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Jacques Sesiano

Liber Mahameleth

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The *Liber mahameleth*

A 12th-century mathematical treatise

Part One

General Introduction, Latin Text

Jacques Sesiano

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Jacques Sesiano
Département de Mathématiques
Ecole polytechnique fédérale
Lausanne, Switzerland

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Preface

I began studying the *Liber mahameleth* in 1974/75, while still a PhD student at Brown University in the United States. There, I shared my first impressions with Prof. E. S. Kennedy, who was very interested in what I considered at the time to be a translation from the Arabic. The present work was finally ready for the publisher in 2012, a couple of years after I retired from teaching at the Ecole Polytechnique Fédérale in Lausanne. That is, it took me almost forty years to complete it, no doubt more than its author spent writing it. I am not ashamed to admit this; after all, the publication of Euler's complete works began in 1911 and still remains to be finished a hundred years later, a length of time which far exceeds Euler's active life.

I have to admit, though, that my work on the *Liber mahameleth* was not continuous. There were times when it was a relief to put it aside for a while. Yet sooner or later I felt the need to return to it, either as a duty to its author or on being urged to do so by someone else. First of all, there was my former professor at Brown University, Gerald Toomer, to whom I owe my whole training in the history of mathematics, my interest in Greek, Latin and Arabic mathematical manuscripts, and my awareness of the need for caution when dealing with critical editions. Second, my colleague Ahmed Djebbar gave me on many occasions the opportunity to present my findings, in France or in Algeria. Meanwhile, Janice McLennan read and reread the English text until she could not find anything more to change, from which I gathered that I had reached a presentable version. Finally, my former student Christophe Hebeisen was of considerable help in establishing the text: he initiated me in the use of TeX and EDMAC and was always available in case of problems.

During all those years, I had to make frequent visits to the Bibliothèque Nationale in Paris and the Biblioteca Capitolare in Padua in order to check my reading of the manuscripts. There I always met with a cordial reception, particularly in the second, smaller library. Finally, all possible facilities were granted me during my career at the EPFL and, if I no longer have the privilege of teaching there, the present work, the fruit of a career-long study, will remain as a reminder of that pleasant time.

Geneva,
Switzerland,
June 2012.

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General Introduction

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General introduction

1. The rebirth of mathematics in mediaeval Europe.

1.1 Early mediaeval times.

The European Middle Ages are often considered to have seen no, or hardly any, scientific achievements, at least in the field of pure science. Besides being a sweeping judgment, this overlooks the circumstances which led to the demise of science in early mediaeval times. Apart from the politically unfavourable situation, access to Greek culture was no longer possible since contact with the Eastern empire of Byzantium had been cut off. Now what late antiquity had left of Greek science in Latin translation was very little. Mathematics, since that concerns us here, was particularly affected. From higher mathematics, by which is meant the geometrical and mechanical works by Archimedes, the study of conic sections by Apollonios, and the algebra of Diophantos, nothing had been translated. Of the fundamental work for mathematical training and thinking, the *Elements of geometry* by Euclid, only extracts remained, namely definitions and propositions taken from the first five Books but, except for the first three theorems, without the demonstrations; thus the main mathematical treatise of ancient Greece was reduced to little more than a primer of properties and formulae. Next, a book by Nicomachos on the elementary properties of integers, thus on number theory, which had a certain success in antiquity, inspired Boëtius, who around 500 made a Latin adaptation of it; that was the only Greek mathematical work to have survived in its entirety. Otherwise only land-surveyors' formulae were transmitted along with various problems, dealing with mensuration of areas and recreational mathematics, from which Alcuin, then at the court of Charlemagne, drew his 'Propositions to sharpen the wits of young people' (*Propositiones ad acuendos iuvenes*). In short, nothing that reflected the major achievements of Greek science was extant in the early Middle Ages. On the other hand, there occurred in Italy around 1500 the most impressive step forward in algebra since ancient times, namely solving the third-degree equation.

1.2 The twelfth and thirteenth centuries.

In Europe, the rebirth of mathematics began in the 12th century, when the partial reconquest of Spain gave the Christians access to scientific manuscripts in Arabic (of texts originally either in Greek or in Arabic), while contacts with the Byzantine Empire resulted in the transmission of some Greek manuscripts. Thus, Euclid's *Elements* were translated, from the Arabic several times, once from the Greek also, together with commentaries of Arabic origin.* In the field of arithmetic, the introductory work

* For an overview, see Murdoch's article *Euclid* in the *Dictionary of Scientific Biography*.

written ca. 820 by Muḥammad al-Khwārizmī, which had played a pioneering rôle in the Islamic world by spreading the use of the Indian numerals, was twice translated into Latin, thus coming to play the same rôle in the Christian world four centuries later.[†] In the field of algebra, the work by al-Khwārizmī (also intended for a general public) was translated —once again several times— but not in its entirety: the two sections following the algebraic reckoning and its application to trade, namely the use of algebra in geometry and inheritance calculation, were left out. We also find translations of a few collections of problems, either of a computational or geometrical nature, with or without the use of algebra. In addition to these translations, the 12th century saw the writing of two works which were original but heavily relied on Arabic sources. The first is an introduction to arithmetic, with a few complements, notably on algebra, by Johannes Hispalensis (John of Seville).[‖] The second is the *Liber mahameleth*, which now deals not only with the basics of reckoning and use of algebra but also with their application to daily and commercial life (trade, hiring, sharing).

This was one of the two ways arithmetic and algebra reached Christian Europe. The other, later, one was through the work of Leonardo Fibonacci, who flourished around 1220. As he himself tells us at the beginning of his *Liber abaci*, he went while still a child for a short time to Bejaia, in Algeria, where his father was in charge of the Pisan merchants. There he was taught elementary reckoning with the Indian numerals (from which we gather that towards 1180 the works translated in Spain cannot have been widely known in Italy). As a merchant, Fibonacci was able to travel all around the Mediterranean, in particular in the Moslem East and the Byzantine Empire. There he apparently met with mathematicians and was thus able to widen his scientific knowledge. The result was a certain number of works, five of which are still extant. Two are of considerable length: the *Liber abaci* already mentioned, devoted to arithmetic, algebra and their applications to commercial and daily life problems, and the *Practica geometrie* on geometry and geometrical problems. Fibonacci's sources were Eastern Arabic (apparently not Hispano-Arabic), and also Byzantine; his predilection for solving linear systems of n equations with n unknowns, which occupy a sizeable, almost disproportionate, place in his *Liber abaci*, seems to have originated in the contacts he had in Byzantium, in particular with a mathematician named Moschos.[°]

The fate of these two transmissions, one from Spain in the mid 12th century, the other from Italy in the early 13th century, was to be more or

[†] First edited by B. Boncompagni, *Trattati d'aritmetica, I*; reproduction of the manuscript in a new edition by K. Vogel; reedition, together with edition and reproduction of the manuscript of another version, by M. Folkerts.

[‖] On John of Seville, see the excellent study by L. Thorndike. The *Liber algorismi* was first edited by Boncompagni, *Trattati d'aritmetica, II* (but with many errors).

[°] See our *Introduction to the History of Algebra*, p. 103.

less as follows. Arithmetic developed in Europe from the Spanish heritage originating with al-Khwārizmī. In the 13th century, John of Holywood (Johannes de Sacrobosco), an Englishman, wrote in Paris an elementary treatise, based (directly or not) on al-Khwārizmī's *Arithmetic*, which happened to be widely read, as was the *Carmen de algorismo* —a mnemonic in verse form on the various arithmetical rules— by Alexandre de Villedieu (Alexander de Villa Dei). Incidentally, note that by the 13th century the name al-Khwārizmī had already become a common noun, and up until the end of the 15th century *algorismus* continued to designate reckoning, while another transcription, *algoritmus*, paved the way to the modern 'algorithm'.

For the application of mathematics to daily and commercial life, however, the *Liber abaci* was almost the only source for the next three centuries. This is attested not only by the number of problems which later on are drawn from Fibonacci but also, and again, by a common noun: *abaco* came to mean 'commercial mathematics' in Italy, as we see with the denomination of *trattati d'abaco* for the treatises teaching it and *botteghe d'abaco* for the places where it was taught. Only a few sections in a few Italian treatises escape that influence, perhaps on account of some originality in their authors, more probably through access to other sources, including the *Liber mahameleth*. But, on the whole, the Spanish influence on this branch of mathematics remained negligible and so also that of the *Liber mahameleth*, which even fell into oblivion right up until the present time.⋆

2. The *Liber mahameleth.*

2.1 Contents.

Like the *Liber abaci* and its successors, the *Liber mahameleth* presents first the arithmetical theory and then a variety of application problems such as buying and selling, profit, partnership, hiring and other questions related to daily and commercial life. This is quite in line with Arabic treatises on the same subject, where the teaching of elementary reckoning is followed by its application to the science of 'transactions' (Arabic *mu'āmalāt*). Whence their designation as 'books on mu'āmalāt', and that of our treatise with *mahameleth* roughly transcribing the Arabic. Arithmetical reckoning and applications thus form two distinct parts of the treatise. We shall call them Book A and Book B.‡ Book A is divided into nine chapters (our A–I to A–IX), and Book B, a notably larger part, into twenty-three chapters of very unequal length. We shall now indicate their subjects, and in addition (except for the short chapter A–I) our numbering of the problems

⋆ Namely until the time of our first researches on it. See Kennedy's study of Bīrūnī's *On shadows*, II, p. 57 and our first publications in 1987, 1988, 2000. Dr. Anne-Marie Vlasschaert has recently published the whole Latin text (2010). Her edition differs in many respects from ours.

‡ In the text itself the second part is headed *pars secunda*, while in one problem (B.278) Book A is referred to as *liber primus*.

they contain (or the theorems for A–II); this gives a rough indication of the respective sizes of these chapters. A detailed conspectus of contents and mathematical methods to be used opens each of these chapters in our mathematical commentary.

Book A

A–I (and **Introduction**): Numbers: General characterization; formation of the successive positive integers and their verbal expression in the decimal system.

A–II: Premisses (our P_1 to P_9) and application of the first theorems from Book II of Euclid's *Elements* to numerical quantities (our PE_1 to PE_{10}). All this is said to be required for later demonstrations, and indeed is.[†]

A–III (A.1–51): Multiplication of integers.

A–IV (A.52–89): Division of integers.

A–V (A.90–144): Multiplication of expressions containing fractions.

A–VI (A.145–186): Addition of expressions containing fractions; solving linear equations (by means of one false position); summing consecutive natural integers, squares, cubes.

A–VII (A.187–214): Subtraction of expressions containing fractions; further linear equations.

A–VIII (A.215–274): Division of expressions containing fractions.

A–IX (A.275–326): Operations with square and fourth roots; extraction of the square root of a binomial expression.

Book B

Introduction: Rule of three.

B–I (B.1–53): Buying and selling (relation between quantities and prices). We are also taught the fundamentals of solving problems using proportions and algebra, recurrently used in the subsequent chapters.

B–II (B.54–103): Selling with profit.

B–III (B.104–116): Partnership and profit.

B–IV (B.117–120): Sharing according to prescribed parts.

B–V (B.121–125): Metallic masses: price and weight, for pure and composite masses; crown problem (of Archimedes).

B–VI (B.126–138): Pieces of cloth of various sizes.

B–VII (B.139–154): Pieces of linen of various sizes.

B–VIII (B.155–174): Cost of grinding.

B–IX (B.175–189): Reducing a quantity of must by boiling.

B–X (B.190–194): Borrowing with a certain capacity measure and returning with another.

B–XI (B.195–243): Wages of workers hired (kind of work not specified).

[†] Except for P_9 and $PE_8 - PE_{10}$.

B–XII (B.244–253) : Wages in arithmetical progression.

B–XIII (B.254–263) : Hiring a carrier (with wage depending on quantity carried and distance).

B–XIV (B.264–275) : Hiring stone-cutters (with wage depending on quantity of stones, time, number of workers); other kinds of work (tending sheep, digging holes).

B–XV (B.276–285) : Consumption of lamp-oil (with quantities of oil depending on number of lamps and nights).

B–XVI (B.286–299) : Food consumed by animals (with quantities of bushels depending on number of animals and days). B.298–299 are about the conversion of capacity units.

B–XVII (B.300–306) : Consumption of bread (with quantities of loaves depending on number of men and days). B. 306 is about the conversion of capacity units.

B–XVIII (B.307–330) : Exchanging moneys.

B–XIX (B.331–341) : Cistern problems.

B–XX (B.342–362) : Ladder and other problems, mainly involving the Pythagorean theorem.

B–XXI (B.363–367) : Bundles of rods of various sizes.

B–XXII (B.368–374) : Pursuit and travelling.

B–XXIII (B.375–381) : Mutual lending within a group of partners.

From B–XIX on, many problems tend to be of a recreational nature. This had become traditional at the time for mathematical treatises.

Remark. In Book B the problems are thus mostly grouped by subject and not by mathematical methods (exceptions are B.153–154 in B–VII). Therefore, when a particular treatment or a formula has been seen in a previous chapter, the problem is reformulated in terms of the types of quantities involved there. This reformulation of problems, with bushels becoming men or inversely (B.269, B.302) or meals becoming wages (B.297), may seem strange to us. But such a way of proceeding is in itself hardly surprising: in the absence of symbolism, a formula cannot be expressed in synthetic form, and the author thus has recourse to analogy.*

2.2 Origin and authorship.

As for mathematical treatises of the time, the basic theory, in particular for demonstrations, is Greek (Euclid's *Elements*), whereas the arithmetical reckoning and applications are unmistakably Arabic. This latter influence becomes evident in some places. First, the metrology and the coinage are mostly Arabic, or more precisely Hispano-Arabic. Secondly, and more characteristic, there is the Islamic background of some of the subjects; thus B–IX, on reducing must, originates in the prohibition of alcoholic beverages, while B–IV, on sharing parts, has to do with the Koranic

* Numerous examples; see Index (Part Three), 'analogy of formulae'.

rules on heritages. Thirdly, some assertions by our author are true for the Arabic language, but do not hold for Latin; thus,[°] saying (*ll.* 73–74 & 138–39) that all numerals are formed verbally from only twelve names does not apply to Latin (*viginti* is a thirteenth name); nor is in Latin 10^{3k} expressed by repeating k times *mille* (note 214); again, the alleged ambiguous formulation in problems involving multiplication, addition or subtraction of fractional expressions (A.135–135′, A.138–139, A.154, A.201) may be true for Arabic, but the wording in Latin is quite unequivocal. Finally, in the problems on computing with fractions we often find pairs of examples of the same kind, the first involving 'elementary' fractions (denominators ≤ 10) and the other 'non-elementary' fractions (denominators > 10); now the way of expressing these two classes of fractions is indeed different in Arabic, but not at all in Latin.

The *Liber mahameleth* thus clearly relies on Arabic sources, and this is hardly surprising for the time and place of its writing. But we may even affirm that the *Liber mahameleth* was (initially) *written in an Arabic environment*. Indeed, its reader was supposed to have, or to be able to have, access to some Arabic treatises and read them in the original. For the author refers him five times to the *Algebra* of Abū Kāmil (not yet translated, see below, p. lxiii) and once to a book pertaining to '*taccir*', that is, on practical geometry (below, p. lxiv). It is thus clear that the author, when writing the book, must have been living in the (still) Arabic part of Spain. On the other hand, he was not himself an Arab, since on one occasion he refers to what 'the Arabs' mostly do, telling us that, although finding it illogical, he will follow their example (in the introduction to A–III, *l.* 632); finally, it is hardly likely that a Moslem would propose a problem dealing with the quantity of wine in a cask (B.340).

It is from late mediaeval Italian sources that we gather the author's name. During the second half of the 14th century, the *Liber mahameleth* was studied and commented on by Grazia de' Castellani. In the early 15th century, Domenico d'Agostino Vaiaio used this commentary for his teaching. All this we know from the manuscript written by one of the latter's students around 1450, which is this time preserved. For this manuscript contains a set of problems taken from the *Liber mahameleth* (or, rather, from Grazia de' Castellani *via* Domenico d'Agostino), in the last of which (our B.185) we read: *Benché in molti modi si possa asolvere tale chaso, chome nella 16 quistione della* 4^a *parte del libro di Maestro Gratia sopra l'Arismetricha d'Ispano, niente di meno Domenicho l'asolve per l'algebra, in questo modo diciendo* (...).[‖] Now *Ispano* is no doubt translating His-

° References are to notes in the Translation and, when preceded by *l.* or *ll.*, to the lines in the Latin text (or the corresponding critical remarks). For the location of specific words, see the Glossary in Part Two.

‖ My Milanese colleague Ettore Picutti drew my attention to this passage on fol. 422^{r-v} of the Florentine MS Palat. 573. A second mention of Grazia de' Castellani in connection with the *Arismetricha d'Ispano* is found in the Siena MS L.IV.21; see Arrighi's note on it, p. 147 in his paniensis ('Spanish') for Hispalensis ('Sevillian'): the confusion between these two similar words is frequent. And both are commonly found in connection with the man we have already mentioned (p. xiv) as being a well-known author and translator of the mid 12th century. In his *Liber algorismi*, on practical arithmetic, there are in fact numerous passages almost identical to parts of the *Liber mahameleth*.[‡] Of him, we know that he was working in Toledo. But his name indicates that he was originally from, or lived at some time in, Seville, where there was at the beginning of the 12th century a large Christian community.[**] Our treatise might thus have been originally written for the educated Christians who lived in the Moorish part of Spain and could read Latin —just as Jews wrote, or translated Arabic treatises, in Hebrew for the benefit of their own community— and later have accompanied its author to Toledo.

But two problems remain, both to do with the form of the text. First, whereas the Latin language in the *Liber mahameleth* is that of an educated person, that of other treatises attributed to a 12th-century author or translator named Johannes is of uneven quality; true, all these 'Johanneses' from the Iberian peninsula may not necessarily be one and the same person. Second, it would seem that the author of the *Liber mahameleth*, whilst in Toledo, could not find the time to complete it; at least, there are numerous indications that the version we have did not receive its final touch: the text is in disorder and there are important omissions, as we shall see in the coming section.

3. Manuscripts of the *Liber mahameleth.*

Our knowledge of the *Liber mahameleth* relies on only four manuscripts, two of which —our $\mathcal{A}$ and $\mathcal{B}$— are almost complete, one —our $\mathcal{C}$— contains less than a half, and another —our $\mathcal{D}$— has only a small selection of problems. Of these, $\mathcal{A}$ is by far our best source. The two manuscripts $\mathcal{B}$ and $\mathcal{C}$ contain a sequence of *disconnected* parts of the work, without any unity of length or of content (listed below, 3.5). Indeed, the text may break off at the end of a paragraph or even do so in the middle of a reasoning; at best, a fragment may be linked to the next but one. There is, though, quite an elaborate if not very helpful device to direct the reader to what should come next: a system of marginal signs, each of which occurs at the end of a piece and at the beginning of the one which is supposed to follow it. There is no doubt that these signs were already in the ancestor of MSS

Scritti scelti. Finally, another mid 15th-century Italian manuscript, which has three problems from the *Liber mahameleth*, confirms this connection, for we read (MS Vat. Ottob. lat. 3307, fol. 138^v): *E molti chasi se possono proporre, chome scrive Maestro Gratia nella praticha d'arismetricha dello Spano, nella 4ª parte del suo trattato.*

‡ See Translation, notes 15, 18, 28, 34, 119, 121, 123–126, 135, 138, 141, 142, 182, 198, 492, 521, 670, 691, 694, 845.

** See Lévy-Provençal, *Séville musulmane*, p. 160.

$\mathcal{B}$ and $\mathcal{C}$: the sequence of many pieces corresponds and the reference signs are the same.

Since $\mathcal{C}$ preserves only part of the work while such signs have been inserted in only half of $\mathcal{B}$, these would be of limited help had we not manuscript $\mathcal{A}$ which represents, except for a few misplaced parts, what must correspond to the intended disposition. The disordered version was, it seems, the only text transmitted, and it was then put in order in MS $\mathcal{A}$: indeed, at times we find the copyist starting to reproduce a fragment of this disordered text (see below, p. xxiii).

The origin of this disorder is not easily accounted for. Clearly, some is due to later additions and complements inserted by the author himself. But reworking alone does not explain why some problems break off in the middle; in one case the proof of the treatment even precedes the treatment itself (A.151). Furthermore, there are misplaced problems even within the disordered fragments (note 500 in the Translation). This explains our belief, expressed at the end of the last section, that the *Liber mahameleth*, or at least the version transmitted, was not in its final form. There are further indications in favour of it, this time to do with the presentation of the subject (below, 3.6).

Remark. MS $\mathcal{D}$ (older than $\mathcal{A}$), with its few extracts, follows the correct order since it places B.352 and B.368 as the reordered text does. This is certainly not an initiative taken by the copyist, who did his work without any kind of discernment. So the text at his disposal might have been a reordered one —at least in part since no general conclusions can be drawn from such a tiny selection.

Finally, note that, as was traditional in (classical, non arithmetical) Arabic mathematical texts, the numerals are written in words; numerical symbols appear rarely, at most —again as in Arabic texts— in illustrations (beginning with the table in A–I). This verbal expression still occurs mostly in MSS $\mathcal{B}$ and $\mathcal{C}$, but not in $\mathcal{A}$. In some places where it does not, we have clear indications that some copyists themselves changed words to numerals (see below, 3.1 to 3.4). But we see that in a few other places the three main manuscripts all have the numeral form, which suggests that this was already the case in their common progenitor. The numerals are then either Roman (e.g. *ll.* 919, 920, 2959, 10719–20) or Indo-Arabic (examples in A.124–138, A.145 *seqq.*).[‖] In one case, the Indo-Arabic numeral is wrong in all three MSS (*ll.* 3088–89), and this points to an error in the common progenitor.

3.1 Manuscript $\mathcal{A}$.

Our $\mathcal{A}$ is the manuscript Fonds latin 7377A of the Bibliothèque Nationale de France; the part concerning the *Liber mahameleth* covers fol. 99^r–204^v (end of the manuscript), roughly the second half. Manuscript $\mathcal{A}$ is

‖ See also *ll.* 5822, 7579, 8117, 8680, 10717, 10727, 15410–11.

a quarto codex written in the 14th century on paper.[†] Between quires, parchment leaves are inserted: for our part, these are fol. 108, 109, 118, 119, 129, 130, 139, 148, 149, 157, 158, 167, 168, 177, 178, 187, 188, 196, 197, 204 and fly-leaf; 122 and 190 (parchment) are inserted fragments. Pages have been numbered —in the 19th century— twice, first in pencil and then in ink. These two foliations differ only at the very end of the codex, where pencil gives a very small inserted piece of parchment its own number (fol. 190) whereas ink makes it fol. 189^{bis}. We have opted for the first numbering, since in a similar situation another fragment receives a number in both foliations (fol. 122). Note also that the paper is, particularly at the beginning of our treatise, of rather poor quality; as a consequence, many stains give the false impression that there are corrections. Finally, we may observe that the leaves must have been clipped since some words are no longer visible (*ll.* 1070, 1425).

MS $\mathcal{A}$ has been copied by various hands, surely in Italy considering the writing. In any event, by the second half of the 17th century we find it in France, in the Library of J.-B. Colbert. From there it went to what was known at the time as the King's Library, but subsequently underwent various (and recurrent) changes in name according to the political situation —with *royale* being replaced then replacing *nationale* or *impériale*— to become what is now the Bibliothèque Nationale de France, where it remained. It contains mathematical treatises of Spanish origin, all but the last translated from Arabic texts. The texts translated are: part of the commentary on Book X of the *Elements* attributed to Anaritius (al-Nairīzī) (fol. $1^r - 33^v$); the partial translation of al-<u>Kh</u>wārizmī's *Algebra* (fol. $34^r - 43^v$, 7); the *Liber mensurationum Ababuchri*, thus by Abū Bakr (fol. 43^v, 8 – 56^v, 32); the *Liber Saydi Abuohtmi* (*sic*), thus by Saʿīd Abū ʿU<u>th</u>mān (fol. 56^v 32 – 57^v, 2); the *Liber Aderameti* (fol. 57^v, 2 – 58^v, 15); the *Liber augmenti et diminutionis* (fol. 58^v, 16 – 68^r, 23); part of the commentary by Pappus on the tenth book of the *Elements*, in an *editione ab Othmen damasceni*, that is, from the Arabic translation by Abū ʿU<u>th</u>mān al-Dima<u>sh</u>qī (fol. 68^r, 24 – 70^v); the partial translation of Abū Kāmil's *Algebra* (fol. $71^v - 97^r$; 71^r, 97^v and 98 are blank).[⋆] These translations are followed by the *Liber mahameleth*; the last and larger part of the codex thus consists of the *Liber mahameleth*, which ends on line 7 of fol. 204^r.

3.1.1 <u>The two copyists.</u>

The text of the *Liber mahameleth* is written by two different hands, one copying the first part (our Book A) and the other the second (Book

[†] Short description of this manuscript in the fourth volume of the old *Catalogus* (1744), p. 349.

[⋆] The texts translated from the Arabic have now all been published: al-Nairīzī's Commentary by Curtze (pp. 252–386 in *Euclidis Opera*, Suppl.); al-<u>Kh</u>wārizmī's *Algebra* by Libri in his *Histoire des sciences mathématiques en Italie*, I, pp. 253–297, then by Hughes; the three mensuration texts by Busard; the *Liber augmenti et diminutionis* by Libri, *ibid.* pp. 304–371; Pappus's *Commentary* by Junge; Abū Kāmil's *Algebra* by R. Lorch *et al.*

B). A blank page (fol. 147) separates the two parts (and now contains, on the verso, a reader's note). The number of lines on each page varies: from 28 to 42 (1^{st} hand) and 38 to 46 (2^{nd} hand).

This first copyist (referred to in the critical notes as 1^a *m.*) is certainly not ignorant of mathematics, but not very competent either (see, e.g., *ll.* 427, 4032, 4067 *seqq.*). He cannot have been very attentive since he has copied the end of A.210 again at the end of A.212 only half a page further on.

The second copyist (our 2^a *m.*) is also the principal one. For he has not only copied the longer part but also revised (though not systematically) the work of the first hand, filling in lacunas and amending the text where it was unclear or contained errors. See *ll.* 256, 929, 938, 1171, 1317, 2045, 2055, 2319, 2348, 2350, 2351, 2556, 3195, 3411, 4283, 4346–47 (unnecessary, see Translation, note 608), 4531–32, 4724, 4996, 5005–6 (note 701), 5012, 5013–14, 5069, 5111, 5120, 5169, 5175, 5177, 5179, 5241, 5348 (incorrect addition, note 757), 5461, 5552, 5567, 5574–75, 5756. It appears from this enumeration that he was particularly interested in the chapter on roots, the most difficult part of Book A. In Book B, he shows himself to be a careful copyist. He corrects the text as he goes along, for computational (*l.* 6068) or textual reasons (*ll.* 6277 (cf. 6286), 6789, 8258); thus he does not blindly reproduce the progenitor, at least not as a rule (*ll.* 6574, 6676). He abbreviates frequently repeated expressions, like *d. v.* for *demonstrare voluimus* (*l.* 6012; cf. *l.* 15439), or drops the case-endings of recurring words such as *longitudo* and *latitudo* (*ll.* 8623, 8626), *sextarii* (*ll.* 6760, 9571), *caficii* (*ll.* 7870, 13479), *morabitini* (*l.* 13604–5), *solidi* (*l.* 13625).[‡] But he mostly does it in such a way that the case-ending may be inferred from the context; for instance, *cafic̄* is followed by *ignoti* (*l.* 9073), by *ignotis* (*l.* 9066), or preceded by *molendo* (*l.* 9143), or *sextar̄* is followed by *ignotos* (*l.* 9573).[⋆] Still, towards the end of his work (from around B.208 on), the second copyist becomes less conscientious; there is an increasing number of omissions, and, more significantly, repetitions (see *ll.* 8030–32, 8225–26, 9476–77, 9506–7, 10139–40, 10262–63, 10272–73, 10887–88, 10978–80, 11125–27, 11166–67, 11198–99, 11401–2, 11517–18, 11605–6, 11980–1, 12092, 12097–98, 12112–14, 12220–21, 12357–59, 12475–76, 12628, 12737–38, 12756, 12800–1, 12802–3, 12877, 14276, 14379, 14724–25, 14773–74, 15126–28, 15135, 15168–69, 15225, 15458–59, 15557–59, 15572–73, 15629–30); in *l.* 9927, he seems to have copied one line of his exemplar twice. Obviously, he then did not check the solutions nor did he reread and check his copy with the original.

This second copyist is also responsible for adding the figures in the spaces left blank. This explains the use of our numerals, with which the first hand is not very familiar, in the figures found in Book A (e.g. tables in

‡ We did not note these occurrences systematically in the critical apparatus.

⋆ Note that abbreviations are also found in the other MSS: *ll.* 1760, 7586, 7870, 7902, 7903, 7906, 8565–66, 12647, 13627.

A–I, in A.40 and A.80). This also explains why a figure was first copied in the wrong place (A.148). At times he checked the figures against the text (figure of the second proof in A.124a) but not always (B.346, 2nd figure). But, unlike the copyist of $\mathcal{B}$ and $\mathcal{C}$, he draws the figures carelessly and rarely uses, if ever, a ruler.

As already mentioned (p. xx), the numerals must have been originally written in words. In MS $\mathcal{A}$, however, numbers are mostly written in symbols. The difference between the two hands is also illustrated by this: in the first case, lack of familiarity with Indo-Arabic numerals may explain why the first hand prefers to use Roman numerals and has thus transcribed many numerals (sometimes partly, see e.g. *ll.* 2561, 3036–37). The second hand, on the contrary, seems to be quite at ease with our numerals: practically all integers, and also the fractions, are expressed in them.[‖] Obviously, the second hand of $\mathcal{A}$ is responsible for the transformation: there are numerous instances confirming the original use of words, either because the copyist initially wrote the first letter(s) of the number expressed in words (*ll.* 6641, 8782, 9126, 12412, 13554) or because he initially transcribed only the first word(s) of the numerical expression: characteristic examples for integers in *ll.* 5949 (200 for 252), 7567, 9578, 9953, 9957, 10028, 10033–34, 10513–14, 11819–20, 11953, 12003, 12013, 12395, 12464–65, 12489, 12666, 12698, 12705, 13492–93, 13597, 13695, 14295, 14606, 15199, 15220; and for fractions in *ll.* 7358, 8979, 10898–99, 11149, 13590, 13764, 13803, 14126, 14135–36, 14139, 14178, 14241–42, 14244–45, 14291, 14605, 14872, 14873, 14876, 14892, 15331–32, 15780; note also the difference when one passage is erroneously repeated (*l.* 10888). The fractions are written numerically only in $\mathcal{A}$, and always by the second hand; the case-ending is often found in the numerator when the fraction is aliquot, otherwise as exponent.

As we mentioned earlier (p. xx), $\mathcal{A}$ presents, unlike MSS $\mathcal{B}$ and $\mathcal{C}$, the subjects in the right order. That its progenitor was also in disorder is apparent in three places. First, after the heading *Capitulum de impositione note*, in A–III, the copyist wrote, then deleted, the somewhat contradictory heading *Item aliter de multiplicatione integrorum numerorum inter se absque nota* (*ll.* 781 & 1089–90); now $\mathcal{B}$ (not $\mathcal{C}$) does indeed jump from one subject to the other (fol. 9^{rb}, 9–10). Second, at the end of B.321, $\mathcal{A}$ has omitted a few lines and jumped to similar words in B.325 (*l.* 14472); now B.325 indeed immediately follows B.321 in the disordered version (in $\mathcal{B}$, the gap would correspond to a jump from line 10 to line 17 on fol. 70^{va}). Finally, at the end of B.359, $\mathcal{A}$ has copied two words which belong to the demonstration in B.360 (*l.* 15422); now this corresponds exactly to the disordered text. The disorder in the transmitted text may also account for other similar errors (not verifiable using $\mathcal{B}$ since the text there is lacunary). Indeed, we see the second hand of $\mathcal{A}$ correcting a few misplace-

[‖] Exceptions e.g. in *ll.* 6281, 6319, 9754, 10567, 14831, 14913. A few errors, as in *ll.* 8346, 11798, 11811, 12073, 15593, 15594; see also a few instances of Roman numerals in *ll.* 5822, 8680, 10719–20 (corrected; clearly already in the progenitor).

ments, such as putting B.211 just after B.201 (see *ll.* 9865, 9866, 9902, 10216) and B.329 after B.326 (see *ll.* 14618, 14632, 14652). This copyist further omits the whole first section of B–XVI (B.286–293), jumping directly to the section mentioned in the chapter's introductory words. But this is exceptional, and on the whole $\mathcal{A}$'s rearrangement can be considered as reliable. We have therefore followed it in the edition, departing from its order only for A.264 (placed with A.124), B.118–120 (which conclude MS $\mathcal{A}$), B.181 (note 1288).

Thus the text in MS $\mathcal{A}$ has been reordered according to the reference signs while, as we have seen, the second copyist corrects here and there the first part. Therefore, unlike the copyists of MSS $\mathcal{B}$ and $\mathcal{C}$, he does more than merely reproduce what he sees regardless of the content. All this suggests that he was not only a copyist but also a mathematician interested in the subject of the treatise. It further appears that he was also a translator of Arabic mathematical texts, for he is the author of the (autograph) Latin version of the *Algebra* of Abū Kāmil found in the same codex (fol. 71^v–97^r). His notes and complements there when the Arabic text of his exemplar is lacunary enable us to guess his identity, for they are signed. His name (written there in full or abbreviated) is Guillelmus.* Finally, since this same (Italian) hand has also amended here and there the other treatises found in the codex, it is reasonable to suppose that Guillelmus himself may have instigated and directed the writing of MS $\mathcal{A}$.

3.1.2 The two readers.

The text of the *Liber mahameleth* in MS $\mathcal{A}$ had, already in the 14th century as it seems, at least two readers. (We find traces of their studying also in other parts of the manuscript.)

• The first reader (our 'other hand', *al. m.*) is responsible, as far as our text is concerned, for

— two glosses: a problem inserted on the (originally left blank) fol. 147^v (*l.* 5801) and a remark to the demonstration in B.351*a* (*l.* 15280);

— the repetition of an intermediate result in B.124 (*l.* 8333) and of the results of B.381 (*l.* 15781), also the correction of an omission in the solution of this same problem (*l.* 15761);

— writing *nota* about the need to know Euclid's *Elements*, just before PE_1 (*l.* 440);

— numerous signs in the margins, mostly ¶ (or, more precisely, ¢) —with a weaker form (⌈) and a stronger one (⫽)—, and †, all drawing attention to new subjects, remarks, rules, certain problems or, generally, points of particular interest to him. In Book A, these occur at the following places (marked with ¶ if not indicated otherwise): *Primus autem*, referring in A–I to the first (integral) number (*l.* 52); *ad similitudinem priorum*, about the successive orders of numbers (*l.* 114); *Postquam autem*, closing A–II (strong form, *l.* 621); in A–III, *Capitulum de impositione note* (strong, *l.*

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* We conjectured that this might be the translator Guglielmo de Lunis. See the introduction to our partial edition of the Latin Abū Kāmil.

781); *Hoc autem capitulum multum utile est* (before A.44, *l.* 1478); *Capitulum de divisione* (heading of A–IV; strong, *l.* 1633); ¶ in the text, and *nota istud capitulum* at the bottom of the page, both for the *Capitulum de denominationibus* (*l.* 1882); ¶ and *nota* for A.99 (*l.* 2339); heading before A.159 (*l.* 3444); A.169 (weak, *l.* 3554); heading before A.170 (strong, *l.* 3568); A.184–186 (weak; *ll.* 3678, 3684, 3688); beginning of the chapter A–IX (*l.* 4928) and † for the definition of root (*l.* 4941). In Book B we find ¶ in the (reader's) interpolated problem (147^v, *l.* 5801), then in B.1 (strong, *l.* 5860); ¶ and † for the heading of B–III (*ll.* 7957–58); again ¶ and † at the beginning of B–V (*ll.* 8253–54), then ¶ and †, with the word *nota* added and ¶ repeated within the text (B.124, *l.* 8289), next † in the margin and ¶ within the text (*Quasi ergo inveneris*, *l.* 8306), and finally ¶ and † for the *de scientia cognoscendi* (B.125, *l.* 8334); ¶ for the heading of B–XVIII (*ll.* 13604–5) and for two problems in it, namely B.316 and B.318 (which are indeed particularly interesting problems; *ll.* 13979, 14264); same sign again for the cistern problems B.331–333, B.334 (here †) and B.335 (*ll.* 14745, 14757, 14768, 14780, 14811); for the beginning of the *capitulum de scalis* and thus B.342 (strong), then B.342*b*, B.343, B.344 (*ll.* 14972, 14989 (weak), 15003, 15025); finally for the very last chapter, beginning with B.375 (*l.* 15645), and † at the end of B.381 (*l.* 15781). The same hand has also drawn a vertical line in the margin for A.145 *b*, one short vertical line in the margin for A.178 and, also in the margin, for square root approximation formulae (*ll.* 3204, 3630, 4948, 4951); finally another mark — namely: (— is found twice in B.118*b* (*ll.* 8180, 8196), thus at the end of the *manuscript*.

It appears therefore that the first reader must have been particularly interested in denominating and searching for integral divisors, in recreational problems (cisterns, ladders), but above all in problems on masses —which is in keeping with his main gloss; of equal note is his study of the last problem in Book B (B.381; see above), which shows him to be a fairly competent reader.

Remark. His interventions in the remainder of the codex are of the same kind, for instance in the Latin Abū Kāmil (fol. 73^r, 81^v, 94^r, 94^v; the last two comments are taken from, or inspired by, Leonardo Fibonacci's *Practica geometrie*†).

• The second reader, our *lector*, has left his mark here and there, in the form of marginal notes and vertical lines drawing attention to what he deemed particularly important, or else underlining parts of the text.

It is he who numbers the theorems in A–I, namely for P_1 (also introducing a numerical example), P_3, P_3', P_7, P_7' (*ll.* 176 & 182, 228, 254, 350, 382); he draws attention to the proof in A.32 (*probatio multiplicationis*, *l.* 1335), repeats, but with numerals, the numbers expressed in words in A.33 and in A.72 (*ll.* 1351, 1919); in this latter section, where divisibility is considered, he repeats in the margin the divisors 6, 5, 4, 3 for rules *xxiii–xxvi* (*ll.* 1969, 1978, 1982, 1987) and writes *partes* for rule *xxxi* (*l.* 2025); he

† See Lorch's Abū Kāmil, p. 225*n*.

again draws attention to A.90*b* (*nota*, *l.* 2166), to A.108–111 (vertical line in the margin and *Nota* next to the heading, *ll.* 2513–14, 2516) and works out numerical examples in A.108–109 (*ll.* 2525–26, 2532); to A.117*b* and following rule *i*, he draws a vertical line in the margin and adds, about the rule, *Nota* and *cum duobus et pluribus* (*ll.* 2580, 2585); he again draws a vertical line for the rules and problems A.225–226 (*l.* 4264); to A.230 and previous heading, he notes *de fractionum denominatione* (*ll.* 4312–13); he draws a hand with a finger pointing to the title before A.236 (*ll.* 4348–50); for A.265 and preceding rules *i* and *ii* we again find vertical lines (*ll.* 4780–811), and once again a hand and a mark at the beginning of A–IX (*l.* 4928).

That he had some mathematical background is seen from his referring to theorems of Euclid (*prima secundi* in PE_1, *septima secundi* in the proof of A.44; *ll.* 443, 1506), or to known rules (*per regulam quatuor proportionabilium* in A.108, *l.* 2520); he illustrates an assertion in A.288 (*l.* 5071).

He has also underlined, and in a rather careless way, various headings, presumably when the subject was of particular interest to him, namely: those of A–II, A–VI, A–VII, A–VIII and A–IX, and the initial words of some rules on divisibility of integers, namely *xix–xxii* (*ll.* 1913, 1916, 1925, 1939) and the subsequent *xxiii–xxix* and *xxxi* (*ll.* 1969, 1978, 1982, 1987, 1991, 1995, 2010, 2025 —writing in this last case *partes* in the margin, see above); the headings of the sections preceding, and/or the initial words of, A.90 (*ll.* 2141–42), A.90*a, alia causa* (*l.* 2158), A.90*b* (*l.* 2166), A.90*c* (*l.* 2170), A.92 (*l.* 2187), A.93 (*ll.* 2203–6), A.95 (*l.* 2278), A.96 (*ll.* 2299–300), A.108 (*l.* 2515), A.112 (*ll.* 2539–40), A.115 (*ll.* 2558–59), A.117 (*ll.* 2571–72), A.121 (*ll.* 2616–17), A.123 (*ll.* 2635–36), A.124 (*ll.* 2655–56), A.124*b* (*l.* 2713), A.124*c* (*l.* 2731), A.125 (*l.* 2743), A.126 (*l.* 2758–59), A.126*c* (*l.* 2795), A.127 (*ll.* 2803–4), A.128*b* (*l.* 2827), A.129 (*l.* 2845), A.129*b* (*l.* 2860), A.130 (*ll.* 2877–78), A.131 (*ll.* 2899–900), A.131*d* (*l.* 2928), A.132*d* (*l.* 2944), A.138 (*l.* 3077), A.145 (*ll.* 3193–94), A.145*b* (*l.* 3204), A.159 (*l.* 3444), A.187 (*ll.* 3694–95), A.189 (*ll.* 3720–21), A.190 (*ll.* 3734–35), A.191 (*l.* 3748–49), A.193 (*ll.* 3813–14), A.194 (*ll.* 3834–35), A.195 (*ll.* 3858–59), rule before A.225 (*l.* 4264), A.230 (*ll.* 4312–13, see above), A.236 (*ll.* 4348–50), A.239 (*l.* 4371), A.240 (*ll.* 4398–99), A.242 (*l.* 4441), A.255 (*ll.* 4604–5), A.257 (*l.* 4631), A.258 (*ll.* 4647), A.260 (*ll.* 4673–75), A.262 (*ll.* 4718–19), A.267 (*l.* 4829), A.284 (*l.* 5032), A.291 (*l.* 5148), A.293 (*l.* 5187), A.303 (*l.* 5302), A.311 (*l.* 5407).

Sometimes only the first problems or pages on a particular subject bear traces of his passage: he seems to lose his enthusiasm rather quickly. Nevertheless, we see the attention of this second reader caught by the conversion of fractions and the criteria for denominating numbers or fractions. All of this concerns Book A. Indeed, in Book B there is no sign of him, whereas such traces are seen earlier in the MS (as in 2^v, 55^v, 57^v).

3.2 Manuscript $\mathcal{B}$.

Our $\mathcal{B}$ is the manuscript D.42 of the Biblioteca Capitolare in Padua;

the *Liber mahameleth* covers fol. $1^r - 86^v$ (end of the manuscript). Manuscript $\mathcal{B}$ is a folio codex written on parchment, in the 14th century.† MS $\mathcal{B}$ is written in double columns of 48 lines to a page (47 on fol. $5^v - 6^r$). The foliation is by a 15th-century hand. The end of the manuscript is missing. There are two fly-leaves at the beginning, just one at the end. The leaves containing the text are stitched in eleven quires of eight leaves each, except for the fifth (fol. 33–38). As already mentioned (p. xix), the *Liber mahameleth* in $\mathcal{B}$ consists of fragments, the logical sequence of which is only partly indicated by reference signs; apart from the occasional omission of reference signs, figures, headings (below, 3.2.1), it is a faithful copy of what must have been its progenitor. It was once in the possession of Bishop Pietro Barozzi, and went to the Biblioteca Capitolare after his death (1507).‡

3.2.1 The copyist.

Unlike the other manuscripts, $\mathcal{B}$ seems to have been entirely devoted to the *Liber mahameleth*. The writing, by a single hand, is good and the copying, meticulous. The pointing finger drawing attention to certain places in the progenitor has thus been conscienciously reproduced; we find it in $1^{rb}, 7$ (*Unitas autem non est numerus*; *l.* 41), $1^{rb}, 23$ (*Primus autem numerus*; *l.* 52); then in $4^{rb}, 33$ (with *nota*; between P_9 and PE_1, about knowledge of Euclid's *Elements* as prerequisite; *l.* 440), $33^{vb}, 31$ (B.10, *fiet ei facilius qui ignorat*; *l.* 5989), $41^{vb}, 32$ (within B.171*a*; *l.* 9170), $42^{va}, 44$ (B.175; *l.* 9235), $45^{ra}, 18$ (B.199; *l.* 9762), $51^{vb}, 44$ (*Experientia*, B.233*c*; *l.* 10896), $72^{rb}, 13$ (*quoniam ab Antiquis*, B.341; *l.* 14965). So also the *nota*, reproduced from the progenitor on fol. $1^{rb}, 7$ (*Unitas autem non est numerus*; *l.* 41), some references to Euclid left in the margins (P_5, P_9, PE_1, PE_5, demonstration following A.13 and corollaries, A.44; *ll.* 322–23, 324–25, 425, 453, 515, 1004, 1039, 1050, 1506). All these additions go back

† Description in Ferdinando Maldura's handwritten catalogue of 1830, *Index codicum manuscriptorum qui in Bibliotheca Reverendissimi Capituli Cathedralis Ecclesiæ Patavinae asservantur*, where we read on p. 168: "D.42…Mahameleth: De numeris. Opus sic incipit: *Omnium que sunt, alia sunt ex artificio hominis, alia non.* Folia aliquot extrema desiderantur. Codex Sęculi XV membranaceus, duplici columna exaratus". In the chronological list of the MSS at the end, we read: "Mahameleth De Numeris". Obviously, Mahameleth (or 'Mahamelet', as written in the *index nominum*) was taken to be the name of the author while the heading of the first chapter was considered to be the title of the whole work. This description is now superseded by Bernardinello's, pp. 598–599 of the first volume of his *Catalogo*. Bernardinello puts it in the 13th century, as had also the present writer (in *Le Liber mahameleth* and *Survivance médiévale*). Some additions by the same copyist rereading his text, where he departs from his calligraphy, point to a later time.

‡ As recorded in the list of Barozzi's books; see Govi's edition, p. 147, No 61 (60 in the original manuscript).

to a reader of one progenitor and cannot be attributed to our copyist. For he is no mathematician and at times seems to be quite unaware of what he is supposed to be copying. A few instances will illustrate this. He read *pro omnibus* for *probationibus* (*lect. alt. ll.* 621–25), *ita* for *quinta* (*l.* 1151), *ZG* for 45 (*l.* 2217), *sex agrega tres* for *sexaginta tres* (*l.* 4864), *seculorum* for *secundorum* and *nominum* for *hominum* (*ll.* 4898–99), *ita dicunt* for *radicum* (*l.* 6753), *incognita* for *octoginta* (*l.* 8392), *quasi tam* for *quartam* (*l.* 8795), *parentes* for *partes* (*l.* 8895), *ter eius* for *tertius* (*l.* 8965), *Rex* for *Res* (*l.* 9724), *numis* for *mensis* (*l.* 10599), *evasit in numis* for *evasit immunis* (*l.* 10768), *propinque* for *pro quinque* (*l.* 11273), *et erit tres viginti sex* for *et erit res viginti sex* (*l.* 11340).[‖] Still, in many cases this first hand did eventually correct his initial errors or omissions.

Indeed, most miscopying was corrected whenever this same hand went back to revise the text. The first revision checked the copy and added various signs. We thus find, in a different (light-brown) ink, first, the addition of the characteristic reference signs (those putting order in the disordered fragments) which had not yet been inserted, and, second, the indications of where a heading should later be written in red ink (the sign is commonly $\mathcal{R}$, for *rubrica*, a few times †), with these headings noted elsewhere on the page (top, bottom, sometimes margin); most of them are still, wholly or partly visible, but some have disappeared altogether when the pages were clipped for binding (see, e.g., *ll.* 2258–59). Checking the copy during this first revision meant correcting within the text (as in *ll.* 797, 1018, 1475, 1507, 1826–27, 3815, 4062, 4639, 6329) or in the margin (as in *ll.* 1458, 1463, 1477, 1621, 1727, 4142, 4152–53, 4188–89, 4448–49). Very occasionally, these corrections actually give a better reading than the other MSS (except when the second hand in $\mathcal{A}$ has corrected the error); see *ll.* 938, 1272–73, 1681, 3137, 3743, 6869, 9767.

In a second revision, the figures were added in the spaces left blank. Once, when no space had been left, the word *Figura* was used to indicate the appropriate place and the figure added in the margin (figure before A.22, fol. 15^{rb}). That the figures were added later than the first revision is again clear from the difference in ink. Note also that in three cases this led to some confusion: twice, during different revisions, a figure was repeated on the following page (those to the Premisses P_1 and PE_1), once it was inserted in the wrong place (that to A.139 in A.138). A few things omitted during the first revision were also added during the second, e.g. signs for connecting the disordered fragments (there are ones on fol. 10^{va}), marginal notes (such as the allusion to Euclid before PE_1, *l.* 440) or words (*l.* 2671).

A change occurs from fol. 25^r on, that is, at the beginning of the fourth quire: the two revisions become one, with figures and corrections being both in the same ink. This, however, is only the first indication that the copyist was becoming less thorough: first, after fol. 32^r, he stopped indicating the headings (*l.* 4371 is the last); then, from fol. 39^r on (beginning of the sixth

[‖] Other examples: *ll.* 300, 1150, 2548, 3477, 3480, 4872, 4902, 6001, 6872, 7337, 9766, 11346, 13383.

quire), he gave up revising altogether. Only in the last extant leaves (fol. 71–86), and even then not always, did he add, whilst doing the copy, a few of the figures (to B.30 on fol. 79^{va}, B.31 on 79^{vb}, B.32 on 80^{rb}, B.35 on 81^{ra}, B.41*a* on 82^{ra}), subheadings in the margin (B.66–70, B.72–76 and B.78–79 on 83^{vb}–84^{va}), rubrication signs (74^{ra}–84^{va}) and reference signs for the fragments (81^{va}–84^{vb}). He also indicated two main headings by their intended place in the text (*l.* 14745, 71^{ra}, 29; *l.* 14972, 72^{rb}, 23), as he had done once before (*ll.* 900–1, 6^{rb}, 14).

There was no final revision. As a result, the main headings are never inserted in the text, nor are the initial letters of new chapters, all of which were supposed to be added in red. This was thus not done, except for the title of the work on the first line. The only other use of red lettering is by a later reader (see below).

Since the copyist did not understand much of the text anyway, it is hardly surprising that he incorporated into the text what were originally marginal glosses; we shall discuss these when we come to the history of the text.

Although inferior to $\mathcal{A}$ and incomplete at the end, $\mathcal{B}$ appears to be a reliable source for the text, and indeed our only source for a major group of problems (B.286–293) and some minor ones (A.141–144, A.236, B.216–218).

3.2.2 The reader.

The manuscript had apparently just one reader, towards the end of the 15th century, who made various annotations in two different inks (black and pink). The handwriting suggests that it was Pietro Barozzi himself, the owner of the manuscript, whose interest in mathematical questions is attested by his contemporaries.* His interventions are of the following kinds:

— He puts marks, such as

the sign +, when passages are of interest to him: A.19 (*Duo igitur*, 9^{va}, 45; *l.* 1150); B.217*b* (*Vel aliter*, 47^{vb}, 32; *l.* 10341); B.227*a* (*Oportet igitur*, 49^{vb}, 20 —barely visible; *l.* 10570); B.230*a* (*qui sunt triginta*, 50^{rb}, 30; *l.* 10653); B.230*b* (*nam dictum est*, difficult reasoning, 51^{ra}, 13; *l.* 10743); B.232*b* (*maius est eo*, about the explanation of the meaning of the problem —text misplaced—, 51^{rb}, 16; *l.* 10811); B.233*c* (*Experientia*, 51^{vb}, 44 —barely visible; *l.* 10896; there is already a hand by the copyist);

the sign ¶, within the text: 1^{ra} (*Practice autem species*; *l.* 24); 1^{rb} (*Primus autem numerus*, just like the *al. m.* in MS $\mathcal{A}$; *l.* 52); in the enumeration of the successive *ordines* and *differentie* (*ll.* 75 to 111, 121 to 131 (*sicut*)) and of the *note* (*ll.* 158 to 161) and *nomina* (*ll.* 162 to 166). This same sign is also commonly found marking off paragraphs or important steps, or problems of particular interest to him, such as: A.165–167 on 11^{vb} – 12^{ra}; A.205*b* and A.211 on 13^{ra}; A.214 on 13^{rb}; the beginning of the chapter on division of integers (13^{va}, 20; the reader of MS $\mathcal{C}$ did the

* See Gaeta's short biography of Barozzi.