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Multiscale Modeling of Pedestrian Dynamics

Emiliano Cristiani • Benedetto Piccoli • Andrea Tosin

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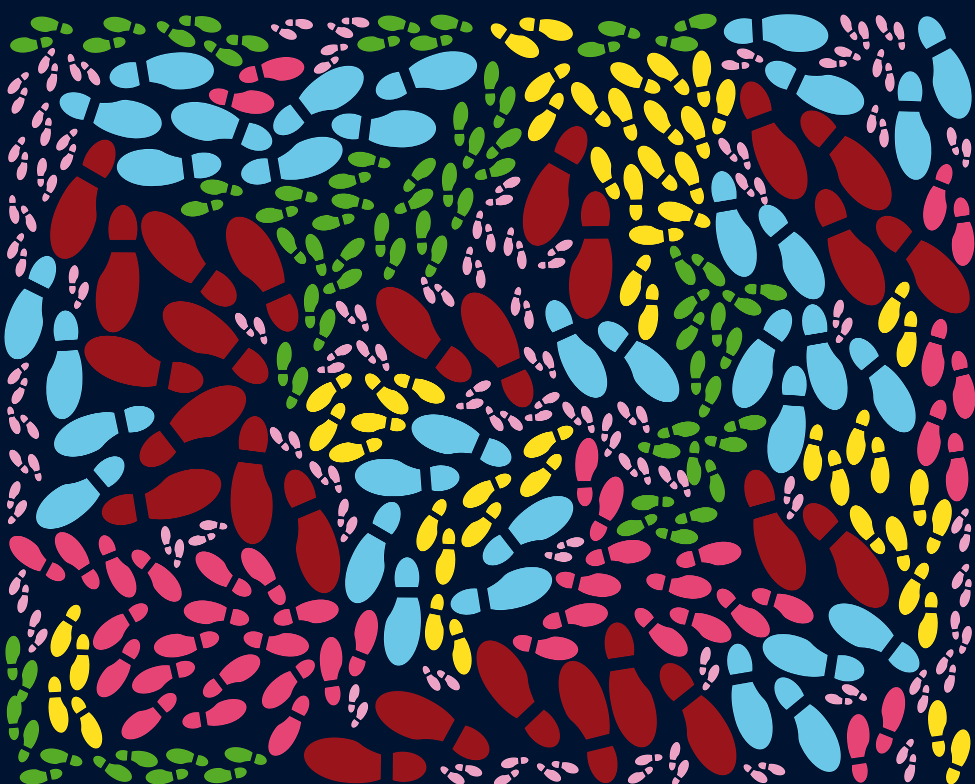

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If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

John von Neumann

To my newborn son Dario, who will soon be a pedestrian

To my son Edoardo

To my parents Rosanna and Dario

Preface

This book presents to researchers and graduate students mathematical models and numerical simulations of crowd dynamics. The book is addressed to scholars and professionals with different backgrounds, in particular applied mathematicians, physicists, engineers, system biologists, and psychologists, who can, for various reasons, be interested in mathematical modeling of crowd behavior and, more in general, of granular flows in living and nonliving complex systems from a multiscale point of view.

In a broader sense, this book is about the *science of mathematical modeling*, seen in action under the particular perspective of pedestrian dynamics modeling. The *leading idea* is that Applied Mathematics does not just consist in the application of existing models to practical case studies but, first and foremost, in the construction of *original mathematical approaches* motivated by often nonstandard problems continuously posed by the real world. In this respect, the cultural path followed in the book encompasses rigorous procedures of mathematization of reality, analysis of the mathematical structures thereby derived, and simulation of realistic scenarios which can constitute a basis for a fruitful dialogue with non-mathematical practitioners.

Research about crowd dynamics is fostered by both theoretical and practical reasons. On the one hand, many scholars want to understand the basic principles of pedestrian motion. Their insights can often be translated into mathematical models, which can be validated through simulations. On the other hand, practitioners are interested in faithful simulations of self-organized phenomena arising in pedestrian flows, especially in complex-shaped two-dimensional built environments. Indeed, it is well known that neglecting group behaviors can lead to major safety issues. Therefore, our interest is mainly focused on models which reproduce the spontaneously emerging *self-organized collective patterns* out of an accurate and realistic design of *individual interaction rules*. Namely, without resorting to the artificial inclusion of empirical features in the mathematical equations with the primary aim of reproducing target phenomena. In this respect, we note that crowd dynamics are often nicely visualized in computer graphics animations: Our approach is rather different, since we aim at a deeper understanding, through mathematical models, of the basic dynamical principles ruling crowd behaviors. To this goal, we consider two points of view which have been classically taken in crowd dynamics modeling: The *microscopic* one, in which pedestrians are tracked individually, and the *macroscopic* one, in which pedestrians are assimilated to a continuum and observed through their average density. We present in detail and critically analyze selected existing models. Then, as a *core topic*, we develop a *multiscale paradigm*, which allows one to bridge the various scales, taking the most from each of them in terms of capturing the relevant clues of complexity of crowds. Our background idea is indeed that most of the complex trends exhibited by crowds are due to an intrinsic interplay between *individual* and *collective behaviors*, which are capable of affecting each other. The modeling approach we promote in this book pursues actively this intuition and profits from it for designing a general multiscale mathematical method susceptible of application also in fields different from the inspiring original one.

The book is divided in two parts and eight chapters, plus two appendices. The first part, mainly introductory, is dedicated to a broad audience. It features virtual experiments pointing out, on the one hand, the phenomenology of pedestrian behaviors we are interested in and, on the other hand, the ability of our multiscale model to address such phenomena. The second part, characterized by a more technical content, presents an overview of single-scale models and the details of our multiscale approach, together with analytical and numerical results, plus its generalization to different application fields.

The hallmarks of the present work, which make it different from other books on the same topic available in the literature, can be summarized as follows:

- This book promotes a true interplay among modeling, theory, and numerics for a cutting-edge multidisciplinary research topic. One of the leading principles is that models should originate from a correct interplay between real world and mathematics.
- This book offers an accurate review of models of crowd dynamics: Both seminal and the most relevant descending works are presented with a sufficient detail to allow readers to be quickly up-to-date with the state of the art in the field.
- This book focuses on a new multiscale description of crowd dynamics, based on measure theory, which covers the full path of Applied Mathematics: Model derivation, qualitative analysis, construction and analysis of numerical schemes, and application of the algorithms to the simulation of more and less standard benchmarks in pedestrian dynamics.
- Numerical tests highlight the effects of the interplay between small and large scales on pedestrian dynamics, suggesting that crowd modeling requires definitely a multiscale approach, in which scales truly integrate and complement.
- This book provides a ready-to-implement pseudo-code version of the multiscale algorithm.

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July 4, 2014

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Contents

Part I
Pedestrian Behavior: Phenomenology and Simulations

Chapter 1
An Introduction to the Modeling of Crowd Dynamics

Abstract In this chapter we begin the discussion about crowd dynamics from an informal phenomenological point of view. In particular, we put in evidence how simple interaction rules adopted independently by pedestrians generate, at a collective level, complex group behaviors featuring various forms of self-organization. Bearing in mind the ultimate goal of the book, which is mathematical modeling, we promote the idea that understanding such basic behavioral rules contributes to the modeling at all scales, also those not directly focused on single individuals. In the light of these arguments, we critically analyze the main scales of observation and representation which are typically used in mathematical modeling, namely the microscopic, the macroscopic, and the mesoscopic (or kinetic) scale. For each of them we discuss the advantages/drawbacks in catching/losing specific features of crowd dynamics, with a view also to the interplay with the available experimental knowledge about crowds. Finally we elucidate the role of the book in this cultural framework and we give reading directions through the various chapters targeted to a few different kinds of readerships.

1.1 Modeling-Oriented Phenomenological Issues

In this section we review the most important pedestrian behavioral rules which are usually taken into account in crowd modeling. As a complement, we discuss the concept of *self-organization* as a collective (and hardly predictable) result of those behavioral rules.

1.1.1 Behavioral Rules

Modeling crowd dynamics requires to identify at least the most important behavioral rules pedestrians are subject to. It is plain that a pedestrian, as a complex living being, is basically unpredictable. Nevertheless, some guidelines can be drawn.

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1.1.1.1 Target

In most of the cases, people move in a bounded space and have a desired destination to be reached. This destination, together with the geometry of the space, defines a *desired velocity field* which is exactly the velocity people would keep if they were alone in the domain. The desired velocity can be very different whether the pedestrian under observation knows the domain or moves in a unfamiliar environment. In the latter case, an exploration phase has to be taken into account. In the rest of the book, we will denote by $\Omega \in \mathbb{R}^d$ the walking area, $\Gamma \in \Omega$ the target, and $v_{\mathrm{d}} : \Omega \to \mathbb{R}^d$ the desired velocity field, where d is the dimension of Ω, usually $d = 2$.

The final velocity field pedestrians actually follow will be given by a suitable combination of the desired velocity field and the *interaction velocity field*, defined taking into account the following features of pedestrians.

1.1.1.2 Repulsion

People want to avoid collisions, so they stop when they are too close to other people. Moreover, they have a tendency to avoid crowded regions, as well as to stay clear of walls and obstacles. Often mathematical models take into account this behavior by assuming the existence of a fictitious *repulsive force* which drives people toward clear spaces.

1.1.1.3 Attraction

Sometimes people have the tendency to follow other people or simply stay in touch. This is the case of *social groups* like friends, families, tourist groups, and so on. For example, small groups of walking friends want to reach their destination all together, while keeping eye-contact and speaking with each other. Instead, tourist groups want primarily stay in touch with their guide (i.e. the sole person who knows the destination) and then keeping the group itself cohesive.

1.1.1.4 Keeping Direction

People have the tendency to keep the same direction of motion, since changing direction is tiresome and usually inefficient. This is one the reason which makes walking through a crowd an annoying task.

1.1.1.5 Visual Field

People have a limited visual field. It is usually assumed to be an angle of 170° or 180°, where the central area is sharper than the lateral ones. The line which divides in two equal parts the visual field can coincide with the actual direction of motion or, instead, with the desired direction of motion, and it is obviously related to the head orientation. If, on the one hand, the assumption that people can see only in front is reasonable, on the other hand it must be noted that people can turn their head, thus perceiving almost all the space around, and that other senses than sight can be involved, like, e.g., hearing and touch. Visual field is also limited by any obstruction people can perceive, like walls, columns, and other pedestrians themselves.

1.1.1.6 Sensory Regions

In normal conditions, people do not interact with the others by contact, as mechanical particles do. Rather, they observe the surrounding space and take decisions. Sensory regions, *which are in general different from the visual field*, represent the portion of the space effectively considered before taking a decision, and can be different from need to need. For example, pedestrians are mainly repulsed by other people walking both close and in front to them, or by people walking on a "collision course", while they are little or no repulsed by far-away pedestrians, even if they are in the visual field. Then, repulsive sensory region is usually short-range and anisotropic. Attraction, instead, can be much more extended in space, even up to the whole visual field.

Sensory regions are one of the main ingredients of the mathematical models and sometimes make the difference among them. Indeed, changing the shape of the sensory regions defined for the various tasks leads to major differences in the simulated pedestrian behavior.

In the rest of the book, we will denote a generic sensory region by $\mathscr{S}(x)$, where x is the position of the pedestrian under observation.

1.1.1.7 Metric vs Topological Sensory Regions

People have limited capabilities in processing information. They cannot perceive (and then respond to) many stimuli at the same time. This means that concurrent stimuli are processed one after the other, and complex situations are actually "simplified". In particular, people do not interact (by means of a repulsion force, for example) with more than a few people contemporaneously. If a *topological* definition of the sensory region $\mathscr{S}$ is applied, $\mathscr{S}$ is continuously enlarged or shrunk in order to include exactly the number of pedestrians who can reasonably be taken into account. This choice effectively models the limitations of the human brain in terms of processing information. On the contrary, if a *metric* definition of $\mathscr{S}$ is applied, the size of $\mathscr{S}$ is fixed once and for all, and the number of the observed pedestrians can be either less or more than the possible one. The latter choice is usually made for simplicity, but it can lead to unrealistic results, especially in case of large crowds.

1.1.1.8 Panic

In case of panic, some of the behavioral rules described above change or cease to apply. The main difference with respect to the normal conditions is that interactions become *physical* and people start pushing. This obviously change the shape of the sensory regions. In addition, people move faster, change directions more often, are attracted to people who have a clear direction (in the hope they have found a safe way out), become more selfish stopping any kind of collaboration, and coordinated movements are lost.

1.1.2 Self-Organization

We say that a crowd is "organized" when a *leader* (inside or outside of the crowd) decides its spatial distribution in order to maximize some global utility. Instead, we say that a crowd is "self-organized" when a coordinated spatial distribution arises by simply applying some local behavioral rules or one-to-one interactions among pedestrians. In the latter case, no one really decides the final shape of the crowd, and the shape which is finally assumed does not necessarily maximize some global utility. Nevertheless, it can happen that people take advantage of the organization which is spontaneously created.

Self-organization in pedestrians shows some important differences with respect to the same phenomenon observed in other biological systems, like animals or cells. As an example, let us consider the well known self-organizing phenomenon of trail formation in ants. In that case, the trail arises from two simple local rules followed by the ants: First, if an ant has found some food and it is coming back to the nest, then it drops on the ground a particular scent, which slowly evaporates. Second, if an ant smells the scent, it follows the trace. These two rules do not require any direct interaction among ants, and no long-range information about food location is spread around the ant colony by an informed individual. It must be noted that ants *are not aware* of forming or following a trail, which can be quite long if compared with the ant size. Then, the overall organization is not perceived by the ants.

It is very difficult to find among humans a phenomenon which shows a perfect analogy with the one mentioned above. Indeed, humans usually perceive and understand the global phenomenon they are part of, even if they did not want or was not able to forecast its emergence. Then, if a single pedestrian is not comfortable with the self-organized phenomenon she is contributing to form, she can rationally change the local rules which are the cause of that phenomenon. This makes pedestrians able to control the overall system, at least partially.

Having this in mind, we list here some self-organized patterns which are observed in crowds and often attempted to be reproduced by mathematical models, see Figs. 1.1–1.3.

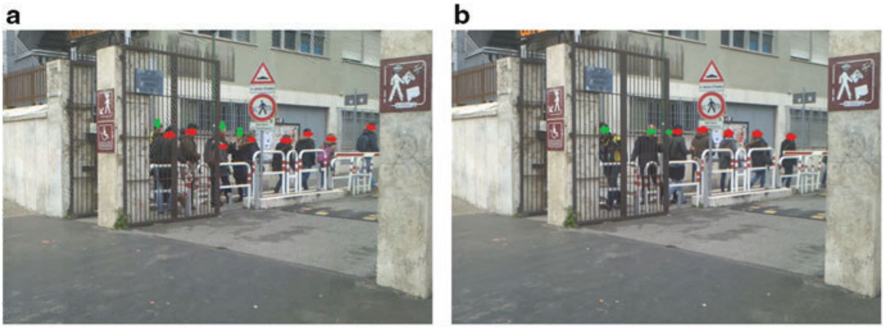

Fig. 1.1 Intermittent flows. Photos taken on February 5, 2014 (Wednesday), lunch time, Sapienza – University of Rome. The gate does not allow the passage of two pedestrians at the same time. (**a**) The first group, marked with *green arrows*, has to wait for the passage of the second group, marked by *red arrows*; (**b**) as soon as the first group passed through, the first group moves ahead (©Emiliano Cristiani)

Fig. 1.2 Arching. Photos taken on January 8, 2013 (Tuesday) in Tivoli (Rome), 8:00 a.m., train station. As soon as the train reaches the station, people run toward the nearest door. They are aware that last people in likely have to be standing for the entire 1-h journey because of lack of seats (©Emiliano Cristiani)

- Intermittent flows at shared bottleneck (Fig. 1.1). When two populations of pedestrians walking in opposite directions have to share a bottleneck, a sort of traffic light effect is observed: Some people of the first population passes the bottleneck, then they stop to allow some people of the other population to pass, and so on. The break of symmetry arises naturally, since there is no leader who settles the flows.
- Arching at bottlenecks (Fig. 1.2). When a large number of people has to pass quickly through a bottleneck (a small door, for example), the formation of a semi-circular arc just before the bottleneck is observed. In particular, people do not queue neatly one after the other. This arching effect can actually drop the efficiency of the overall dynamics.
- Lane formation in crossing flows (Fig. 1.3c). Two populations of pedestrians which walk in opposite directions self-organize in lanes. The space is divided in stripes, each of which is occupied by pedestrians moving in one direction only. This way diminishes the probability of encounters among pedestrian having opposite directions, thus improving the overall efficiency of both flows.

Fig. 1.3 (**a**) V-like pattern. Photo taken on November 14, 2013 (Thursday), University of Rome "Tor Vergata". Three post-docs come back to their office after lunch; (**b**) teen wall. Photo taken on November 9, 2013 (Saturday) in Tivoli (Rome), 6 p.m., main shopping street; (**c**) lanes in crossing flows. Photo taken on February 5, 2014 (Wednesday), lunch time, Sapienza – University of Rome. Note that the crosswalk is partially obstructed by improperly parked cars, and the pedestrian traffic light is *red* (!). Forming lanes is the only way for pedestrians to cross the street sufficiently fast (©Emiliano Cristiani)

- V-like (Fig. 1.3a). Small social groups of walking friends or family members often assume a configuration which resembles a V, where the vertex is pointing against the direction of motion. This terminology comes from the biological literature, in particular that regarding migrating geese which show a V-like configuration where the vertex is directed toward the flying direction. By means of the V-like configuration each pedestrian finds a comfortable walking position supporting visual and verbal communication with the other group members. When the group has more than four members, it often split up in smaller groups.
- River-like and wall-like configuration (Fig. 1.3b). In social groups, V's are a good compromise between a short-range repulsion (to avoid collisions) and attraction (to stay together and communicate). When the surrounding crowd density reaches high levels, physical constraints prevail over social preferences and communications stop. People then start walking one behind the another,

forming a river-like pattern. An interesting exception is represented by groups of (hand-holding) teenagers. They are less susceptible to physical constraints and more interested in communicating and being noticed. Then, they can form outright moving walls orthogonal to the direction of motion, forcing the others to slow down and circumvent them.

In Sect. 3.2.3 we will come back on self-organization issues with a more detailed discussion which follows some psychological considerations about pedestrians as individuals.

1.2 Preliminary Reasonings on Mathematical Modeling

1.2.1 Crowds as a Living Complex System

When attempting to describe new real world systems by mathematical equations one is normally faced with two possible approaches. The first one can be called the approach *by analogy*. One tries to figure out whether a more familiar system exists, which has already been successfully modeled, showing qualitatively comparable trends to those observed in the new system. Then one uses the models set up for the familiar system as a starting point for the mathematization of the new one. The second approach is instead based on the idea that the novelty itself of the new system should induce a mathematization ab initio, i.e., from very basic first principles.

The validity of either approach depends strongly on the peculiarities of the new system at hand. If there are reasons to suspect that the way in which it works is phenomenologically different from other better known systems then the first approach should be rejected in favor of the second one, despite the possible similarity of the observable behaviors. In fact, mathematical models should not limit themselves to reproducing observable behaviors. They should mainly identify the underlying less visible causes leading to such behaviors, so as to provide an essential explanation of the basic mechanisms ruling the system. Indeed, it is on this basis that reliable simulations and predictions can be grounded, also in scenarios not yet empirically tested. On the other hand, if the new system is clearly structurally similar to another one then the second approach should be rejected in favor of the first one, because it runs the risk of being uselessly time-consuming. In fact, there is no need for rediscovering from the beginning well consolidated mathematical structures. However, in both cases a preliminary careful analysis of the distinguishing features of the new system is the essential first step.

Human crowds can be classified, with good reasons, among the *new* systems which mathematics has started to deal with in relatively recent times. At the beginning, the main modeling approach was by analogy with particle systems of gas dynamics. Pedestrian dynamics were assimilated to those of gas particles described as a continuum flowing in space, the inspiration being drawn mainly from the similarity of the observed qualitative flow patterns in the two cases. Some authors also borrowed a terminology proper of fluid dynamics, using expressions such as *laminar* and *turbulent* flow, for describing different regimes of crowd movement. However, more recently the idea of crowds as a *living complex system* has begun to impose itself, suggesting that an approach ab initio may be preferable for constructing more targeted mathematical models.

To say that crowds are a complex system means basically that one cannot expect to predict the behavior of *many* pedestrians from the detailed knowledge of the behavior of *one* pedestrian (as it happens instead for e.g., fluid particles). Indeed individual pedestrians modify continuously their local walking program due to interactions with neighboring people. This way they generate spontaneous collective trends not directly contained in the simple behavioral rules followed by each of them. It is as if repeated mutual interactions "amplified" the effect of the individual behavioral rules in a hardly controllable way. Even more important, all of this is made possible largely by the fact that crowds are *living* systems, i.e., they are not *passively* subject to the inertia law like the inert matter (e.g., again fluid particles). Actually, this does not mean that pedestrians elude the usual laws of Physics. Rather, they are able to influence the latter *actively* through personal decisions, whose impact is not necessarily assimilable to that of external force fields.

In view of the reasonings just proposed, it is of some importance to discuss a few basic complexity clues of crowds that mathematical models should cope with. In order for model to comply with them as much as possible, it can be guessed that methods traditionally used for describing the inert matter have to evolve, as already implied, in new mathematical ideas.

1.2.1.1 Interactions and Multiscale Effects

Interactions among pedestrians pertain specifically to the scale of *single individuals*. Indeed, they are usually one-to-one, or at most one-to-few, as they involve immediate neighbors. On the other hand, the probably most striking effect of such interactions is the spontaneous emergence of self-organized *collective* flow patterns, clearly visible at a group level. This kind of influence of smaller on larger scales can be viewed as an *individuality-to-collectivity* scaling. Nevertheless, also the opposite influence is possible, namely the local collective state of the crowd (e.g., the local crowding of an area) can modify the individual interaction rules. Mathematical models should provide a way to link the individual point of view, which generates the dynamics, to a large-scale collective representation not necessarily focused on single pedestrians.

1.2.1.2 Perception Ability and Expression of Behavioral Strategies

Pedestrians react to the neighboring crowd according to the *perception* they have of it. This is a subjective ability, which corresponds to the expression of a behavioral strategy. For instance, depending on the state of the surrounding environment (including, but possibly not limited to, the number and localization of other nearby people) and on the travel purpose (leisure, commuters, rush hours, ...), they can feel themselves facing either well-focused individualities or more blurred "packages" of walkers. Consequently, even if the elementary interaction rules are always the same, their global effect can be extremely different because of the *filtering* operated by pedestrian psychological perception. It is worth pointing out that the latter is an *active* ability of pedestrians as living agents, which can greatly impact on the effect of basic physical laws. From a complementary point of view, perception is the psychological consequence of the physical fact that crowds are extremely *granular* systems, i.e., systems in which the real dynamics originate at the level of single individuals. Mathematical models should explain how perception influences the usual dynamical laws and contributes to originate different observable outcomes.

1.2.1.3 Large Deviations, Loss of Determinism, and Panic Onset

The expression of the aforesaid behavioral strategies, as well as their impact on standard laws of Physics, can be considered under either a deterministic or a stochastic point of view. The former is appropriate in *normal* conditions, i.e., when a standard rational attitude can be identified over which large deviations are not expected to occur. Conversely, the latter is suited for addressing cases in which irrational behaviors cannot be excluded, which might induce large deviations over the normal trends even up to *panic* onset. Mathematical models should account, at least at a qualitative level, for the transition from normal to panic conditions, namely explain how it can be triggered and how normal behavioral rules are modified in extremely critical situations. However, we anticipate that in this book we will be concerned only with crowd dynamics in normal conditions, so that an essentially deterministic approach is admissible.

At last, it is worth mentioning that models conceived for treating complex systems should deal with the above, and possibly also other, complexity clues within the standards of the mathematical reductionism. That is, the modeled system should not be as complex as the real one, for otherwise models are mostly ineffective for practical purposes. Therefore, a balance has to be sought between following the aforesaid guidelines and envisaging suitable strategies of complexity reduction by means of proper mathematical structures.

1.2.2 Scaling and Representation

The first step of the modeling approach is the choice of the most appropriate scale for describing, by mathematical equations, the system at hand. This book is concerned with crowds as ensembles of interacting individuals, who, as discussed above, generate complex dynamics involving various scales. Therefore, its background idea is that the modeling approach should necessarily pursue a *multiscale* perspective.

However, various models exist in the literature which adopt a single-scale viewpoint. Hence it is important to explore the main characteristics of each modeling scale, also for a preliminary assessment of their possible links.

Traditionally, three types of mathematical descriptions are considered, corresponding to as many observation and representation scales.

1.2.2.1 Microscopic Scale

The microscopic scale is the one at which the minimal entities composing a system, henceforth called *particles* for brevity, are visible. Here the adjective "minimal" stands for "atomic". That is, particles are regarded as the very fundamental constituents, further levels of detail being unnecessary for explaining the genesis of the dynamics exhibited by the system. In crowd dynamics, the microscopic scale corresponds to the level of single pedestrians, who can indeed be regarded as the atoms of a crowd. Mathematical models at the microscopic scale describe the movement of each single walker by means of proper *state variables*, normally position and velocity (at least in purely mechanical contexts). Since pedestrians are tracked one by one from their initial positions, this kind of description is also called *Lagrangian* (although such a terminology has no direct connection with the modeling scale).

Depending on the reference mathematical framework, microscopic models can be formalized in a few different ways. For instance, *differential* models use systems of ordinary differential equations which express the variation in time of the state variables attached to each pedestrian (much as in Rational Mechanics). On the other hand, *agent-based* models, such as e.g., Cellular Automata, update the microscopic states of pedestrians at discrete times according to mainly algorithmic evolution rules.

1.2.2.2 Macroscopic Scale

As the name itself suggests, the macroscopic scale is just the opposite of the microscopic one. The focus is no longer on single particles, viz. pedestrians, but rather on their average distribution, which is described by means of a *density* in space, usually denoted by ρ, evolving in time. An immediate technical difference with the microscopic scale is that now space is, together with time, an independent variable. Indeed, one is not labeling anymore pedestrians one by one in order to track them along their paths. Rather, the space variable refers to arbitrary positions in the geometric space possibly crossed by different walkers at different times. This viewpoint is also called *Eulerian* as opposed to the Lagrangian one characteristic of the microscopic scale.

Ideally, the macroscopic picture is what is seen by a sufficiently far observer, who cannot distinguish individual pedestrians but detects just their collective mass. In order for this point of view to make sense, it is conceptually necessary that the