d\omega dt d\nu}. \quad (1)$$



The right side of this equation is the energy that flows through an element of area  $dA$  in the unit of time  $dt$ , in the unit of solid angle  $d\omega$ , and in the unit of frequency  $d\nu$ . Its physical dimensions are, for example,  $\text{erg rad}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}$ .

Integrating Eq. 1 over all the directions, we obtain the so-called **mean intensity**:

$$J_\nu = \frac{1}{4\pi} \oint I_\nu d\omega, \quad (2)$$

where the integral is calculated over the whole solid angle.

## 2.2 Flux

**Flux** represents the total energy passing across an element of area  $\Delta A$  over the unit of time and frequency. As the specific intensity, we can consider the limit of all the small quantities diminishing toward zero. In this case we will have:

$$F_\nu = \lim \frac{\sum \Delta E_\nu}{\Delta A \Delta t \Delta \nu} = \frac{\oint \Delta E_\nu}{\Delta A \Delta t \Delta \nu}, \quad (3)$$

where again we consider a complete integration over all directions. Flux and intensity could be easily related to each other. If we replace in the left side of Eq. 3 the relation for the energy derived from Eq. 1, we obtain:

$$F_\nu = \oint I_\nu \cos \theta d\omega, \quad (4)$$

that represents the component of the net flux in the direction  $\theta$ .

We can develop this equation for an emitting point on the physical boundary, i.e. the stellar surface. In this case the flux coming in from the outside is null, and if we suppose that there is no azimuthal dependence for  $F_\nu$ , we get:

$$F_\nu = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_\nu \sin \theta \cos \theta d\theta = 2\pi \int_0^{\pi/2} \sin \theta \cos \theta d\theta = \pi I_\nu. \quad (5)$$

This is the equation that we must solve if we want to compute a theoretical spectrum of a particular star. The importance of this equation is then obvious.

## 2.3 K-integral

It is useful to define another equation using the second moment of  $\theta$ , that is, the so-called **K-integral**:

$$K_\nu = \frac{1}{4\pi} \oint I_\nu \cos^2 \theta d\omega. \quad (6)$$

It represents the  $z$ -component of the radiation stress tensor written in Cartesian coordinates. Physically this integral is linked to the radiation pressure, and it is easy to show the validity of the following equation:

$$P_R = \frac{4\pi}{c} \int_0^\infty K_\nu d\nu. \quad (7)$$

### 3 Absorption Coefficient and Optical Depth

Let us consider a slab of plasma and let  $I_\nu^0$  be the specific intensity of the light before the interaction with the slab and  $I_\nu + dI_\nu$  the intensity after the interaction. Let us suppose that only true absorption and scattering give contribution to  $dI_\nu$  while no emission is present. In this case, we can write:

$$dI_\nu = -\kappa_\nu \rho I_\nu dx, \quad (8)$$

where  $\kappa_\nu$  is the absorption coefficient that has units of area per mass ( $[\kappa_\nu] = \text{cm}^2 \text{g}^{-1}$ ) and is therefore a mass absorption coefficient,  $\rho$  is the density in mass per unit volume and  $dx$  is the slab thickness, that has units of length. At this point I have to stress an important concept: the way in which the radiation propagates through the stellar material depends both on the physical conditions of the plasma at a given frequency and on the length of the path. We can say that at a given frequency, the radiation sees the combination of these two factors, namely  $\kappa_\nu \rho dx$ . Define the **optical depth** along the photon direction of propagation as follows:

$$d\tau_\nu = \kappa_\nu \rho dx, \quad (9)$$

which, integrated over some path length  $L$ , becomes:

$$\tau_\nu = \int_0^L \kappa_\nu \rho dx, \quad (10)$$

where  $\tau_\nu$  is the optical depth at a given frequency  $\nu$  and  $x$  is the geometrical depth. It measures a characteristic of matter and radiation coupled together, and corresponds, for a given frequency and absorption coefficient, to the distance at which the intensity is reduced by a factor of  $1/e$ . Using optical depth, Eq. 8 can be written as:

$$dI_\nu = -I_\nu d\tau_\nu, \quad (11)$$

for which the trivial solution is given by

$$I_\nu = I_\nu^0 e^{-\tau_\nu}. \quad (12)$$

In a plasma of astrophysical interest, we distinguish from optically thick, for which  $\tau_\nu \gg 1$ , and optically thin, for which  $\tau_\nu \ll 1$ . I would like to stress again here the importance of frequency: the same plasma (same chemical composition and physical conditions) could be optically thick at a certain frequency, say  $\nu_1$ , but optically thin for another frequency, say  $\nu_2$ .

## 4 Emission Coefficient

Like we did in the previous section, we consider the increase  $dI_\nu$  undergone by the radiation when passing through a slab of plasma. We suppose now that the processes contributing to  $dI_\nu$  are true emission and photons scattering into direction of propagation, with no absorption. In this context, scattering refers mainly to photons previously absorbed and then immediately re-emitted in the direction from the same atomic transition.

If we denote by  $j_\nu$  the emission coefficient (units  $[j_\nu] = \text{erg rad}^{-1} \text{ s}^{-1} \text{ Hz}^{-1} \text{ g}^{-1}$ ), we define the increment of the radiation as:

$$dI_\nu = j_\nu \rho dx. \quad (13)$$

## 5 Source Function and Its Physical Meaning

We can now introduce a new quantity given by the ratio between the absorption and emission coefficients and called the **source function**:

$$S_\nu = \frac{j_\nu}{k_\nu}. \quad (14)$$

This quantity has the same units of the specific intensity and can be seen as the specific intensity of a radiation emitted in some point in a hot gas.

To better understand the meaning of  $S_\nu$ , we can refer to Hubeny (1996) and consider this example: Let us write the number of photons emitted in an volume element  $dV = dx \cdot dA$ , in all directions. From the definition of the emission coefficient, it follows that:

$$N_{em} = \frac{4\pi}{h\nu} (j_\nu \rho dx dA dv dt), \quad (15)$$

where the quantity in parenthesis represents the energy emitted in the volume  $dV$ , the factor  $4\pi$  comes from an integration over the solid angle, and  $h\nu$  transforms energy to the number of photons. By using the definition of the optical depth and the source function, and after some elementary algebra, we obtain

$$N_{em} = S_\nu d\tau_\nu \frac{4\pi}{h\nu} \rho dA d\nu dt. \quad (16)$$

In other words, we have:

$$S_\nu \propto \frac{N_{em}}{d\tau_\nu}. \quad (17)$$

Hence, the source function is proportional to the number of photons emitted per unit of optical depth interval.

### 5.1 Two Simple Cases

In two “extreme” cases, the algebraic form of  $S_\nu$  is simple: pure isotropic scattering and pure absorption.

### 5.2 Pure Isotropic Scattering

All the emitted energy is due to photons being scattered into the direction under consideration. In this case the contribution to the emission  $dj_\nu$  is proportional to the solid angle  $d\omega$  facing the observer and to the energy “absorbed”  $\kappa_\nu I_\nu$ :

$$dj_\nu = \frac{1}{4\pi} \kappa_\nu I_\nu d\omega, \quad (18)$$

where  $\frac{1}{4\pi}$  is the normalization factor for unit solid angle, valid under the hypothesis that the energy is isotropically re-radiated.

To obtain all the contributions to  $j_\nu$ , we proceed with an integration over the solid angle, keeping in mind that  $\kappa_\nu$  is independent of  $\omega$ , and using Eq. 2, we can write:

$$j_\nu = \frac{1}{4\pi} \oint \kappa_\nu I_\nu d\omega = \frac{\kappa_\nu}{4\pi} \oint I_\nu d\omega = \kappa_\nu J_\nu. \quad (19)$$

From this equation it is straightforward to show that:

$$S_\nu = \frac{j_\nu}{k_\nu} = J_\nu. \quad (20)$$

In short, in the simple case of pure isotropic scattering, the source function is the mean intensity. Moreover, when thermodynamic equilibrium holds, the radiation intensity is equal to the Planck function, i.e.  $J_\nu = B_\nu$ .

### 5.3 Pure Absorption

Now we are assuming that all the absorbed photons are destroyed and all the emitted photons are newly created with a distribution governed by the physical state of the gas. The source function for this case is given by Planck’s radiation law:

$$S_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} = B_\nu(T). \tag{21}$$

This is the specific intensity emitted by a gas of a temperature  $T$  and for a given frequency  $\nu$ .

## 6 The Transfer Equation

In the previous sections we have discussed separately the cases of radiation travelling in a slab of stellar material in which it is affected either by losses, expressed in the absorption coefficient  $\kappa_\nu$ , or gains, expressed in the emission coefficient  $j_\nu$ . Now we consider the general case in which the change in specific intensity,  $dI_\nu$ , over an increment of linear path length  $ds$ , is the sum of those losses and gains, expressed as:

$$dI_\nu = -\kappa_\nu \rho I_\nu ds + j_\nu \rho ds. \tag{22}$$

This equation can be written in a more useful form, by dividing both sides by  $\kappa_\nu \rho ds$ , and using the definition of source function (Eq. 14):

$$\frac{dI_\nu}{\kappa_\nu \rho ds} = -I_\nu + \frac{j_\nu}{\kappa_\nu} = -I_\nu + S_\nu. \tag{23}$$

Finally, we have the differential form of the **equation of radiative transfer**

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu. \tag{24}$$

The integration follows from a standard integrating-factor scheme. After some manipulation, we obtain the so-called integral form of the radiative transfer equation:

$$I_\nu(\tau_\nu) = \int_0^{\tau_\nu} S_\nu(t_\nu) e^{-(\tau_\nu - t_\nu)} dt_\nu + I_\nu(0) e^{-\tau_\nu}. \tag{25}$$

The meaning of this equation can be easily understood: radiation along the line at the point  $\tau_v$  is composed of the sum of intensities,  $S_v$ , originating at the generic points  $t_v$  along the line, but suffering extinction according to the optical-depth separation  $\tau_v - t_v$  (first term of the sum), plus the radiation due to the original intensity  $I_v(0)$  that has suffered an exponential extinction  $e^{-\tau_v}$  (second term of the sum).

Equation (24) holds along a line. In stellar atmospheres applications, it is useful to define the optical depth relative to the star along a stellar radius, and not along the line of sight. We are also assuming, in the following discussion, that as the atmosphere is thin with respect to the radius, a plane-parallel approximation can be used.

Assuming spherical coordinates originating in the centre of the star and with the  $z$  axis toward the observer, we write the transfer equation in the form:

$$\frac{1}{\kappa_v \rho} \frac{dI_v}{dz} = -I_v + S_v. \quad (26)$$

Let us write  $\frac{dI_v}{dz}$  according to spherical geometry; if we assume  $I_v$  has no azimuthal dependence, we obtain:

$$\frac{1}{\kappa_v \rho} \left( \frac{\partial I_v}{\partial r} \frac{dr}{dz} + \frac{\partial I_v}{\partial \theta} \frac{d\theta}{dz} \right) = -I_v + S_v. \quad (27)$$

We know, from geometrical consideration, that,  $dr = \cos \theta dz$  and  $r d\theta = -\sin \theta dz$ . Then, by substitution of these expressions, and keeping in mind that for a plane-parallel atmosphere  $\theta$  does not depend upon  $z$ , the transfer equation becomes:

$$\frac{1}{\kappa_v \rho} \left( \frac{\partial I_v}{\partial r} \cos \theta \right) = -I_v + S_v. \quad (28)$$

Adopting the convection of using a new geometrical depth variable, defined as  $dx = -dr$  and writing  $d\tau_v$  for  $\kappa_v \rho dx$ , we have the basic form of the radiative transfer equation used in the stellar atmosphere applications:

$$\cos \theta \frac{dI_v}{d\tau_v} = I_v - S_v. \quad (29)$$

## 6.1 Elementary Solutions

Following the outline depicted in Hubeny (1996), in this section, we describe the simplest solutions of the 1-D plane-parallel radiative transfer equation.