## Saeed Eftekhar Azam

# Online Damage Detection in Structural Systems Applications of Proper Orthogonal Decomposition, and Kalman and Particle



**Filters** 



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### Saeed Eftekhar Azam

# Online Damage Detection in Structural Systems

Applications of Proper Orthogonal Decomposition, and Kalman and Particle Filters





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### **Foreword**

Monitoring the health of structures and infrastructures exposed to aging or extreme loadings is nowadays recognized as a societal need. The pervasive use of miniaturized sensors, recently developed through microelectronics-driven technological processes, has forced people to look for smart monitoring strategies tailored to handle the large amount of data provided by densely deployed sensor networks. Moreover, as each health monitoring procedure relies upon a theoretical/numerical model of the considered structure, the more accurate the model the more powerful the monitoring scheme; such increased accuracy also entails additional monitoring burden.

If a structure undergoes a damaging process reducing its load-carrying capacity, the health monitoring procedure should be able to identify the damage itself in terms of location and amplitude. It is then necessary to filter out the possible noise terms and provide meaningful information from the structural response. Due to the presence of damage, robust procedures able to deal with a nonlinear system evolution are obviously to be envisioned.

The two topics discussed above, i.e., the size of the model to be handled and the nonlinearities in its evolution law, might be difficult to manage simultaneously in a common frame. It may happen that the filtering algorithm, which is supposed to compare the responses of the real structure and of a fictitious, linear-comparison one (featuring no damage evolution in a predefined time window), provides estimations affected by drifts or biases, sometimes also diverging. It may also happen that by increasing the size of the numerical model, e.g., due to a required finer space discretization in case of numerical (e.g., finite element) procedures supplying the model itself, the aforementioned bias and divergence issues get amplified.

It is also worth noting that structural health monitoring systems should be able to provide results in real-time or, at least, close to such target, so that warnings can be provided as soon as critical conditions are approached during the life cycle of a structure.

The research activity reported in this book moved from all the aforementioned critical aspects, with the aim of providing a robust, accurate, and easy to implement methodology for the health monitoring of civil structures and infrastructures, possibly suffering damage inception and growth. Two main challenging topics are

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specifically dealt with: the derivative-free filtering of the response of nonlinear systems; a time-varying, reduced-order modeling able to self-adapt to a changing system dynamics. As for the former issue, results are known to be not satisfactory if one does not properly account for the statistics of noise terms and structural state, and for the nonlinear evolution of the last ones. Here, the author shows that a wise combination of Kalman and particle filtering can indeed provide a very efficient (in terms of computational costs) and robust (in terms of avoidance of output divergence) framework. As for the latter issue, a snapshot-driven proper orthogonal decomposition methodology is known to work well in case of timeevolving linear systems; on the other hand, it is still disputed whether proper orthogonal decomposition can be adopted for a nonlinear time evolution of the system, linked, e.g., to damage growth. Here, the author shows that a further exploitation of Kalman filtering can provide, if governed by a partial observation of the system, a very efficient way to continuously tune the reduced-order model, thereby avoiding time-consuming re-training stages suggested by others in the past.

This book thus introduces a novel, hybrid approach to damage identification and health monitoring of structural systems. As such, it has been written mainly focusing on the theoretical and implementational aspects of the approach, partially leaving experimental validations aside. In my opinion, readers can find in it all the details necessary to adapt the methodology to many, if not all the real-life situations to be practically envisioned.

Stefano Mariani

### **Preface**

The aim of this monograph is to present the key ingredients of a still-in-progress research discipline within the structural engineering realm, namely online damage detection. The material of the text offers detailed explanations on recursive Bayesian filters (e.g., Kalman filters, particle filters), proper orthogonal decomposition methods (POD) (e.g., singular value decomposition, principal component analysis), and a combination thereof, i.e., a synergy of reduced order modeling and recursive Bayesian filtering. Illustrations accompanied by the theoretical description allow the reader to intuitively comprehend the notions. Therefore, this book can serve as a tutorial for scientists and engineers who want to apply and implement proper orthogonal decomposition and/or Bayesian filters to a specific problem.

Throughout the book, the focus of the numerical examples is on structural systems. The techniques presented in this research monograph are well established in fields like automatic control, statistics, etc. However, they are rather new to civil and structural engineers; hence, the algorithms are presented in enough details so that the reader can easily implement them on any structural state-space model. At first, the ease of implementation has been the main concern; however, the author believes that the way the main notions are analyzed makes this book an inspiration for conducting further research and development of these methods.

The objective of the study presented in this monograph is to develop techniques for vibration-based non-destructive damage identification of the structures. In fact, the major emphasis is on the development of quick and robust recursive damage detection algorithms in order to facilitate the task of online, real-time continuous monitoring of civil structures, such as, e.g., residential buildings, bridges, and other similar structures. This goal can be accomplished only through mixing different disciplines of science and technology, including automatic control, applied mathematics, and structural engineering.

It should be emphasized that though Bayesian filters have been extensively studied in the automatic control field, their applications in structural engineering are yet to be investigated. The applications of extended Kalman filter (EKF), sigma-point Kalman filter (SPKF), and particle filter (PF) to simplified and low-dimensional models are suggested in the existing literature; nevertheless, to the best of my knowledge, applying the extended Kalman-particle filter (EK-PF) has never been reported when dealing with a structural engineering problem.

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The algorithms for all the Bayesian filters used in this book are derived using the same notation; this can allow the reader to easily understand the similarities and ideas behind each one of them. Their performances dealing with different identification tasks are scrutinized in detail, and the reason for their success and failure in each case is highlighted.

It is perceived that as the number of the degrees-of-freedom increases, the adopted methods in the literature lose their accuracy in system identification, and thus in damage detection process. This problem is created due to the high dimension of the parameter space, i.e., by so-called curse of dimensionality. To manage this issue, in this study I make recourse to reduced order modeling of the systems. The aforementioned task is accomplished by using the proper orthogonal decomposition. Before using POD-based models in the Bayesian filters, the performance of such methodology is thoroughly investigated to ensure accuracy, speed-up, and robustness when different sources of excitation shake the structures.

The major contribution of the present research is the development of a recursive stochastic algorithm by a synergy of dual estimation concept, POD-based order reduction, and a subspace update. The proposed methodology takes advantage of Bayesian filters (like EKF and EK-PF) for dual estimation of state and parameters of a reduced order model of a time-varying system. A Kalman filter is employed within each iteration period to update the subspace spanned by the POMs of the structure. The efficiency and effectiveness of the algorithm are verified via pseudo-experimental tests conducted on multi-storey shear buildings. It will be shown that the procedure successfully identifies the state, the model parameters (i.e., the components of the reduced stiffness matrix of the structure) and relevant proper orthogonal modes (POMs) of the reduced model. Unbiased estimates furnished by the algorithm permit the health monitoring of the structure.

By reading this monograph, one could learn how the family of Kalman filters and particle filters are connected; compare their performances when dealing with a structural dynamics problem; see through detailed examples why and when they fail; figure out which filter can better fit a certain problem; and know how to tune the parameters of the filters. Moreover, the way the filters are presented renders the task of implementing more complicated filters easy and even developing ad hoc filters for structural engineering possible. Concerning reduced order modeling, possible limitations caused by POD-based reduced models are shown via numerous graphs and tables. The nature and extent of the inaccuracies caused by abridging the full mathematical model of the structures are carefully studied and analyzed. Finally, the use of such reduced models in the Bayesian filters is studied for the case in which the model can change (sustain damage) and also when it is a priori known that the model remains undamaged.

To follow the contents of this monograph, the reader is expected to have a background in statistics and calculus, and to be familiar with linear algebra and fundamentals of signal processing.

The material covered in this book is derived from the doctoral dissertation of the author, which was submitted to the scientific faculty of the doctoral program at the Department of Structural Engineering of the Politecnico di Milano. The author Preface

sincerely acknowledges the role of his supervisor Prof. Stefano Mariani in shaping this monograph; his encouragements, friendliness, rational way of thoughts, mindful directions, and scientific attitude had been the main elements without which this work would never have been in its present state. The author wants to thank all his friends in the department and outside it, whose presence made those years so memorable.

Saeed Eftekhar Azam

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# Chapter 1 Introduction

**Abstract** In the current Chapter, the fact that a significant portion of the existing civil structures and infrastructures in the developed and industrialized nations was constructed at the early period of twentieth century is discussed. It is also expressed that a notable part of the existing structures have been subject to deteriorations. Moreover, the need to develop damage identification techniques for vibration based non-destructive damage identification of the structures is briefly debated. Then, the major emphasis of the monograph and the type of the target structures is explained. In the end, the major disciplines covered inside the book are highlighted.

### 1.1 Background and Motivation

A significant portion of civil structures and infrastructures was constructed at the early period of twentieth century in the developed and industrialized nations; consequently, they have been subject to deteriorations. To illustrate this issue, over 50 % of the bridges were built in the U.S.A prior to 1940 (Stallings et al. 2000); moreover, over 42 % of all the aforementioned bridges are structurally deficient as reported by Klaiber et al. 1987. In Canada, over 40 % of the present functional bridges were built prior to 1970 and majority of these Canadian bridges demand prompt rehabilitation, strengthening or replacement (ISIS Canada 2007). The Canadian Construction Association estimated nearly 900 billion US dollars as the cost to rehabilitate global infrastructures (ISIS Canada 2007). In the next years, it takes a great deal of budget to rehabilitate the global infrastructure which highlights the significance of developing reliable and cost effective methods for the investments required for rehabilitation. Moreover, in seismically active zones, the deterioration due to degradation in the structures may be combined with the damage due to extreme seismic actions.

In recent years, civil engineering community has globally focused their attention on structural health monitoring with the purpose to identify the damage

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occurred in civil structures at the earliest possible stage, and to estimate the remaining lifetime of the structures. Structural damage caused by corrosion leads to degradation of the mechanical properties of the affected components; therefore, it changes the response of the structure as well. Moreover, the failure of the structural components such as shear walls, bracings and connections clearly changes the mathematical system which is defined to mimic the behavior of the structure. Hence, the goal of structural health monitoring can be perceived by structural system identification. The system corresponding to healthy state should be primarily identified; moreover possible changes which occur in the system with respect to the healthy state of structures are indications of structural damage in next planned system identifications. This task is realized within the frames of nondestructive vibration-based damage identification either by direct identification of the system or an alternative indirect scheme. Moreover, several dynamic characteristics of the system are identified, and possible variations in their value are employed to update the system. Instances of former methods include dual estimation of states and parameters of the structure via Bayesian inference techniques (Chatzi et al. 2010), while latter methods utilize modal properties of the structure to detect the damage (Moaveni et al. 2010).

To prevent the possible casualties and losses caused by sudden collapse of the structure, timely detection of the structural damage is essential.

The collapse of the bridge on Minneapolis I-35 W highway is one of the recent structural catastrophe. The steel truss bridge, constructed in 1967, collapsed during rush hour which led to dozens of causalities on August 1, 2007 (French et al. 2011). Beyond philanthropic issues, the economic impact of the collapse has been substantial: road-user costs due to the unavailability of the river crossing imposed a financial burden of \$220,000 US dollars per day (Xie and Levinson 2011). These statistics highlight the economic significance of the civil infrastructure; and therefore substantiate the demand to monitor their safety: St. Anthony Falls Bridge on the I35 W, constructed to replace the collapsed steel truss bridge, includes over 500 instruments to monitor the structural behavior (French et al. 2011). To detect the damage at the earliest possible stage, long-term monitoring systems are required to process the data sensed by these instruments.

### 1.2 Objectives and Scope

The objective of the study presented in this monograph is to develop damage identification techniques for vibration based non-destructive damage identification of the structures. In fact, the major emphasis is on the development of quick and robust recursive damage detection algorithms in order to facilitate the task of online real-time continuous monitoring of civil structures, such as e.g. residential buildings, bridges and other similar structures. To accomplish this end, four Bayesian filters, namely the extended Kalman filter (EKF), the sigma-point Kalman filter (SPKF), the particle filter (PF) and a hybrid extended Kalman particle filter

(EK-PF) are adopted to identify the structural system. To avoid shadowing effects of the structural system, performance of the filters is benchmarked by dual estimation of state and parameters of a single degrees-of-freedom structure featuring nonlinear behaviors namely: an exponential softening and a bilinear (linear-softening, linear plastic and linear hardening) constitutive laws are examined. It will be observed that the EK-PF outperforms all the other filters studied in this research. It should be emphasized that though Bayesian filters have been extensively studied in the automatic control field, their applications in structural engineering is yet to be investigated. The applications of EKF, SPKF, and PF to simplified and low dimensional models are suggested in the existing literature; nevertheless, to the best of our knowledge, applying EK-PF has never been reported when dealing with a structural engineering problem. After the performance of the filters is benchmarked when working with a single degree-of-freedom system, multi degrees-of-freedom structures are handled. Consequently, EKF for its computational efficiency and EK-PF for its excellent performance working with single degree-of-freedom systems are adopted. It will be indicated that the performance of EKF and EK-PF is identical when engaging with a two degrees-offreedom system; nevertheless, moving to three and four degrees-of-freedom structures, the EK-PF outperforms the EKF in terms of the bias in the estimation. It is perceived that as the number of the degrees-of-freedom increases, the adopted methods lose their accuracy in system identification and thus in damage detection process. This problem is created due to the high dimension of the parameter space, i.e. by the so-called curse of dimensionality. To manage this issue in this study, we make recourse to reduced order modeling of the systems. Regarding the model order reduction technique, a method based on the proper orthogonal decomposition (POD) is adopted. Such method utilizes POD to define a subspace in which the main dynamic evolution of the system occurs; the vectors that span the POD subspace are called proper orthogonal modes (POMs). Once such a subspace is attained, a projection method onto the POD subspace is employed to reduce the order of the set of governing equations of the system; subsequently the speed of calculations is increased. In addition to the expediting the calculations, another striking property of the so-called POMs is that they are sensitive to changes in the system parameters; thus in this study this property is exploited to identify the damage in the structure.

The major novel contribution of this monograph is to develop a recursive stochastic algorithm by a synergy of dual estimation concept, POD-based order reduction and subspace update. The proposed methodology takes advantage of Bayesian filters (e.g. EKF and EK-PF) for dual estimation of state and parameters of a reduced order model of a time-varying system. A Kalman filter is employed within each iteration period to update the subspace spanned by the POMs of the structure. The efficiency and effectiveness of the algorithm is verified via pseudo-experimental tests conducted on a ten-storey shear building. It will be indicated that the procedure successfully identifies the state, the model parameters (i.e. the components of the reduced stiffness matrix of the structure) and relevant POMs of

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the reduced model. Unbiased estimates furnished by the algorithm permits the health monitoring of the structure.

### 1.3 Organization of the Content

The present research is categorized into three major topics in this monograph, namely: (a) online and model-based system identification of dynamic systems; (b) model order reduction of dynamic systems; and (c) reduced order model identification of dynamic systems. The content of the monograph is derived from the PhD dissertation of its author which has been presented to the faculty of the doctoral course of the department of structural engineering at the Technical University of Milan (Eftekhar Azam 2012).

In the Chap. 2, the first research topic is extensively examined. Dual estimation of state and parameters of structural state space models is considered; moreover, the EKF, SPKF, PF and EK-PF are employed for parameter identification and state estimation. First, the performance of the filters is benchmarked by applying a single degree-of-freedom nonlinear system; subsequently, application of the filters to multi degrees-of-freedom systems is considered. Therefore, a multi storey shear building is assessed. Limitations for applicability of this approach in the identification of e.g. the stiffness matrix of multi storey structures are highlighted. It is concluded that due to bias in the estimates, these approaches are not suitable for system identification of shear building structures with more than three storeys.

Model order reduction of multi storey buildings is presented in the Chap. 3. Proper orthogonal decomposition is employed to extract the minimal subspace which features the dominant characteristics of the structure, via information contained in the response of the structure itself. The subspace discovered by POD is obtained by mathematical manipulation of the samples of the response of the structure (gathered in the so-called snapshot matrix), thus it can be load dependent. In case the external excitation is formerly known, load dependency of the reduced model will not be a problem; nevertheless in case of seismic excitations, such condition is not always true. To address this issue and build the snapshot matrix, the samples are selected from the response of a case-study structure to the El Centro accelerogram; furthermore, the obtained reduced model is subsequently employed to simulate the response of the case-study structure to the Friuli and the Kobe earthquake records. It is observed that POD-based reduced models are robust to changes in input seismic load. Afterwards, efficiency of the method in expediting the calculations, with high level of fidelity, is numerically examined.

Chapter 4, investigates the statistical properties of residual errors induced by POD-based reduced order modeling. Such errors enter the state space equations of the reduced systems in terms of system evolution and observation noise. A fundamental assumption made by recursive Bayesian filters, as exploited in this study, is the whiteness of the aforementioned noises. In this chapter, null hypothesis of the whiteness of the noise signals is tested by utilizing the Bartlett's whiteness test.