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Editors

The Art of Science

From Perspective Drawing to Quantum Randomness

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ISBN 978-3-319-02110-2 ISBN 978-3-319-02111-9 (eBook)
DOI 10.1007/978-3-319-02111-9
Springer Cham Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014943908

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Every truth requires some pretence to make it live.

Joseph Conrad

Preface

The Renaissance is famous for its discovery of linear perspective, complex numbers, and probability. History has been quick to recognize the power of perspective that gave form to a "classic" style in painting, but has failed to acknowledge the true significance of complex numbers and probability. Both were treated with a great deal of suspicion by the scientific establishment and as a result were overlooked for many years. Linear perspective was already four centuries old, when quantum theory first showed how probability might be moulded from complex numbers and went on to create the notion of "complex probability amplitude". Yet, from a theoretical point of view, the space opened by linear perspective to painting and the space opened by complex numbers to science are equally important and share many characteristics. This book explores that shared field.

It may well seem challenging, or even inappropriate, to relate notions belonging to contemporary science with inventions and themes of the Renaissance. But we want to make it clear that we have no wish to antedate the findings of the contemporary science back into the Renaissance, nor to trace the history of science from the fifteenth century to the present. Instead, our purpose is to extend the "ideal" style Leonardo conceived for painting to science. In Leonardo's view, painting must recreate the geometry of nature through the harmony of form; "the mind of the painter must transmute itself into the very mind of nature and be the interpreter between it and art" (*Trattato*, I, 24v). Our ambition is to encourage the reader to see science as an "art" and art as a form of scientific knowledge.

As to the title, *The Art of Science* reverses *The Science of Art* by Martin Kemp as it envisages a "complementary" view. *The Science of Art* (1992) rests on the premise that there *were* special kinds of affinity between art and science from the Renaissance to the nineteenth century, and on the observation that "the affinities centred upon a belief that the direct study of nature through the faculty of vision was essential if the rules underlying the structure of the world were to be understood." Consequently, Kemp's book focuses on *optically minded* theory and practice of art. Its primary concern is to examine the extent to which artists' work and ideas were scientifically founded. *The Art of Science*, instead, rests on the premise that there *are* special kinds of affinity between art and science and sees the affinities emerging from a conception of art and science as "symbolic forms". Accordingly, the faculty of vision is essential as it turns "imagination" into visual and graspable forms. In Leonardo's words: "[The eye] triumphs over nature, in that the constituent parts of nature are finite, but the works which the eye commands of the hands are infinite, as is demonstrated by the painter in his rendering of numberless forms." To master the rules of Albertian perspective allows the painter not only to depict the world as it appears but also to *see and draw other possible worlds*. This is the main lesson that science gains from the Renaissance art.

The philosophical concerns underlying our project are sympathetic to attempts to revise the "picture theory of science" (*Bildtheorie*) and, in a broad sense, to a "structuralist" view of science. While we will not enter into the contemporary debate about the themes of structuralism, we want to pay tribute in retrospect to two leading figures: Ernst Cassirer and Hermann Weyl.

The heritage of Cassirer's *Philosophy of Symbolic Forms* (1923–1929) cannot be confined within the main stream of the neo-Kantian philosophy, *tout court*. His revision of the "transcendental" approach highlighted a common denominator among a variety of "forms" arising in remote disciplines and cultural areas. This shared term, which manifests a "symbolic" character, allows Cassirer to compare the extraordinary variety of products of human spirit (myth, language, art, science) and to understand all of them as symbolic constructions in the general frame of a "science of culture" (*Kulturwissenschaft*): "The fundamental concepts of each science, the instruments with which it propounds its questions and formulates its solutions, are regarded no longer as passive images of something given but as *symbols* created by the intellect itself." The search for a theory of artwork within a comprehensive *Kulturwissenschaft* and the attempt to deduce the meaning of symbols created by art from their iconographic content and style may bring to mind the iconological researches developed by Aby Warburg and his Circle (joined, among the others, by Fritz Saxl and Ernst H. Gombrich). The Warburg programme, however, was more historical than theoretical. Even when an interpretation was advanced—such as Erwin Panofsky's *Perspective as Symbolic Form* (1924–1925)—the paradigm was borrowed from a theory of knowledge external to the artistic work and style under consideration. Hence, the artistic representation was to provide evidence for a previously accepted theory. By contrast, our goal is to focus on the artistic "invention" at the beginning, not at the end, of a theoretical path that leads to the scientific representation. In this way, through the medium of mathematical thought, a "visual" form can be used as a model for scientific knowledge.

As Leonardo's pictorial style is related to the geometry of nature, so is Hilbert's mathematical "style" related to his vision of geometric forms. The "ideal" style Hilbert conceived for mathematical knowledge results in a "general theory of forms". In particular, if we look for evidence of our claim that "the faculty of vision is essential as it turns 'imagination' into visual and graspable forms", we should pay attention to his essays on "intuitive geometry". To fully appreciate the potentialities embedded in Hilbert's picture of mathematical theories and their impact on the development of physical concepts, we should look at Hermann Weyl's writings. While his refined works on mathematical physics—such as *The Theory of Groups* *and Quantum Mechanics* (1931)—disclose a “visual understanding” of science only to scientists, his *Philosophy of Mathematics and Natural Science* (1927–1949) enhances mutual understanding between humanities and science as it shows the symbolic form of their specific contents. Finally, his *Symmetry* (1952) is a model to follow for an “art guide” to science.

This book has a long story. A shared interest in conceptual and epistemological issues relevant to art and science prompted us to conceive a project on *Reality and Its Double. Perspective and Complex Numbers Between the Renaissance and Quantum Physics*, awarded by the *Istituto di Studi Avanzati* (ISA) of the University of Bologna in 2009. During the events connected with the project including a series of lectures and a closing conference on *The Art of Science*, we had the opportunity to discuss and clarify issues and consequently select the most relevant topics. This volume includes papers delivered both as lectures and as contributions to the conference plus some that were specially commissioned.

We are grateful to the *Istituto di Studi Avanzati* for its generous support of our project and to all the participants for their valuable contributions. In particular, we want to thank John Stillwell for his unwavering confidence in the idea of this book. We are also grateful to the reviewers for their comments on the manuscript. Finally, it is a pleasure to thank Giuseppe Longo and Wilfried Sieg for their gentle encouragement and David Deutsch for his valuable comments and suggestions.

Bologna, Italy
September 2013

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Annarita Angelini

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Part I
Ways of Perspective

What does artificial perspective tell us about scientific knowledge? How does it enhance "visual understanding" of mathematic forms? How does it refine the notion of "observability"? By addressing such questions from different points of view—mathematical, philosophical, historical—the essays collected in Part I lead to the depiction of linear perspective as an art of seeing, projecting, and measuring.

In the opening chapter, John Stillwell directs us to view *perspectiva pingendi* with a mathematician's eye. The discovery of the *costruzione legittima* for perspective drawing—namely, of a method for projecting the three-dimensional space on the pictorial surface and, more in general, of a "scientific", optical system allowing any *imaginary* scene to be represented as if it were *real*—led to interest in a new kind of geometry, *projective geometry*, in which points and lines are the main ingredients. Thanks to the possibility of creating perspective drawings without measurement, projective geometry freed itself completely from coordinates and became a system in which all theorems were derived by reasoning about points and lines alone. Seemingly, geometry and algebra had diverged completely. But a surprising development was on the horizon: when geometry is freed from numbers, addition, and multiplication, it becomes feasible to reconstruct algebra on a purely geometric foundation by means of the Pappus theorem, the Desargues theorem, and the little Desargues theorem. Even more surprisingly, these purely geometric theorems were found (by David Hilbert and Ruth Moufang) to control which kind of algebra is possible in two, four, and eight dimensions.

If, on the one hand, linear perspective encouraged mathematicians to see a new kind of geometry, on the other, the medieval interpretations of the Euclidean geometric optics encouraged Renaissance painters to play with its rules. After flying towards the eighth dimension, Nader El-Bizri takes us back to see "reality" in perspective. His essay (Chap. 2) contrasts dialectically the "art" of optics with the "science" of painting. The pictorial structure is intrinsically implied within the visual elements of the science of optics and geometry, while it simultaneously depends on these sciences for the projections and constructions needed to render spatial depth in artificial perspective. The "science of painting" is set against the principal classical theories of optics and geometry. Even though Renaissance authors were more often theoretically inclined to follow Euclid, Ptolemy, and Vitruvius, they nonetheless paid much attention to the transmitted traditions that were associated with the eleventh century Arab polymath al-Hasan Ibn al-Haytham (known as Alhazen). Adaptively mediated by medieval European opticians and mathematicians, they led ultimately to the transformation of "natural" perspective into "artificial" perspective; hence, to teach painting how to *imitate* "reality" bypassing the natural vision.

It was Filippo Brunelleschi's work, particularly in architecture, that inaugurated a most radical deviation from late medieval tradition. It endorsed, in the most striking way, the widespread humanistic intolerance of Scholastic scientific conception (Chap. 3). The internal organization of Brunelleschi's buildings showed entirely new optical unity of space, precisely defined architectural elements, emphasis on the visible manifestation of proportions and, what is most radical, lack (negation) of paintings and colours on walls. The white surface works as a "transcendental" light, providing a background for the primary elements of the buildings (columns, arches, architraves) made of darker stone (*pietra serena*). Dalibor Vesely explores the meaning of Brunelleschi's primary architectural elements, their relation to Alberti's *lineamentum*, and also to Zuccaro's and Mannerists' *disegno interno*. These relations appeared to be supported by a neo-Platonic metaphysics of light and its epistemological consequences. As a "universal formative power", *disegno interno* may be viewed as a general source of creativity underlying modern forms of knowledge and, consequently, modern European culture as a whole.

From a mathematical point of view, the method of 'ideal elements' demonstrates that universal formative power. Indeed, Hilbert traced the origin of the ideal elements to the points at infinity of plane geometry. Such ideal points where parallel lines meet on the projective plane originate as vanishing points on the pictorial plane. Before the independence of the parallel postulate was 'logically' proven, an ideal non-Euclidean geometry was 'visually' presented by perspective drawing. Therefore, taking art's imagination to its limits, mathematics has produced new forms of 'visual' geometry. Tristan Needham (Chap. 4) not only drives us to see visual differential geometry as an artwork, but also depicts its forms with a painter's hand. Beltrami's interpretation of the hyperbolic geometry comes to life with rare intensity in the figures accompanying the text.

Although artistic and scientific knowledge may seem to go hand in hand in the Renaissance, their relationship may appear controversial as much in modern and contemporary culture as in ancient thought. In Victor Stoichita's *Short History of Shadow* (1997), both the myth regarding the birth of artistic representation, in Pliny's *Natural History*, and the myth regarding the birth of cognitive representation, in Plato's cave, are traced to the motif of shadow. According to Pliny the Elder, painting originated from the idea of circumscribing shadows by lines (*omnes umbra hominis lineis circumducta*). It was a young woman in love who, when her lover was going abroad, "drew in outline on a wall the shadow of his face thrown by the lamp" (*Natural History*, XXXV, 35,151). For Plato, however, a shadow has a "negative" connotation turning "what is observable" into a dark spot, a "phantom" (*eídolon*). Seeing nothing but projected shadows, the prisoners in the cave took shadows for reality. Their "cognitive" representation may be compared with that of the painter whose art is directed to the imitation of appearances (*phantasma*) not of truths (*aletheia*): "the mimetic art is far removed from truth," observes Socrates in *The Republic*, "and this, it seems, is the reason why it can produce everything, because it touches or lays hold of only a small part of the object, and that a phantom" (*Rep*. 598b). Even the Eleatic Stranger, reporting Plato's thought in *The Sophist* (236c-e), distinguished a "fantastic art"—the art of producing appearances and presenting them as if they were real things (*tékne phantastiké*), i.e., painters' and Sophist's art—from a *less imperfect* "likeness-making art", the art of producing copies (*tékne eikastiké*). Thus painting was confined to the bottom of Plato's cave, while "science", as an imitation of the truth, aimed at producing copies of reality as it is.

Since Plato, "scientific knowledge" has not been concerned with the description of "shadows", and even less with the production of fantastic images. Its principles cannot be reconciled with the essentially plural and "sophistic" character of painting. Its images must convey a "veritable" and "realistic" *mimesis*. Indeed, it ascribes to them the same quality of "specular-reflection" that Socratic sapience ascribed to the self-knowledge of soul. A scientific representation is conceived as a mirror of reality and distinguished from deceptive appearances. Accordingly, a scientist plays the role of a neutral observer and assesses the degree of similarity between the "truth" of the observed reality and the "truth" of the scientific representation. Yet, a mirror image immediately evokes Narcissus' metamorphosis which, on reflection, is a consequence of a deception. Narcissus falls in love with his own specular-image, believing it to be the "shadow" of someone else. "That which you behold is but the shadow of a reflected form (*ista repercussae, quam cernis, imaginis umbra est*)" (*Metamorphoses*, 3, 436). The seduction of the *other* becomes a first step towards the recognition of one's own *self* reflected in the mirror. "I burn with love of my own self; I both kindle the flames and suffer them" (*ibid*., III, 464). Though scientific knowledge aims at mirroring reality, in its historical development, the awareness of the "action" of mirrors has been oscillating: from the maximum of illusion, according to which scientific representation is the faithful image of reality reflected in a mirror, to the maximum of narcissistic disenchantment (or enchantment), according to which scientific representation mirrors scientist's "vision", unveiling the logic underlying the construction of knowledge.

In the Albertian perspective, the artistic representation born from shadow is engaged with the "mirror-reflection" pursued through the scientific tradition. The art of painting then performs a dual magic: as a *shadow*, a simulacrum of a lack of sensibility, it is more "powerful" than a direct (sensory) vision depending on the body's "measurability" constraints; as a *reflection* of a point of view, namely, of a subjective criterion (*ratio*), it relates the "resemblance" between the original and the copy to the artist (Fig. 1). As Narcissus is at the same time subject and object of his desire, so the artist, or that "layman of wisdom" which dominates the Renaissance

Fig. 1 Giorgio Vasari: *The Studio of the Artist, c.* 1563. Florence, Casa Vasari

scene, is aware that "the object is now something other than the mere opposite the—so to speak—*ob-jectum* of the Ego. It is that towards which all the productive, all the genuinely creative forces of the Ego are directed".[1]

Simon Altmann (Chap. 5) sheds light on the action of mirrors in art and science. Indeed, since Narcissus was seduced by his mirror image and turned into a flower, humanity has been both fascinated and concerned by mirrors. Although art proceeded rather slowly from mirror symmetries (notable examples can be seen Greek pottery decorations) to more complex rotational patterns, it was not until the nineteenth century that the mathematics of rotations was understood. From specular and rotational patterns emerged the mathematics of *quaternions* and *spinors*, which eventually, influenced profoundly our knowledge of physics, especially quantum physics.

[1]E. Cassirer, *Individuum und Kosmos in der Philosophie der Renaissance* (1927). English translation: *The Individual and the Cosmos in Renaissance Philosophy*, Univ. of Pennsylvania Press, Philadelphia 1963, p. 143.

Chapter 1
From Perspective Drawing to the Eighth Dimension

John Stillwell

1.1 Problems of Perspective

The *Arnolfini Portrait* (Fig. 1.1), by Jan van Eyck (1434), is an acclaimed example of the new realism in Flemish painting in the early fifteenth century. It seems to be an accurate depiction of a three-dimensional space and of three-dimensional objects. However, van Eyck's treatment of perspective is not mathematically correct.

Take a closer look at the chandelier (Fig. 1.2).

If the six arms of the chandelier are identical, then the lines connecting corresponding points on the two arms must be parallel. We consider such points on the two leftmost arms. Figure 1.3 shows the lines connecting three pairs of corresponding points—one line through the tops of the candle holders, and lines through the first and second crockets.

Parallels can't look like this! They should either look parallel or else converge to a common point "at infinity."

There are fifteenth century artworks with far more blatant errors in perspective than Jan van Eyck's. Figure 1.4 shows one, from the unknown illustrator of a book by Savonarola (ca. 1497).

By trying to make parallels "look parallel" when they should meet at infinity, the artist has lost control of another set of parallels, which are not even straight! This error brings to light a key problem in perspective—drawing a tiled floor.

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R. Lupacchini and A. Angelini (eds.), *The Art of Science*,
DOI 10.1007/978-3-319-02111-9_1,

Fig. 1.1 Jan van Eyck: *The Arnolfini Portrait*, 1434. London, National Gallery

Fig. 1.2 The chandelier in the *Arnolfini Portrait*

1.1.1 Drawing a Tiled Floor in Perspective

One of the first artists to understand the mathematics involved in perspective drawing was Piero della Francesca, whose *Flagellation of Christ*, from around 1460 (Fig. 1.5), includes a meticulously drawn tiled floor.

Fig. 1.3 *Lines* through corresponding points on two arms

Fig. 1.4 Bartolomeo di Giovanni: Illustration from Savonarola's *Dell'Arte di Ben Morire*

Fig. 1.5 Piero della Francesca: *Flagellation*, *c.* 1460. Urbino, Galleria Nazionale delle Marche

Fig. 1.6 Piero's floor, with a diagonal added

Unlike the unknown illustrator above, Piero allows parallels in the floor to meet on the horizon, which enables him to get the diagonals right. Figure 1.6 shows a close-up of the floor in the picture, with a diagonal superimposed. Notice how the diagonal passes precisely through the corners of tiles. (The contrast has been heightened to show the tiles more clearly.)

In fact, *getting the diagonals right* is the whole secret of drawing a tiled floor in perspective. It is the basis of a method which may be called the *diagonal method*, first appearing in the book *De pictura* (*On painting*) of Leon Battista Alberti in 1436 (Fig. 1.7).