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Abraham Lincoln's Cyphering Book and Ten other Extraordinary Cyphering Books

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Foreword by Fred Rickey

 Springer

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Chapter 6, which deals with Abraham Lincoln's cyphering book, was written by Nerida F. Ellerton, Valeria Aguirre Holguín (New Mexico State University), and M. A. (Ken) Clements.

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Overall Book Abstract, and Individual Abstracts for the 12 Chapters

Overall Abstract

This book analyzes 11 extraordinary cyphering books prepared by students in Great Britain or in North America between 1630 and 1836. Altogether, there are 150 figures, most of them full-color images of pages, or sections of pages, in the cyphering books. Changes in school mathematics curricula and in the teaching and learning of mathematics in Europe and North America during the period from 1630 to 1880 are discussed in ways consistent with the authors' main thesis, originally presented in *Rewriting the History of School Mathematics in North America 1607–1861* (Springer, 2012), that school mathematics was controlled by expectations associated with the cyphering tradition, especially the business-oriented *abbaco* curriculum. The 11 extraordinary cyphering books which are analyzed were prepared by:

1. An anonymous person in a New England settlement in the 1660s;
2. Thomas Prust and John Collings (England and North America, 1702–1721);
3. Martha and Elisabeth Ryan (North Carolina, 1776–1781);
4. Mary Walters (Maryland, 1820);
5. Abraham Lincoln (Indiana, 1819–1826);
6. Thomas Dixson (England, 1630);
7. George Bickham (London, c. 1740);
8. An anonymous British midshipman (c. 1791);
9. Charles Page (Royal Mathematical School, Christ's Hospital, London, 1825); and
10. William Beattie (who prepared two of the cyphering books—one around 1810 in Scotland, and the second, in 1835 and 1836, possibly as a special gift for his friend, King William IV, of England).

Chapter 6, which was co-authored by Nerida Ellerton, Valeria Aguirre Holguín and Ken Clements, offers the first major analysis of the 22 extant pages of Abraham Lincoln's cyphering book—which is the oldest surviving Lincoln manuscript. The analysis reveals that between 1819 and 1826 the future President, with much determination, prepared his cyphering book. What made his cyphering book extraordinary was the fact that he prepared it at all, given the circumstances which made it difficult for him to study.

In the final chapter, six major lessons for 21st-century school mathematics are drawn from the analyses set out in the preceding 11 chapters. The six lessons relate to relationships between the intended, implemented and attained curricula associated with the cyphering era, to the great importance attached to the cyphering books by those who prepared them, to how ethnomathematical circumstances influenced the intended, implemented and attained curricula, and to differences between the cyphering traditions in Great Britain and North America.

Individual Chapter Abstracts

Chapter 1: Cyphering Books and the Cyphering Tradition in North America and Great Britain, 1630–1880

Abstract: The cyphering tradition controlled school mathematics in Western Europe and North America during the seventeenth and eighteenth centuries and the first half of the nineteenth century. In this book the authors analyze 11 extraordinary cyphering books—7 arithmetic cyphering books, 3 navigation cyphering books, and 1 mensuration/surveying cyphering book. Of these, 6 were prepared

entirely in Great Britain, 4 entirely in North America, and 1 partly in Great Britain and partly in North America. After defining key terms—such as “cyphering book,” “cyphering tradition,” “intended curriculum,” “implemented curriculum,” and “attained curriculum”—this first chapter summarizes characteristics of the 11 extraordinary cyphering books that will be considered. Finally, six research issues—which will frame subsequent analyses—are stated.

Chapter 2: Primitive Beginnings, Circa 1667

Abstract: Although the name of the person who prepared this seven-page manuscript, and of the place where it was prepared, are unknown, the last owner indicated that the manuscript was purchased from a very long-established family in Maine, New England. Importantly, the paper is clearly watermarked, and the watermark indicates the paper was made in 1666 or 1667. We have assumed that the paper was brought from Europe to New England during the second half of the 1660s, and that the writing in the manuscript was done in New England. If that was indeed the case then this is easily the oldest of known extant North American cyphering books and, in that sense, is extraordinary. A second notable feature of the manuscript is that the mathematics and spelling are at a very low level. Two other interesting features of the manuscript are: (a) the person who prepared it seemed to want to be able to claim that he or she had cyphered to the rule of three because, all of a sudden, that rule is mentioned on the last page; and (b) one of the topics mentioned was “halving,” which was *not* a normal part of the *abbaco* curriculum sequence for arithmetic, but *was* part of the Sacrobosco curriculum sequence for arithmetic.

Chapter 3: “Thomas Prust his Booke Amen 1702”

Abstract: The person who prepared most of the entries for the cyphering book studied in this chapter was Thomas Prust, who was born in England in 1690 and migrated to New England in 1720. There is a handwritten note in the cyphering book indicating that in 1720 Thomas passed it on to James Collings. Thomas became a shopkeeper and the training he received when he was preparing his beautiful, largely *abbaco*-inspired, cyphering book would have been beneficial to him during his time as a shopkeeper. He used the galley algorithm when performing division calculations, and was determined to master the rule of three. Many of the problems in the cyphering book were linked to real-life situations. The record indicates that James Collings became a sea captain, and he probably took this cyphering book with him on his voyages. Although we do not know how many later students were guided by this extraordinary manuscript, of one thing we can be certain: this was originally Thomas Prust’s “booke,” and over 300 years later most of it still remains intact.

Chapter 4: Daughters of the Revolution: Martha and Elisabeth Ryan’s Cyphering Book, Circa 1780

Abstract: There are three independently remarkable characteristics of this manuscript, and there is a fourth feature which is also of special interest. The first extraordinary characteristic is the patriotic fervor of the two sisters who prepared it. Illustrations of ships and patriotic inscriptions reflecting the Revolutionary times—such as “Liberty,” “George Washington,” and “Poor freedom is better than rich slavery”—are to be found on many of the pages. The second extraordinary aspect of this manuscript is that although it was prepared by two teenage sisters around 1780 the arithmetic went well beyond the rule of three to topics like alligation, fellowship, equation of payments, and single and double false position—it was very unusual for females at that time to study such topics. The third, and most obvious, remarkable aspect of the manuscript is the extremely high quality of the

penmanship, calligraphy and artwork. Martha and Elisabeth Ryan lived in North Carolina, and this manuscript is an example of an early “southern” cyphering book—of which relatively few of those prepared by girls have survived.

Chapter 5: With a Tinge of Green: Mary Walters’ Cyphering Book, 1820

Abstract: In 1820 Mary Walters was based in Baltimore, Maryland. She had emigrated from Ireland to North America and her cyphering book, prepared in 1820, provides evidence that in Maryland some Irish-heritage school teachers were responsible for engaging students in a form of mathematics education that had more than a tinge of green. Certainly, like all who followed the *abbaco* arithmetic curriculum, Mary was expected to learn how to add, subtract, multiply and divide counting numbers, and to apply elementary measurement principles in a range of everyday situations. When dealing with tasks involving the measurement of quantities, she needed to be able to change from one set of units to another. She also needed to learn to apply the direct rule of three in a wide range of real-life scenarios in which ratio and proportional thinking were needed. But the entries in Mary’s cyphering book also made it clear that her teacher(s) wanted her to inject herself into problem-based scenarios by which she would learn to apply elementary arithmetic principles to tasks concerned with human-interest, life-and-death situations which often generated emotions like jealousy, love, and desire. In particular, she was expected to appreciate the power of stories, poetry and song to convey interesting problem scenarios.

Chapter 6: He would be Good: Abraham Lincoln’s Early Mathematics, 1819–1826

Abstract: This chapter, jointly written by Nerida Ellerton, Valeria Aguirre Holguín, and Ken Clements, offers a comprehensive analysis of the earliest extant handwritten manuscript of Abraham Lincoln—22 pages (11 leaves), from a cyphering book that Lincoln prepared between 1819 and 1826. The pages are examined from historical, mathematical and educational perspectives, and an order in which they were written is conjectured, with justifications provided. This is the first time all 22 pages have been examined, it having become received tradition that there were only 20 surviving pages. The authors argue that if the corpus of all extant U.S. cyphering books were to be studied, then the future President’s cyphering book would not be particularly outstanding from calligraphic, penmanship, or abstract mathematical points of view. Analysis reveals, nevertheless, that Lincoln succeeded in his quest to prepare an attractive cyphering book in which almost all entries were arithmetically correct—despite his having to cope with rough frontier circumstances.

Chapter 7: Sacrobosco’s Heritage: Thomas Dixson’s Cyphering Book, 1630–1632

Abstract: The 302-page handwritten manuscript by Thomas Dixson which is the focus of attention in this chapter is 46 years older than the oldest of the British cyphering books mentioned by John Denniss (2012). The analysis presented here suggests that the curriculum sequence put forward by Johannes de Sacrobosco early in the thirteenth century, for the study of arithmetic, may have been more influential in seventeenth-century England than Leonardo of Pisa’s *abbaco* curriculum sequence. Some attention in this chapter is given to the so-called “galley” algorithm for division, which Dixson used. In keeping with the standard Sacrobosco curriculum sequence, Thomas Dixson included, early in his arithmetic studies, sections on arithmetical and geometrical progressions, and on common fractions—those topics would have come much later if the *abbaco* curriculum had been followed.

Chapter 8: Writing as if Arithmetick: George Bickham’s Cyphering Book, Circa 1740

Abstract: Of the 11 extraordinary cyphering books considered in this volume, George Bickham’s was the only one that was specifically prepared for sale to others. We do not know how many copies of this book were printed, but we do know that the copy which we now hold is one of very few which remain. Bickham was a writing master who was regarded, in his time, as one of world’s best penmen. As was usually the case with writing masters, he paid much attention to the kind of writing and calligraphy needed to prepare a strikingly beautiful cyphering book, and this book provides testimony to that fact. However, there are hardly any model examples, or exercises for the student to do. The emphasis seemed to be on form rather than mathematical substance, and the language of commercial arithmetic is strongly emphasized. As would have been expected, topics are presented according to the normal *abbaco* sequence.

Chapter 9: Bound for Botany Bay? Circa 1791

Abstract: The 240-page navigation cyphering book featured in this chapter includes 80 artistic creations, many of them exquisite water-color miniatures. These miniatures are remarkable in the sense that they depict buildings, towers, hills, streams, etc., on which are labeled heights, widths, distances, angles, etc., relevant to mathematical tasks which are given in the text. Some of the labels are so small that it is difficult to read them with the naked eye. An important part of this chapter relates the beautiful artwork in the manuscript to the 18th- and 19th-century push to include serious drawing education in curricula designed for middle-class students and artisans. Another intriguing aspect of the chapter is that it draws attention to mathematical compromises, by mathematicians, with respect to trigonometric functions and logarithms. The title of the chapter, “Bound for Botany Bay? Circa 1791” was chosen because of the possibility that the diagrams and other artwork in the chapter depict actual scenes that the person who created the navigation cyphering book saw from a ship (or ships) that was (or were) part of the First Fleet from England to Botany Bay (New Holland) in 1787 and 1788.

Chapter 10: From the Royal Mathematical School: Charles Page, 1825

Abstract: Charles Page was born in 1810, and between 1818 and 1825 he was a student at the Royal Mathematical School within Christ’s Hospital, London. Over his final two years as a “mathemat” (i.e., student in the Royal Mathematical School) he prepared the 671-page navigation cyphering book which is the subject of analysis in this chapter. Throughout the manuscript the standard of penmanship, calligraphy, artwork, and mathematics is at a very high level—indeed, much of the mathematics in this cyphering book is at a higher level than the mathematics in any of the other extraordinary cyphering books considered in this volume. Careful examination of the entries in Charles Page’s manuscript suggests, however, that many of them were taken down from dictation. In the section on quadratic equations there are unchecked errors, and we were left with the impression that although the appearance of pages was invariably impressive, the quality of Charles’ learning about mathematics and about navigation may have been less than impressive. Details in this chapter provide a glimpse of the expectations on mathemats in the 1820s.

Chapter 11: Fit for a King? William Beattie, Circa 1810 and 1835–1836

Abstract: William Beattie prepared both of the manuscripts that are analyzed in this chapter. The first has 466 pages and is concerned mainly with mensuration. Many of the notes and examples in that manuscript are from a 1748 London edition of William Hawney’s classic text, *The Compleat*

Measurer. The second manuscript has 309 pages and concerns navigation. It is based almost entirely on a 1796 edition of John Hamilton Moore's *The New Practical Navigator*. It is conjectured that although the title page of the measurement manuscript is dated 1835, in fact the manuscript was prepared by Beattie around 1810, when he was a student at Clarencefield Academy in Dumfriesshire, Scotland. Beattie became a medical doctor, and served as personal physician and then private secretary to the man who would become King William IV—and would be remembered as the “Sailor King.” It is conjectured that the navigation cyphering book, dated 1836 on its title page, was specially prepared by Beattie for his ailing King (who died in 1837). Although the penmanship and calligraphy in both manuscripts are of a high standard, Beattie seemed to value the mensuration manuscript more highly than the navigation manuscript.

Chapter 12: Lessons from Extraordinary Cyphering Books

Abstract: Six lessons about school mathematics, and its history, are drawn from the analyses of the 11 extraordinary cyphering books. These lessons relate to: (a) the intended curriculum that the *abbaco* and Sacrobosco traditions, and the British and North American textbook authors, defined for school mathematics during the period 1600 through 1880; (b) the implemented curriculum, as evidenced by entries in cyphering books; (c) the lack of hard data with respect to the attained curriculum—that is to say, with respect to what was actually learned by students during the cyphering era; (d) the remarkable value that many students placed upon their cyphering books; (e) the ethnomathematical circumstances which surrounded the preparation of the extraordinary cyphering books; and (f) the educational significance of qualitative differences between British and North American cyphering books.

Preface

In Chapter 4 of our *Rewriting the History of School Mathematics in North America 1607–1861: The Central Importance of the Cyphering Tradition* (Ellerton & Clements, 2012), also published by Springer, we summarized our journey which enabled us to recognize the large influence on school mathematics, over many centuries, of the cyphering tradition. Since writing that book we have continued our investigations into the cyphering tradition, and have now analyzed about 1000 handwritten cyphering books—most of them prepared in the United States or in Great Britain between 1630 and 1880.

The cyphering tradition *controlled* the intended and implemented curricula of pre-college mathematics education in Great Britain and North America between 1600 and 1850. And, although the influence of that tradition declined sharply in the second half of the nineteenth century, it lingers almost ghost-like, affecting school mathematics programs in every country in the world. Much has been written about historical circumstances which influenced presidents, prime ministers, great literary figures, large corporations, and about the rise and fall of civilizations. However, very few books have been written on the history of school mathematics—something which has affected, and continues to affect, vast numbers of people, everywhere.

“But,” one might respond, “not everyone is greatly interested in the history of electricity, or of the automobile, or of the internet, and yet electricity, cars, and the internet, are all vitally important in today’s society. One simply does not have enough time to take a scholarly interest in everything that is important. So why should one have an interest in the history of school mathematics?” If you are a mathematics education researcher, or education historian, then you might have wanted to add that scholars must be selective in what they choose to research, and that so far as school mathematics is concerned the most important task is to improve *present* practices. To such an argument we would respond by pointing out that most people have an interest in their family roots—they want to know from whom, and from where, they have come. From that perspective, it would appear to be reasonable to expect mathematics teachers and mathematics education researchers, and those concerned with school curricular issues, to have some interest in the roots of today’s school mathematics curricula and practices.

This book does more than provide a complement to our *Rewriting the History of School Mathematics in North America 1607–1861*. Here, we extend our analysis to the history of the cyphering tradition in the United Kingdom, covering the period from about 1630 to 1850. As well, the book adds flesh to our earlier analysis of the cyphering tradition in North America. The methodology employed is the historical case study. Eleven cyphering books were chosen as cases to be studied, and each was, in its own peculiar way, “extraordinary.” The analyses presented are more qualitative than quantitative.

One of the chapters provides the most detailed analysis of Abraham Lincoln’s cyphering book ever presented—that chapter is the first to examine all 22 surviving pages of what is easily the oldest surviving handwritten manuscript prepared by the future president. Another chapter analyzes the hauntingly beautiful composite manuscript of Martha and Elisabeth Ryan, who, around 1780, studied arithmetic to a level well beyond what was expected of girls in North America. Another manuscript was probably prepared somewhere in New England in the late 1660s—if it was, then it is easily the

oldest extant North American cyphering book. One chapter analyzes the incredible 671-page navigation cyphering book prepared by Charles Page, a boy attending the Royal Mathematical School at Christ's Hospital, London, in the 1820s. Another chapter presents an analysis of a navigation cyphering book that may have been prepared as a gift for the King of England. Space does not permit each manuscript to be mentioned here, but each is, in its own way, remarkable.

Throughout the book a critical stance has been adopted. Some of the analyses reveal that students and their teachers were more interested in preparing attractive books than in developing an understanding of the mathematics that should have been the main focus of their attention. The final chapter begins with a discussion of whether it makes sense to claim that “lessons for today” can sensibly be identified from history. Then six lessons that we believe emerge from our analyses in this book are, in fact, identified.

Those who read this book will, we trust, gain a deeper knowledge of the history of school mathematics. They should also gain new perspectives on today's problems, and challenges, with respect to school mathematics. Perhaps, too, they will be inspired by the “stories” emerging from our analyses of the extraordinary cyphering books. We were.

This book analyzes cyphering books prepared at various times in either North America or Great Britain between 1630 and 1836. During that period there were name changes with respect to both nations in which the books were prepared. The term “United States of America,” or its abbreviated form, the “United States,” will be used to denote the nation comprising those states which, after 1776, comprised the United States of America. The term “Great Britain” will be used to describe the nation comprising England, Scotland and Wales, which was formed by the Act of Union passed by the Scottish Parliament and by Westminster in 1707 establishing the Kingdom of Great Britain. After the Irish Parliament voted to join the Union in 1801 the then Kingdom of Great Britain became the United Kingdom of Great Britain and Ireland.

In this book many of the figures are photographic images of handwritten pages from cyphering books. Often it is difficult to decipher the writing—sometimes (as with the surviving pages of Abraham Lincoln's cyphering book), the pages were very worn, and the writing faded. Also, the ways letters were formed have changed—so that an “s” will often appear to a modern reader like an “f.” The writing is sometimes very small, and on numerous occasions we used enlarged images for analysis.

A careful reader of this volume might notice that, from time to time, there is repetition of points made earlier in the book. Individual chapters of this book will be available for purchase in electronic form, and therefore it makes sense to repeat material important for a chapter, even if that material has been discussed earlier in the book. We have attempted to limit such repetition to cases where what is being repeated represents essential knowledge if the present chapter is to be understood as a stand-alone document. Another feature, in line with opportunities provided by technology, is that there is a reference list at the end of *each* chapter, and a *composite* reference list after all 12 chapters have been presented.

Toward the end of some of the chapters we have acknowledged those who have assisted our research in major ways with respect to those chapters. Although we will not repeat those acknowledgements here, we do wish to thank all who have so generously extended to us their time and expertise as we progressed along “the way.” That general acknowledgement takes into account numerous persons working in archives in various parts of the United States of America and the United Kingdom.

Of course, we would offer a special “thank you” to Fred Rickey, who has written the Foreword to this book. Fred, a fellow researcher in the history of mathematics education, is someone whose work we greatly admire. Fred, and two of his colleagues, are currently in the process of writing a book on the cyphering manuscripts prepared by George Washington in the 1740s. We await the outcome of this research with much anticipation. We also wish to thank Valeria Aguirre Holguín, of New Mexico State University, for her contributions to the chapter on Abraham Lincoln’s cyphering book, and George Seelinger and other colleagues within the Department of Mathematics at Illinois State University.

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January 2014

Foreword

Igitur, eme, lege, fruere.¹

In their previous book, *Rewriting the History of School Mathematics in North America 1607-1861: The Central Role of Cyphering Books* (2012), Nerida Ellerton and Ken Clements convincingly established that the long-forgotten and centuries-old tradition of students preparing their own cyphering books was central to their study of mathematics. Surprisingly few of these cyphering books have survived even though they were highly valued by their creators. Even so, our authors have amassed a personal collection of some 350 cyphering books and examined another 650 in various libraries. Out of this abundance of resources, eleven “exceptional” cyphering books have been selected for detailed examination in this volume. These eleven were selected for a variety of reasons: their age, the lovely handwriting, the beauty of the diagrams, their geographic origins, their mathematical content, and the contexts in which they were written.

The mathematics in these cyphering books is the mathematics of the practitioner, not the theoretician. This is not pure mathematics, but, to use the archaic term, mixed mathematics. Besides the arithmetic of the integers, there is business arithmetic, practical geometry, logarithms and trigonometry, fortification, navigation, and surveying. Of course, none of these cyphering books deals with all of these topics but each has a common arithmetical core which derives from the medieval abacists. This work describes the provenance and mathematical content of the eleven cyphering books, setting each in its historical context.

The mathematics in these eleven wonderful cyphering books is completely algorithmic. Rules are stated which indicate how to solve a class of problems, and the student is expected to memorize and follow them. There is almost no explanation of why these algorithms work the way they are expected to.

Many obsolete mathematical terms will be encountered—alligation, compound addition, evolution, exchange, fellowship, the galley method, reduction, superficies—and they are carefully explained as they arise, thereby giving insight into how mathematics was taught and learned in Great Britain and in North America between 1630 and 1836. The most interesting of these terms are the various forms of the rule of three, an algorithm for dealing with proportions. There is little elementary symbolic algebra in these cyphering books—the rule of three provides a way of avoiding it. Not all students progressed to the rule of three in their study of mathematics, but those who did were truly accomplished. They were regarded as well prepared to run a business or to oversee a plantation. The more advanced were prepared to pilot a ship, to become professional surveyors, or to become schoolteachers who would use their own cyphering books as texts. Of course, not all teachers were well prepared.

At a meeting of the History and Pedagogy of Mathematics group at West Point in April of 2013, Valeria Aguirre Holguín, a graduate student at New Mexico State University, gave a presentation entitled “New Information on the Provenance of Abraham Lincoln’s Arithmetic Copybook.” It was at this time that the three authors began joint work on the Lincoln chapter. Amazingly, in June of 2009, Ellerton and Clements had identified a leaf of Lincoln’s cyphering book in the Houghton Library at Harvard University, a result announced in a June 2013 press release.

¹ “Therefore, buy, read, enjoy [this book],” Copernicus, *De revolutionibus orbium coelestium* (1543).

This brought the number of identified leaves to 11 (that is to say, 22 pages); hopefully someone, someday, will identify more leaves from Lincoln's cyphering book. The 11 extant leaves are carefully examined in a chapter that will undoubtedly become the most studied chapter in the book.

Louis C. Karpinski lists hundreds of elementary mathematics books in his *Bibliography of Mathematical Works Printed in America Through 1850*. But textbooks such as these were expensive and not easy to acquire, so students continued to prepare their own cyphering books until the time of the Civil War when educational philosophy changed. But those that have survived bear witness to the pride that the authors took in preparing their own personal cyphering books.

As with any seminal work, this one will engender numerous future research questions. Paramount among these is one only partially answered here—a question which still perplexes researchers today: Based on their written work, how do we know how much mathematics students have comprehended and retained for their future work? It will be interesting to match the signatures in the corpus of cyphering books with the way those individuals used mathematics later in their lives. One sign of a wonderful book is the further research it encourages.

West Point, New York

V. Frederick Rickey

Chapter 1

Cyphering Books and the Cyphering Tradition in North America and Great Britain, 1630–1880

Abstract: The cyphering tradition controlled school mathematics in Western Europe and North America during the seventeenth and eighteenth centuries and the first half of the nineteenth century. In this book the authors analyze 11 extraordinary cyphering books—7 arithmetic cyphering books, 3 navigation cyphering books, and 1 mensuration/surveying cyphering book. Of these, 6 were prepared entirely in Great Britain, 4 entirely in North America, and 1 partly in Great Britain and partly in North America. After defining key terms—such as “cyphering book,” “cyphering tradition,” “intended curriculum,” “implemented curriculum,” and “attained curriculum”—this first chapter summarizes characteristics of the 11 extraordinary cyphering books that will be considered. Finally, six research issues—which will frame subsequent analyses—are stated.

Key Words: *Abbaco* curriculum; Abraham Lincoln; arithmetic education; attained curriculum; ciphering book; ciphering tradition; copybook; cyphering book; history of education; history of school mathematics; implemented curriculum; intended curriculum; navigation cyphering book; Royal Mathematical School at Christ’s Hospital; Noah Webster.

What is a Cyphering Book, and what was the Cyphering Tradition?

What is a Cyphering Book?

In this book the term “cyphering book” will be used to refer to a handwritten manuscript, usually prepared at some time during the period 1600–1880, which focused on mathematical content and had all of the following properties:

1. Either the manuscript was written by a student who, through the act of preparing it, was expected to learn and be able to apply whatever content was under consideration; or, it was prepared by a teacher who wished to use it as a model which could be followed by students preparing their own cyphering books.
2. Usually, all entries in the manuscript appeared in ink—as handwritten notes, or problem solutions, or as illustrations. Headings and sub-headings were presented in decorative, calligraphic style. Occasionally, water-color illustrations were prepared.
3. The manuscript was dedicated to setting out rules, cases, model examples and exercises associated with a sequence of mathematical topics. Although most cyphering books were specifically concerned with arithmetic, especially commercial arithmetic, some were dedicated to algebra, or geometry, or trigonometry, or to mathematics associated with mensuration, navigation, surveying, fortification, etc.
4. The topics covered were sequenced so that they became progressively more difficult. The content also reflected the expectation that, normally, no child less than 10 years of age would be assigned the task of preparing a cyphering book (Ellerton & Clements, 2012).

In their earlier book, Ellerton and Clements (2012) analyzed 212 North American cyphering books prepared between 1701 and 1861. They now have 350 North American cyphering books in their collection, and have studied another 650 North American cyphering books—all prepared between 1700 and 1861—held in archives at the Phillips Library (Salem, Massachusetts), Harvard University, Yale University, the University of North Carolina (at Chapel Hill), the University of Michigan, the College of William and Mary, and the Rockefeller Library (both in Williamsburg, Virginia), the Library of Congress, and the New York Public Library. They also own 150 cyphering books from other nations, including 100 from Great Britain—the third largest collection of British cyphering books known to exist (Denniss, 2012).

As a result of efforts to standardize American-English spelling, led by Noah Webster during the period 1780 through 1830 (Ellerton & Clements, 2008), the term “ciphering book” (with an “i”) gradually replaced “cyphering book” (with a “y”) in the United States of America. Although the term “copy book” (or “copybook”) was sometimes used to describe a cyphering book, it was a more general term, and was applied to any book in which handwritten texts had been “copied.” It has most commonly been used in relation to handwritten manuscripts in which there was an attempt to use fine penmanship and/or high calligraphy, especially when the copying was modeled on a well-regarded system of penmanship (see, e.g., Bickham, c. 1740).

The *Abbaco* Curriculum

A cyphering tradition which found its way into Europe from Arabic nations after about 1200 more or less controlled, for over 600 years, how European and North American school students who were at least 10 years of age learned mathematics (Ellerton & Clements, 2012; Van Egmond, 1980). From about the thirteenth century onwards, sharp rises in the numbers of international trading and banking companies in Southern European city-states prompted the formation of vernacular schools across Europe in which commercial mathematics, accounting and writing were taught by “reckoning masters” to sons of the merchant class.

During the period 1200–1850 almost all Western European and North American students aged 10 years, or more, who learned mathematics at school or with a private tutor were expected to prepare cyphering books, and the intended curriculum derived from what has been called the *abbaco* tradition. That curriculum defined a well-sequenced set of topics and was an integral part of the approach to business arithmetic adopted by European nations. For many years it was accepted by scholars that the tradition was introduced into Europe through Leonardo of Pisa’s (Fibonacci’s) *Liber Abaci*—which was written around 1200 CE (see, e.g., Yeldham, 1926)—but in recent years that tradition has been challenged. There is evidence that even before 1200 there were features of the tradition already to be found in parts of Southern Europe (Høyrup, 2005).

According to Jens Høyrup (2008), *abbacus* schools, which operated in Italy from the thirteenth century, were

... primarily frequented by merchant and artisan youth for ... two years (around the age of 11), who were taught the mathematics needed for commercial life: calculation with the Hindu-Arabic numerals; the rule of three; how to deal with the complicated metrological and monetary systems; alloying; partnership; simple and composite discount; the use of “single false position”; and area computation. Smaller towns might employ a master; in towns like Florence and Venice, private *abbacus* schools could flourish. In both situations *abbacus* masters had to compete, either for communal positions or for the enrolment of students. (pp. 4–5)

In schools, the cyphering tradition came to be associated with a typical organizational structure, which included three recitation components:

- First, the teacher prescribed what an individual student would do. Sometimes the teacher would write the headings and the problems to be solved into a student's cyphering book (Dickens, 1850; Walkingame, 1802). Each student would then prepare for the next recitation session.
- Second, the student would work individually on finding solutions to set exercises, often on a slate or on scraps of paper. These would be shown to the teacher during another recitation session. If approved by the teacher, the student would then be told to complete his or her cyphering book entries which included introductory statements for the given topic, rules, cases, model examples, and exercises. Sometimes the teacher provided a student with an older cyphering book, or textbook, to show what was required. Writing would be done with a quill and home-made ink (Ellerton & Clements, 2012).
- Finally, the student would show the teacher his or her completed cyphering book entries. It was expected that headings would be in fine calligraphy, and general penmanship would be of a good quality. A cyphering book was to be a guidebook for life—a text that could be consulted if and when the need arose—and it was important for all entries to be correct.

In each of these recitation components, the teacher would talk to individual students, asking verbal questions. At the lowest level, this verbal aspect of recitation may have involved the teacher expecting a student to recite rules which should have been memorized. At the highest level, teachers would ask probing questions that tested a student's understanding (Adams, 1848; Babcock, 1829; Cobb, 1835; Colburn, 1824; Emerson, 1838; Sterry & Sterry, 1795). According to Lyman Cobb (1835), a diligent teacher would talk with each learner, in recitation sessions, as often as once a day, probing the student's understanding of the principles upon which the rules were founded. Cobb (1835) added:

The teacher should not permit him [the learner] to commence a new sum, or engage in a new rule, until he is fully and thoroughly acquainted with the principles of the rule in which he has been working. Young scholars are generally anxious to make rapid progress in passing through the Arithmetick. This propensity, however laudable, should not be indulged at the expense of a partial knowledge of the subject. The teacher should endeavour to convince the scholar that, in order to make his progress advantageous, he must perfectly understand each rule and its principles. (p. 2)

Thus, the cyphering tradition incorporated a form of pedagogy which combined individual, yet supervised, learning, with follow-up evaluative sessions, usually through one-on-one recitations.

Figure 1.1 shows reproductions of two etchings by Abraham Bosse, a French-Huguenot artist who, in the 1630s, depicted the kind of classrooms that existed in middle-class Western European schools at that time. In the early 17th century, the Huguenots, the Dutch, and groups within quite a few other Western European nations (e.g., Italy), valued arithmetic, and saw it as a vital part of the education of all children.

In both of the etchings reproduced in Figure 1.1, a teacher and a student can be seen engaged in the process of recitation—the teacher was checking an individual student's work and, perhaps, asking questions or setting new work for the student to do. Some students were waiting to see the teacher, and others were working, individually, preparing their copybooks for when it would be their turn for recitation. In the North American colonies, most school rooms were less elaborate than those shown in Figure 1.1, and often teachers were not qualified to teach arithmetic beyond numeration

and the four operations with whole numbers. These early North American students rarely owned a commercially-distributed arithmetic textbook (Ellerton & Clements, 2012).

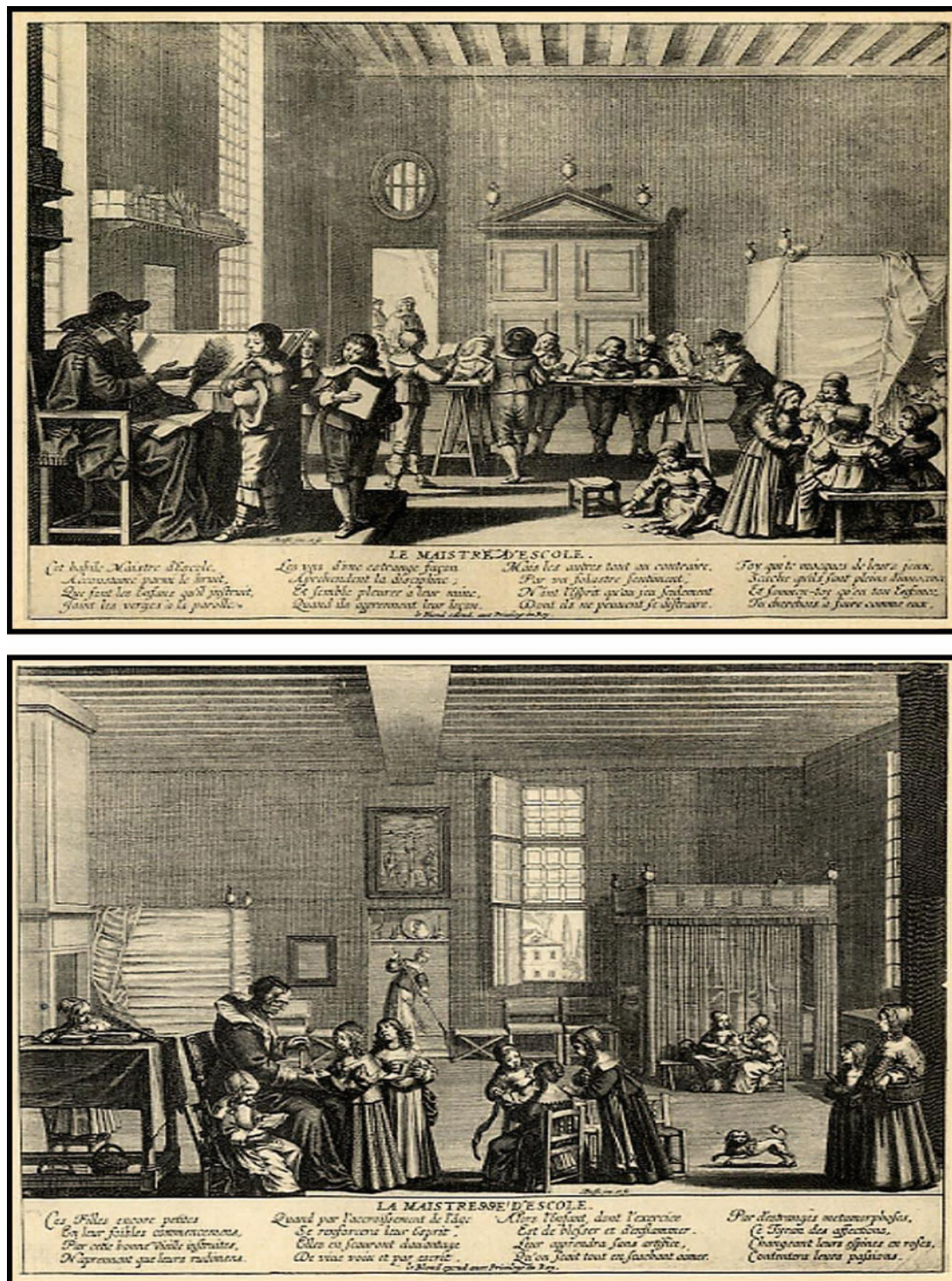


Figure 1.1. *Le Maître d'Ecole* (c. 1635) and *La Maîtresse d'Ecole* (c. 1638) by Abraham Bosse, (1611–1678). Held by the Metropolitan Museum of Art, New York (public domain images).

In Jan Steen’s (1670) depiction of a village school in France, reproduced in [Figure 1.2](#), there was little to be learnt from the teacher who was sharpening a quill and seemed to be unaware of the chaos surrounding him. A schoolmistress (the “usher”) was checking a child’s work—but others fought, sung, made fun of the teachers, and even slept. On the right a child was handing spectacles to an owl—a symbol Steen sometimes used to denote “foolishness.”



Figure 1.2. Jan Steen’s (c. 1670), “The Village School” in France. The original painting is held by the National Gallery of Scotland, in Edinburgh, Scotland (public domain image).

The cyphering tradition was duly translated to North America from Europe, and it quickly gained control over pre-college intended mathematics curricular sequences. Monroe (1917), drawing on research by William Eliot Griffis (1909) and William Heard Kilpatrick (1912), pointed out that cyphering was always included in the curriculum of the Dutch-language schools established in New Amsterdam—which would become New York—between the 1630s and the mid-1660s.

The approach to mathematics education inherent in the cyphering tradition represented a radical departure from the humanist, classical medieval forms of mathematics often found in European universities between 1200 and 1700. At the universities, mathematics was presented to students in Latin. At its worst, the study of mathematics at a university at that time did not go much further than a statement and illustration of rules for determining moveable feast days in the church calendar; at its best, however, it featured rigorous proofs and exercises (Duhem, 1985).

Eleven Extraordinary Cyphering Books

This volume analyzes 11 extraordinary cyphering books in varying degrees of detail. Each of the 11 manuscripts was initially prepared at some time between 1630 and 1835, in either Great Britain or North America. It will be useful here to indicate why each can be regarded as “extraordinary.” A summary of the 11 manuscripts is given in [Table 1.1](#).

Table 1.1
Summary of the 11 Extraordinary Cyphering Books

Chapter in this Volume	When and where the Manuscript was Prepared; Number of Pages	Author(s) of the Manuscript	Present Location of the Manuscript	Type of Cyphering Book	Why is this Manuscript “Extraordinary”?
2	c. 1667; Maine, New England (?); 7 pages.	Unknown	Ellerton/ Clements collection, Illinois	Arithmetic	This is probably the oldest extant North American cyphering book. The level of mathematics is extremely low, but the last page mentions the rule of three.
3	1702–1703; Great Britain, then 1720–1721, Portsmouth, New Hampshire; 88 pages.	Thomas Prust, and John Collings	Ellerton/ Clements collection	Arithmetic	Thomas Prust began preparing this book in England, but brought it to Portsmouth, NH, in 1720, and handed it on to his step-son, John Collings. The manuscript features beautiful calligraphy.
4	1776–1777 and 1780–1781; Bertie County, North Carolina; 151 pages.	Martha and Elisabeth Ryan	Wilson Library, University of North Carolina	Arithmetic	This features amazing penmanship, calligraphy, and artwork. The content goes well beyond the rule of three. It is held in the Southern Historical Collection of the Wilson Library.
5	1820–1821; Maryland (USA); 80 pages.	Mary Walters	Ellerton/ Clements collection	Arithmetic	This features a problem-based approach to arithmetic via Irish ballads, poetry, and song.
6	1819(?)–1826; Spencer County, Indiana (USA); 22 pages.	Abraham Lincoln	11 leaves are in 11 different locations (all in USA)	Arithmetic	This is the oldest extant manuscript of the future President. The content goes beyond the rule of three. A page-by-page analysis of content is presented.
7	1630–1632; Great Britain; 302 pages.	Thomas Dixson	Butler Library, Columbia University	Arithmetic	This is a very early British cyphering book. Penmanship and calligraphy are good, and much attention is paid to “galley” division.
8	c. 1740; London; 24 pages.	George Bickham (the Elder)	Ellerton/ Clements collection	Arithmetic	This cyphering book was prepared by one of the world’s best-known exponents of fine penmanship and calligraphy.
9	c. 1791; Great Britain; 240 pages.	Unknown	Ellerton/ Clements collection	Navigation	This manuscript was probably prepared by a midshipman during one or more voyages. It features exquisite miniature water-color illustrations.
10	1820–1825; London; 671 pages.	Charles Page	Ellerton/ Clements collection	Navigation	This was prepared at the Royal Mathematical School, Christ’s Hospital, London
11	c. 1810; Scotland; 466 pages.	William Beattie	Ellerton/ Clements collection	Mensuration /Surveying	This was probably prepared at Clarencefield Academy, in Dumfriesshire, Scotland
11	c. 1835; London; 309 pages..	William Beattie	Ellerton/ Clements collection	Navigation	This may have been prepared especially for the “Sailor King,” King William IV of England. William Beattie was the King’s private secretary.

Just two of the eleven extraordinary cyphering books analyzed in this book were prepared by females—which is consistent with recent literature indicating that in North America in the eighteenth and nineteenth centuries between 15 percent and 20 percent of cyphering books were prepared by females (Ellerton & Clements, 2012). Although eleven extraordinary cyphering books have been summarized in Table 1.1, two of those were prepared by the same person—William Beattie. Beattie prepared the mensuration/surveying manuscript sometime around 1810, when he was still at school in Scotland, and he prepared the navigation manuscript about 25 years later, when he was a close confidant of King William IV, of England.

Authors of seven of the eleven cyphering books were British. One of these books—that by Thomas Prust—was begun in England in 1702 but was subsequently brought to North America, where it was passed on to a second owner (John Collings), who made additional entries. Three of the other cyphering books—those by Martha and Elisabeth Ryan, Mary Walters, and Abraham Lincoln—were prepared in North America, and the fourth was probably prepared in Maine in New England, on paper made in Europe but brought to the New World in 1666 or 1667.

Cyphering Books, and Intended, Implemented, and Attained Curricula

Throughout this book it will be argued that the preparation of cyphering books was a very important component of what students actually did at school during the cyphering era. Handwritten entries typically included introductory notes on topics, rules, cases, model examples, and completed exercises. Sometimes, the notes were copied from a textbook or from an older cyphering book, and sometimes they were dictated by a teacher (Ellerton & Clements, 2012). For most cyphering books we do not know whether students merely copied material, or whether they copied and memorized it, or whether they attempted to comprehend what they wrote so that they could apply the knowledge in appropriate situations. That is to say, the cyphering books are largely silent on how well students learned what they transcribed.

Entries in cyphering books testify unmistakably to what Westbury (1980) and Travers and Westbury (1989) have called the *implemented* curriculum—that is say the actual mathematics that students studied. By contrast, textbooks, and documents prepared by education institutions like schools and colleges summarizing what students would be expected to learn, indicated the *intended* curriculum—which may or may not have matched the implemented curriculum. The *attained* curriculum is a term denoting what students actually learned. Cyphering books showed what students had studied but did not, and could not, show what they had learned.

During recitations, good teachers attempted to identify, through questioning, what students did and did not understand (McMurry & McMurry, 1903). A cyphering book said a lot about the kinds of mathematics students had studied but very little about how well they had learned it. Yet, if one carefully and systematically reads the entries in a cyphering book one can often catch glimpses of the degrees of understanding of the person who prepared the book. Thus, for example, in the cyphering book prepared by Abraham Lincoln (see Chapter 6), a careful reader can sense that Abraham had a driving determination to know what he was doing. He was not happy merely to copy. By contrast, with George Bickham's (c. 1740) cyphering book (Chapter 8), a reader is likely to feel that the author seemed to care only about the quality of the penmanship and calligraphy. In each of the chapters, comment will be made on the extent to which the person who prepared a cyphering book seemed to want to know, to understand, what he or she was writing about.

Six Prominent Issues

Ellerton and Clements's (2012) original book on the rise and fall of the cyphering tradition in North America between 1607 and 1861 was mainly concerned with establishing, beyond reasonable doubt, that the *abbaco* curriculum, as it found expression in a cyphering tradition inherited from Western Europe, effectively controlled the teaching and learning of school mathematics in the New World. Analyses in that book took advantage of the largest-ever data set of *primary* sources (comprising mainly, but not exclusively, textbooks and cyphering books) ever assembled in relation to the early history of school mathematics in North America.

In that earlier book, Ellerton and Clements (2012) presented a broad-brush analysis of the cyphering tradition as it operated in North America between 1607 and 1861. They reached the conclusion that it was through the cyphering tradition that some citizens became numerate, as they learned that problems demanding quantitative solutions came in categories, and that each category could be subdivided so that particular rules and cases would apply. This realization led Ellerton and Clements to conceptualize what they called the IRCEE ("Introduction, Rule, Case, Example, Exercise") genre. The "promiscuous problem" sets found in many of the cyphering books in their principal data set were intended to force students to ask themselves—"To which type of main problem category does this problem relate?" And, "If I want to solve this problem, which rule and which case will be most appropriate?"

The cyphering tradition provided the glue which held together, but at the same time constrained, the development of pre-college mathematics education curricula and teaching and learning in North America between 1607 and 1861. It was the force of that tradition which explains why the same arithmetic topics were studied, in more or less the same sequence, right across North America—in Boston, in New York, across Canada, and in schools in small rural communities. Ellerton and Clements (2012) concluded that the cyphering tradition "provided teachers with a subconscious base for the intended, implemented and attained aspects of school mathematics," and in particular provided "the backdrop against which the PCA (Problem, Calculation, Answer) genre, that still bedevils thinking and practice in school mathematics in the United States, arose" (p. 151).

This present volume provides qualitative, fine-grain analyses of early school mathematics in Great Britain and North America. The basic framework for analysis will be a concept of curriculum as having intended, implemented, and attained components (Westbury, 1980). It will be assumed that details of intended curricula are most readily available through contemporary textbooks, which in themselves are important primary sources. It will also be assumed that the best available evidence for implemented curricula is provided by cyphering books. This present volume is mainly concerned with analyzing implemented curricula and, because of that, the commercially-published textbooks are regarded as complementary rather than fundamental to learning. It has been useful, nevertheless, to be in a position to compare entries in cyphering books with sections in textbooks.

Denniss (2012), Michael (1993), and Stedall (2012) have provided evidence that between 1750 and 1860 most entries in British cyphering books were copied from textbooks, especially from those authored by the prolific Francis Walkingame. One only has to read the prefaces in the numerous editions of Walkingame's arithmetics (see, for example, Walkingame, 1802) to be convinced that that author assumed that students would use a textbook as the basis for making entries into cyphering books. It was likely that this textbook-driven approach to cyphering would be favored by some teachers in the newly-formed United States of America—especially when arithmetic textbooks written by North American authors became more readily available.

In Benjamin Workman's (1788) adaptation of John Gough's *A Treatise of Arithmetic in Theory and Practice*, there was an attempt to fit Gough's text, originally written for students in Ireland, to

students in North America. Workman emphasized that he had taken great pains to correct all errors in Gough's original text, and that he had also noticed that there was a huge number of errors in the arithmetic by Thomas Dilworth, which was often used in North America. Workman (1788) then commented on the difficulties a teacher of arithmetic in North America who used a textbook as the basis for setting exercises for students to do in their cyphering books might have:

When his pupils find answers different from those in the book, he obliges them to revise the work; for knowing the book to be right, he insists the mistake lies with them; but if he knows the book to abound with errors, he is obliged to work the questions himself before he can certainly tell them their work is wrong. (p. v)

Workman wrote at a time when North American teachers like Nicolas Pike (1788) recognized that preparing an arithmetic textbook for students in the schools of the recently-established United States of America could be a profitable venture. In the United States, many teachers had taught the subject without having access to published texts (Ellerton & Clements, 2012), the only texts available having been handwritten cyphering books—often prepared by the teachers themselves.

Ellerton and Clements hold a large collection of North American school mathematics textbooks dating from 1729 through 1950. In that collection there is at least one edition of most of the mathematics books listed by Karpinski (1980) as having been published in North America up to 1850. They also hold a major collection of 17th-, 18th-, and 19th-century school mathematics textbooks published in Great Britain. Those textbooks, combined with their set of 350 North American cyphering books and 100 British cyphering books, provided them with a solid primary data set which was readily available to them as they conducted the qualitative analyses reported in the following chapters.

The Six Issues

At the outset, we (Ellerton and Clements) identified the following six issues for detailed study. Although our analyses of individual cyphering books were not constrained by the need to take account of these issues, they were, nonetheless, never far away from our thinking.

1. For a particular cyphering book, what was the *intended* mathematics curriculum?
2. For a particular cyphering book, what was the *implemented* mathematics curriculum, and to what extent did that differ from the intended curriculum?
3. Was it possible to get glimpses of the *attained* curriculum merely by examining a cyphering book?
4. For a particular cyphering book, to what extent was there a sense of individual valuing of the mathematics, of ownership, of a “this-is-my-book,” mentality?
5. What were the contextual and ethnomathematical circumstances which surrounded the preparation of the cyphering book, and did those circumstances have an impact on the student who prepared it?
6. To what extent did the cyphering tradition in Great Britain differ from the cyphering tradition in North America?

Following the chapter-by-chapter analyses, these six issues will be explicitly addressed in the final chapter (Chapter 12) of this book.

A Window into the History of School Education

Within the academic mathematical community there has long been a strong sense of the need to pay due attention to the history of the development of mathematical ideas. However, among mathematics educators there does not seem to have been the same kind of determination to investigate, critically, the history of school mathematics. In recent years that deficiency within the international mathematics education community has begun to be addressed—for example, in September 2013 the first author (Ellerton) presented a paper at the Third International Conference on the History of Mathematics Education (3ICHME) held at Uppsala University, Sweden. About 40 people attended the Conference, but interestingly only two other scholars from the United States of America were present. The time is ripe for much greater attention to be paid to the history of school mathematics in all parts of the world, including Europe and North America. From that perspective, this book provides a window into the cyphering tradition which was so important in the early history of school mathematics in Western nations, including that history within the United States of America, Canada, and the United Kingdom.

In the chapters which follow, features of extraordinary cyphering books are identified and used to draw attention to central and, if you like, ordinary, features of the cyphering tradition. Each chapter will be well illustrated, showing pages from the cyphering books under consideration. The aim, throughout, has been to make readers aware of key features of the *abbaco* curriculum and, more generally, of the cyphering tradition, which controlled school mathematics in Western nations for many centuries. The use of the word “controlled” in the last sentence could raise academic eyebrows, but that is the word we feel is most appropriate.

By reproducing many pages and sections of pages from the 11 cyphering books, and by providing accompanying analyses, we hope to make readers fully aware of mathematical, ethnomathematical, pedagogical and individual factors which influenced school mathematics during the cyphering era. Although the related concepts of intended, implemented and attained curricula (Travers & Westbury, 1989; Westbury, 1980) provide the framework for analysis in this book, it is interesting to note that during the cyphering era the word “curriculum” was unknown.

The analyses conducted in the preparation of this book were informed by many primary sources from both North America and the United Kingdom. Six major issues were identified. Thus, for example, it is concluded that commercially-published textbooks played a much more important role in the British version of the cyphering tradition than in the North American version. However, within-nation differences were probably greater than between-nation differences. Thus, for example, in Chapter 6, which deals with Abraham Lincoln’s cyphering book, it is argued that the task of preparing a cyphering book in rural Indiana in the 1820s was an altogether different thing from preparing a cyphering book in an exclusive private secondary school in, say, Massachusetts.

Since the publication of Ellerton and Clements’s (2012) book on the cyphering tradition in North America at least three books which include analyses of cyphering books have appeared in the United Kingdom (Denniss, 2012; Stedall, 2012; Wardhaugh, 2012). Many other similar books are needed, written by scholars from a range of nations, which take account of the cyphering era in the history of school mathematics.

The scholarship which generated this book lies at the intersection of mathematics, mathematics education, and the history of education. It is important that books such as this one be read by scholars in all three of those domains. Mathematicians need to learn to distinguish between the history of mathematics, on the one hand, and the history of mathematics education, on the other, and to reflect on if and when such a distinction is helpful. Mathematics educators need to pay much more attention than they have to the history of their field of study. And, scholars with a general interest in the history

of school education will profit from becoming better acquainted with aspects of the teaching and learning of a subject which has long been a major component of the school curriculum. Too often, textbooks on the history of education have paid scant attention to the history of school mathematics, despite the fact that there can be no doubt that during the seventeenth, eighteenth and nineteenth centuries the cyphering tradition was, increasingly, a vitally important aspect of general school education, not only in Western European nations but also in North America.

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