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Claudia Moreno González
José Edgar Madriz Aguilar
Luz Marina Reyes Barrera *Editors*

Accelerated Cosmic Expansion

Proceedings of the Fourth International Meeting on Gravitation and Cosmology

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Preface

This volume is a collection of papers representing the proceedings of the IV International Meeting on Gravitation and Cosmology, which took place on May 21-25, 2012, in the city of Guadalajara, Jalisco, México.

This meeting was sponsored by ICTP-Trieste (Italy) and COECyTJAL—Universidad de Guadalajara (México). The conference is part of a series of scientific meetings devoted to current and selected topics in gravitation and cosmology. These meetings began at the Universidad Central Marta Abreu de la Villas in Santa Clara, Cuba, in 2004.

The goal of this meeting was to attract the attention of leading experts in the fields of gravitation and cosmology. The subjects discussed provided both an update and an evaluation of the state of alternative theories of gravity, in connection with the issue of the accelerating expansion of the universe. Topics reviewed included f(R) theories, dark matter and dark energy issues, Modified Newtonian Dynamics (MOND) models, scalar tensor theories derived from non-Riemannian geometries, emergent universes and the cosmological constant. Attendees included younger and senior researchers as well as graduate students. All contributions in this volume have been refereed by the scientific committee.

Finally, we would like to thank to all those who helped us to make this meeting such a success. In particular, we would like to express our grateful thanks to Professor Israel Quiros for bringing the event to the Universidad de Guadalajara. Special thanks goes out as well to the authorities of the Universidad de Guadalajara for their support during the meeting.

Guadalajara, Jalisco, México
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Part I
Modified Theories of Gravity

Chapter 1
Recovering Flat Rotation Curves and Galactic Dynamics From f(R)-Gravity

Salvatore Capozziello

Abstract The so-called $f(R)$-gravity addresses issues like dark energy and dark matter from the point of view of gravitational field instead of requiring new material ingredients. In particular, several self-gravitating structures, like spiral and elliptical galaxies, can be consistently described, in the weak-field limit regime, due to the presence of Yukawa-like corrections in the Newtonian potential. We give here some examples related to the observations.

1.1 Introduction

Dark matter issues come from dynamical mass estimates of self-gravitating systems. In several astrophysical observations, there is more matter dynamically inferred than that can be accounted for from luminous components. This mass discrepancy is usually attributed to additional (missing or dark) matter, assuming the validity of Newton law of gravity at astrophysical scales. Oort was the first that posed the 'missing matter' problem [1]. By observing the Doppler red-shift values of stars moving near the plane of our Galaxy, he asserted that he could calculate how fast the stars were moving. He found that there had to be enough matter inside the galaxy such that the central gravitational force was strong enough to keep the stars from escaping, much as Sun's gravitational pull keeps a planet in its orbit. But when the calculation was made, it turned out that there was not enough mass in the Galaxy. The discrepancy was not small: the Galaxy had to be at least twice as massive as the sum of the mass of all its visible components combined. In addition, in the 1960's the radial profile of the tangential velocity of stars in their orbits around the Galactic Center, as a function of their distance from that center, was measured. It was found that typically, once we get away from the Galactic Center, all the stars travel with the same velocity independent of their distance out from the Galactic Center.

There were problems, too, at a larger scale. In 1933, Zwicky announced that when he measured the individual velocities of a large group of galaxies, known as the Coma cluster, he found that all of the galaxies that he considered were moving

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so rapidly relative to one another that the cluster should have come apart long ago. The visible mass of the galaxies making up the cluster was far too little to produce enough gravitational force to hold the cluster together. So not only our own galaxy was lacking mass, but also the whole Coma cluster of galaxies was suffering the same problem at a different scale [2]. Initially, the problem was only approached by leaving Newton's law inviolated and postulating the existence of some invisible dark entities to make up the missing mass. At the beginning, it has never came to mind anyone to go back and examine the basic assumption that only gravity was at work in these cases. It was easier to patch up the theory introducing invisible entities. Many names have been coined to define this invisible entity, a bit as in the days of ether.

There are the MAssive Compact Halo Objects (MACHOs), objects like black holes, and neutron stars that purportedly populate the outer reaches of galaxies like the Milky Way. Then there are the Weakly Interacting Massive Particles (WIMPs), which possess mass, yet do not interact with ordinary matter (baryons such as protons and neutrons) because they are composed by something unknown. Dark (missing) matter (DM) even comes in two flavors, hot (HDM) and cold (CDM). The CDM is supposedly to be in dead stars, planets, brown dwarfs ("failed stars") etc., while HDM is postulated to be fast moving in particles floating throughout the universe. It should be constituted by neutrinos, tachyons etc. But where is all of this missing matter? The truth is that after many years of looking for it, there is still no definitive proof that WIMPs exist, or that MACHOs will ever make up more than five percent of the total reserve of missing dark stuff. Besides, by adding a further ingredient, the cosmological constant Λ, such a model (now ΛCDM) has become the new cosmological paradigm usually called the *concordance model*. In fact, high quality data coming from the measurements of cluster properties as the mass, the correlation function and the evolution with redshift of their abundance, the Hubble diagram of Type Ia Supernovae, the optical surveys of large scale structure, the anisotropies of the cosmic microwave background, the cosmic shear measured from weak lensing surveys and the Lyman-α forest absorption are evidences toward a spatially flat universe with a subcritical matter content and undergoing a phase of accelerated expansion. Interpreting all this information in a self-consistent model is the main task of modern cosmology and ΛCDM model provides a good fit to the most part of the data giving a reliable picture of the today observed universe.

Nevertheless, it is affected by serious theoretical shortcomings that have motivated the search for alternative candidates generically referred to as *dark energy* or *quintessence*. Such models range from scalar fields rolling down self interaction potentials to phantom fields, from phenomenological unified models of dark energy and dark matter to alternative gravity theories.

Essentially, dark energy (or any alternative component) has to act as a negative pressure fluid which gives rise to an overall acceleration of the Hubble fluid. Despite of the clear mechanisms generating the observed cosmological dynamics, the nature and the fundamental properties of dark energy remain essentially unknown notwithstanding the great theoretical efforts made up to now.

The situation for dark matter is similar: its clustering and distribution properties are fairly well known at every scale but its nature is unknown, up to now, at fundamental level.

On the other hand, the need of unknown components as dark energy and dark matter could be considered nothing else but as a signal of the breakdown of Einstein General Relativity (GR) at astrophysical (galactic and extragalactic) and cosmological scales. In this context, Extended Theories of Gravity (ETGs) could be, in principle, an interesting alternative to explain cosmic acceleration and large scale structure without any missing components. In their simplest version, the Ricci curvature scalar R, linear in the Hilbert-Einstein action, could be replaced by a generic function $f(R)$ whose true form could be "reconstructed" by the data. In fact, there is no a priori reason to consider the gravitational Lagrangian linear in the Ricci scalar while observations and experiments could contribute to define and constrain the "true" theory of gravity [3]. Coming to the weak-field limit, which essentially means considering Solar System scales, any alternative relativistic theory of gravity is expected to reproduce GR which, in any case, is firmly tested only in this limit and at these scales [4]. Even this limit is a matter of debate since several relativistic theories do not reproduce exactly the Einstein results in their Newtonian limit but, in some sense, generalize them.

In general, all these efforts can be included in the so called Modified Newtonian Dynamics (MOND), first proposed by Milgrom [5], in the attempt to explain missing matter without dark matter but assuming a change into dynamics at scales larger than Solar System's ones. In general, any relativistic theory of gravitation yields corrections to the weak-field gravitational potentials which, at the post-Newtonian (PN) level and in the Parametrized Post-Newtonian (PPN) formalism, could constitute a test of these theories [4]. This point deserves a deep discussion. Beside the fundamental physics motivations coming from Quantum Gravity and unification theories [3], ETGs pose the problem that there are further gravitational degrees of freedom (related to higher order terms, non-minimal couplings and scalar fields in the field equations) and gravitational interaction is *not* invariant at any scale. This means that, besides the Schwarzschild radius, other characteristic gravitational scales come out from dynamics. Such scales, in the weak field approximation, should be responsible of characteristic lengths of astrophysical structures that should result *confined* in this way.

Here, without claim of completeness, we will try to address the problem to describe some self-gravitating structures without dark matter asking for corrections to the Newtonian potential that could fit data and reproduce dynamics. In this sense, the MOND approach can be reproduced not in a phenomenological way but starting from a field theory [6]. It is possible to show that *any analytic ETG, except GR, presents Yukawa-like corrections in the weak-field limit*. From an astrophysical point of view, these corrections means that further scales have to be taken into account and that their effects could be irrelevant at local scales as Solar System. The emergence of Yukawa-like corrections to the Newtonian potential is discussed in Sect. 1.2 where the weak-field limit of $f(R)$-gravity, the simplest ETG, is worked out. Here, $f(R)$ is a generic function of the Ricci curvature scalar R. A large class of $f(R)$

models are compatible with results of Solar System experiments and constraints coming from Equivalence Principle, as shown in [7]. These results are extremely important because the philosophy is that ETGs are not *alternative* to GR but are just an extension of it that could take into account the whole dynamics of gravitational field. In fact, as soon as $f(R) = R$, GR is fully recovered. In Sect. 1.3, we use the modified gravitational potential to reproduce the rotation curves of spiral galaxies by using simulated data and confronting with the Navarro-Frenk-White model for the halo. Furthermore, we extend the test to elliptical galaxies considering long-slit and planetary nebulae data. It is found that the Yukawa-like potential is able to fit nicely these elliptical galaxies and the anisotropy distribution is consistent with that estimated if a dark halo is considered. The parameter which measures the "strength" of the Yukawa-like correction is, on average, smaller than the one found for spiral galaxies and correlates both with the scale length of the Yukawa-like term and the orbital anisotropy. Conclusions are drawn in Sect. 1.4.

1.2 Yukawa-Like Corrections from the Weak-Field Limit of f(R)-Gravity

The simplest ETG in four dimensions is given by the action

$$S = \int d^4x\sqrt{-g}\left[f(R) + \mathcal{L}^{(m)}\right], \tag{1.1}$$

where $f(R)$ is an unspecified function of scalar curvature R. The term $\mathcal{L}^{(m)}$ is the minimally coupled ordinary matter contribution, considered as a *perfect fluid*. This is the straightforward extension of GR as soon as it is assumed $f(R) \neq R$. Varying with respect to $g_{\mu\nu}$, we get

$$\begin{aligned} G_{\alpha\beta} &= \frac{1}{f'(R)}\left\{\frac{1}{2}g_{\alpha\beta}\left[f(R) - Rf'(R)\right] + f'(R)_{;\alpha\beta} - g_{\alpha\beta}\Box f'(R)\right\} + \frac{T^{(m)}_{\alpha\beta}}{f'(R)} \\ &= T^{(curv)}_{\alpha\beta} + \frac{T^{(m)}_{\alpha\beta}}{f'(R)}, \end{aligned} \tag{1.2}$$

where $T^{(curv)}_{\alpha\beta}$ is an effective stress-energy tensor constructed by the extra curvature terms. We are considering physical units for the gravitational coupling. In the case of GR, $T^{(curv)}_{\alpha\beta}$ identically vanishes and standard, minimal coupling is recovered for matter contribution. In order to deal with standard self-gravitating systems, any theory of gravity has to be developed to its Newtonian or post-Newtonian limit depending on the order of approximation of the theory in terms of power of c^{-2}, where c is the speed of light [8].

In principle, the following analysis can be developed for any ETGs but we will consider only the $f(R)$ case. As discussed in [3], we can deal with the Newtonian and the post-Newtonian limit of $f(R)$ gravity adopting the spherical symmetry.

The solution of field equations can be obtained considering the general spherically symmetric metric:

$$ds^2 = g_{\sigma\tau}dx^\sigma dx^\tau = g_{00}(x^0,r)dx^{0^2} - g_{rr}(x^0,r)dr^2 - r^2 d\Omega \tag{1.3}$$

where $x^0 = ct$ and $d\Omega$ is the solid angle. In order to develop the Newtonian limit, let us consider the perturbed metric with respect to a Minkowskian background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. The metric entries can be developed as:

$$\begin{cases} g_{tt}(t,r) \simeq 1 + g_{tt}^{(2)}(t,r) + g_{tt}^{(4)}(t,r), \\ g_{rr}(t,r) \simeq -1 + g_{rr}^{(2)}(t,r), \\ g_{\theta\theta}(t,r) = -r^2, \\ g_{\phi\phi}(t,r) = -r^2 \sin^2\theta, \end{cases} \tag{1.4}$$

where we are assuming $c = 1$, $x^0 = ct \to t$ and expansion is at order c^{-2} and c^{-4}. Since we want to obtain the most general result, we does not provide any specific form for $f(R)$-Lagrangian. We assume, however, analytic Taylor expandable $f(R)$ functions with respect to a certain value $R = R_0$:

$$f(R) = \sum_n \frac{f^n(R_0)}{n!}(R - R_0)^n \simeq f_0 + f_1 R + f_2 R^2 + f_3 R^3 + \cdots . \tag{1.5}$$

In order to obtain the weak field approximation, one has to insert expansions (1.4) and (1.5) into field Eqs. (1.2) and expand the system up to the orders $\mathcal{O}(0)$, $\mathcal{O}(2)$ e $\mathcal{O}(4)$ (that is, as stated above at order c^{-2} and c^{-4}). This approach provides general results and specific (analytic) theories are selected by the coefficients f_i in Eq. (1.5). It is worth noticing that, at the order $\mathcal{O}(0)$, the field equations give the condition $f_0 = 0$ and then the solutions at further orders do not depend on this parameter. If we consider the $\mathcal{O}(2)$-order approximation, the field equations in vacuum, results to be

$$\begin{cases} f_1 r R^{(2)} - 2 f_1 g_{tt,r}^{(2)} + 8 f_2 R_{,r}^{(2)} - f_1 r g_{tt,rr}^{(2)} + 4 f_2 r R^{(2)} = 0, \\ f_1 r R^{(2)} - 2 f_1 g_{rr,r}^{(2)} + 8 f_2 R_{,r}^{(2)} - f_1 r g_{tt,rr}^{(2)} = 0, \\ 2 f_1 g_{rr}^{(2)} - r\left[f_1 r R^{(2)} - f_1 g_{tt,r}^{(2)} - f_1 g_{rr,r}^{(2)} + 4 f_2 R_{,r}^{(2)} + 4 f_2 r R_{,rr}^{(2)}\right] = 0, \\ f_1 r R^{(2)} + 6 f_2 \left[2 R_{,r}^{(2)} + r R_{,rr}^{(2)}\right] = 0, \\ 2 g_{rr}^{(2)} + r\left[2 g_{tt,r}^{(2)} - r R^{(2)} + 2 g_{rr,r}^{(2)} + r g_{tt,rr}^{(2)}\right] = 0. \end{cases} \tag{1.6}$$

It is evident that the trace equation (the fourth in the system (1.6)), provides a differential equation with respect to the Ricci scalar which allows to solve exactly the system (1.6) at $\mathcal{O}(2)$-order. Finally, one gets the general solution:

$$\begin{cases} g_{tt}^{(2)} = \delta_0 - \frac{Y}{f_1 r} - \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{3\xi r} + \frac{\delta_2(t)e^{r\sqrt{-\xi}}}{6(-\xi)^{3/2}r} \\ g_{rr}^{(2)} = -\frac{Y}{f_1 r} + \frac{\delta_1(t)[r\sqrt{-\xi}+1]e^{-r\sqrt{-\xi}}}{3\xi r} - \frac{\delta_2(t)[\xi r+\sqrt{-\xi}]e^{r\sqrt{-\xi}}}{6\xi^2 r} \\ R^{(2)} = \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{r} - \frac{\delta_2(t)\sqrt{-\xi}e^{r\sqrt{-\xi}}}{2\xi r} \end{cases}$$

where $\xi \doteq \frac{f_1}{6f_2}$, and Y is an arbitrary integration constant. When we consider the limit $f(R) \to R$, in the case of a point-like source of mass M, we recover the standard Schwarzschild solution. Let us notice that the integration constant δ_0 is dimensionless, while the two arbitrary functions of time $\delta_1(t)$ and $\delta_2(t)$ have respectively the dimensions of $length^{-1}$ and $length^{-2}$. The functions of time $\delta_i(t)$ $(i = 1, 2)$ are completely arbitrary since the differential equation system (1.6) contains only spatial derivatives and can be fixed to constant values. Besides, the integration constant δ_0 can be set to zero since it represents an unessential additive quantity for the potential. It is possible to write the general solution of the problem considering the previous expression (1.3). In order to match at infinity the Minkowskian prescription for the metric, one can discard the Yukawa growing mode and then we have:

$$\begin{cases} ds^2 = \left[1 - \frac{2GM}{f_1 r} - \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{3\xi r}\right] dt^2 \\ -\left[1 + \frac{2GM}{f_1 r} - \frac{\delta_1(t)(r\sqrt{-\xi}+1)e^{-r\sqrt{-\xi}}}{3\xi r}\right] dr^2 - r^2 d\Omega \\ R = \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{r} \end{cases} \tag{1.7}$$

At this point, one can provide the gravitational potential. The first of (1.7) gives the second order solution in term of the metric expansion (see the definition (1.4)). This term coincides with the gravitational potential at the Newton order. In particular, since $g_{tt} = 1 + 2\Phi_{grav} = 1 + g_{tt}^{(2)}$, the gravitational potential of $f(R)$-gravity, analytic in the Ricci scalar R, is

$$\Phi_{grav} = -\left[\frac{GM}{f_1 r} + \frac{\delta_1(t)e^{-r\sqrt{-\xi}}}{6\xi r}\right]. \tag{1.8}$$

This general result means that the standard Newton potential is achieved only in the particular case $f(R) = R$ while it is not so for any analytic $f(R)$ models. Equation (1.8) deserves some comments. The parameters $f_{1,2}$ and the function δ_1 represent the deviations with respect the standard Newton potential. To test these theories of gravity inside the Solar System, we need to compare such quantities with respect to the current experiments, or, in other words, Solar System constraints should be evaded fixing such parameters. On the other hand, these parameters could acquire non-trivial values (e.g. $f_1 \neq 1$, $\delta_1(t) \neq 0$, $\xi \neq 1$) at scales different from the Solar System ones. We note that the ξ parameter can be related to an effective mass being

$m^2 = (3\xi)^{-1}$ and can be interpreted also as an effective length L. Equation (1.8) can be recast as

$$\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{L}}\right), \tag{1.9}$$

where the first term is the Newtonian-like part of the potential for a point-like mass $\frac{M}{1+\delta}$ and the second term is a modification of the gravity including a scale length L associated to the above coefficients of the Taylor expansion. If $\delta = 0$ the Newtonian potential and the standard gravitational coupling are recovered. Comparing Eqs. (1.8) and (1.9), we obtain that $1 + \delta = f_1$, and δ is related to $\delta_1(t)$ through

$$\delta_1 = -\frac{6GM}{L^2}\left(\frac{\delta}{1+\delta}\right) \tag{1.10}$$

where $\frac{6GM}{L^2}$ and δ_1 can be assumed quasi-constant. Under these assumptions, the scale length L could naturally reproduce several phenomena that range from Solar System to cosmological scales. In the following section, we show examples of self-gravitating systems where this potential can be used to address the missing matter issues. Understanding on which scales the modifications to GR are working or what is the weight of corrections to the Newtonian gravitational potential is a crucial point that could confirm or rule out these alternative approaches.

1.3 The Cases of Spiral and Elliptical Galaxies

As we said, ETGs can impact on the estimate of missing matter properties at galactic scales. In fact, corrections on the gravitational potential give rise to modifications on the rotation curve [9]. The amount of such modifications can be compared with Newtonian mechanics and then constrains the halo model parameters by fitting the theoretical rotation curve to the observed one. In other words, one can see how modified gravity could be a possible way to solve the cusp/core and similar problems related to the presence of dark matter.

Here we take into account spiral and elliptical galaxies but also larger scales, as galaxy clusters, can be considered within this approach [10]. Let us assume a gravitational potential as Eq. (1.9). It is worth noticing that a Yukawa - like correction has been used several times in the past showing that the observed flat rotation curves of spiral galaxies may be well fitted by this model with no need for dark haloes provided $\delta < 0$ and L adjusted on a case-by-case basis [11]. Equation (1.9) is the starting point for the computation of the rotation curve of an extended system.

One has to remember that, in the Newtonian gravity, the circular velocity in the equatorial plane is given by $v_c^2(R) = R d\Phi/dR|_{z=0}$, with Φ the total gravitational potential. Thanks to the superposition principle and the linearity of the point mass potential on the mass, this latter is computed by adding the contribution from infinitesimally small mass elements and then transforming the sum into an integral over the mass distribution. For a spherically symmetric source, one can simplify this procedure considering the Gauss Theorem to find out that the usual result $v_c(r) = GM(r)/r$ with $M(r)$ the total mass within r. However, because of the Yukawa - like correction, the Gauss Theorem does not apply anymore and hence one must generalize the derivation of the gravitational potential. Alternatively, one can remember that $v_c(R) = RF(R, z = 0)$ being F the total gravitational force and R a radial coordinate [12]. This is the starting point where a general expression can be derived for the case of a generic potential giving rise to a separable force, *i.e.*:

$$F_p(\mu, r) = \frac{GM_\odot}{r_S^2} f_\mu(\mu) f_r(\eta), \tag{1.11}$$

with $\mu = M/M_\odot$, $\eta = r/r_S$ and $(M_\odot, r_s)$ the Solar mass and a characteristic length of the problem. In our case, it is $f_\mu = 1$ and:

$$f_r(\eta) = \left(1 + \frac{\eta}{\eta_L}\right) \frac{\exp\left(-\eta/\eta_L\right)}{(1+\delta)\eta^2}\,, \tag{1.12}$$

with $\eta_L = L/r_S$. Using cylindrical coordinates (R, θ, z) and the corresponding dimensionless variables (η, θ, ζ) (with $\zeta = z/r_S$), the total force then reads:

$$F(\mathbf{r}) = \frac{G\rho_0 r_S}{1+\delta} \int_0^\infty \eta' d\eta' \int_{-\infty}^\infty d\zeta' \int_0^\pi f_r(\Delta)\tilde{\rho}(\eta', \theta', \zeta') d\theta', \tag{1.13}$$

with $\tilde{\rho} = \rho/\rho_0$, ρ_0 a reference density, and we can define

$$\Delta = \left[\eta^2 + \eta'^2 - 2\eta\eta' \cos(\theta - \theta') + (\zeta - \zeta)'^2\right]^{1/2}. \tag{1.14}$$

For obtaining axisymmetric systems, one can set $\tilde{\rho} = \tilde{\rho}(\eta, \zeta)$. Furthermore, the systems we are considering here are spiral galaxies which will be modeled as the sum of an infinitesimally thin disc and a spherical halo, and then the scaling radius r_s will be the disc scale length R_d. Under these assumptions, the rotation curve may then be obtained as:

$$v_c^2(R) = \frac{G\rho_0 R_d^2 \eta}{1+\delta} \int_0^\infty \eta' d\eta' \int_{-\infty}^\infty \tilde{\rho}(\eta', \zeta') d\zeta' \int_0^\pi f_r(\Delta_0) d\theta', \tag{1.15}$$

with

$$\Delta_0 = \Delta(\theta = \zeta = 0) = \left[\eta^2 + \eta'^2 - 2\eta\eta' \cos\theta' + \zeta'^2\right]^{1/2}. \tag{1.16}$$

Inserting Eq. (1.12) into Eq. (1.15) gives rise to an integral which has to be evaluated numerically even for the spherically symmetric case. It is evident that the total rotation curve may be split in the sum of the standard Newtonian term and a corrective one disappearing for $L \to \infty$, *i.e.* when ETGs have no deviations from GR at galactic scales. Assuming that a spiral galaxy can be modeled as the sum of an infinitesimally thin disc and a spherical halo and denoting with **p** the halo model parameters, it is possible to calculate the total rotation curve as:

$$v_c^{2(R,M_d,}\mathbf{p}_i) = v_{dN}^{2(R,M_d)} + v_{hN}^2(R,\mathbf{p}_i) + v_{dY}^{2(R,M_d)} + v_{hY}^2(R,\mathbf{p}_i),$$

where M_d is the disc mass, the labels d and h denote disc and halo related quantities, while N and Y refer to the Newtonian and Yukawa - like contributions [9]. This means that one may model a spiral galaxy as the sum of a thick disc and a spherical halo without dark matter contribution. Therefore to simulate the rotation curves, one can refer

- to spiral galaxies with reasonable values of the model parameters and
- the sampling and the noise should be the same as actual data.

In Fig. 1.1 it is shown an example of simulated curves with superimposed theoretical curves.

It is generated the disc scalelength R_d and the halo virial mass from flat distributions over the ranges:

$$0.5 \leq R_d/R_{d,MW} \leq 2.0 \quad 11.5 \leq \log M_{vir} \leq 13.5$$

with $R_{d,MW} = 2.55$ kpc, the disc scalelength for the Milky Way [13]. In order to set the disc mass, we first define the halo mass fraction (DM) within the optical radius as:

$$f_{DM} = \frac{M_h(R_{opt})}{M_d + M_h(R_{opt})}, \tag{1.17}$$

with $R_{opt} = 3.2R_d$ the optical radius. The disc mass within R_d can be approximated with the total disc mass. One can generate f_{DM} from a flat distribution in the range $(0.9, 1.1) f_{DM,fid}$ and $f_{DM,fid} = 50\%$ (see, *e.g.*, [14] and refs. therein). The halo scale-length R_s is computed as $R_s = R_{vir}/c$ where the concentration c is randomly generated from a Gaussian centered on:

$$c = 16.7 \left(\frac{M_{vir}}{10^{11}\ h^{-1}\ \mathrm{M}_\odot} \right)^{-0.125}, \tag{1.18}$$

and variance set to $10\,\%$ of the mean value. Note that the above relation has been derived by [15] for the mass range $(0.03, 30) \times 10^{12}\mathrm{M}_\odot$ following the method detailed in [16] and updating the cosmological model. Finally, it is necessary to set the modified potential parameters (δ, L) and so one first can set $\delta = 1/3$ and run different simulations randomly generating $\log \eta_L = \log(\lambda/R_d)$ from a flat distribution covering the wide range $(-2, 2)$. In order to explore the impact of δ, one can also consider the extremal case $\delta = 1.0$ thus maximizing the contribution of the Yukawa term. The sample of simulated rotation curves is the starting point of the analysis. Indeed, one can fit each curve with a given (Newtonian) model and compare the output best fit parameters with the input ones [9]. In realistic situations, one has a set of (R, v_c) data

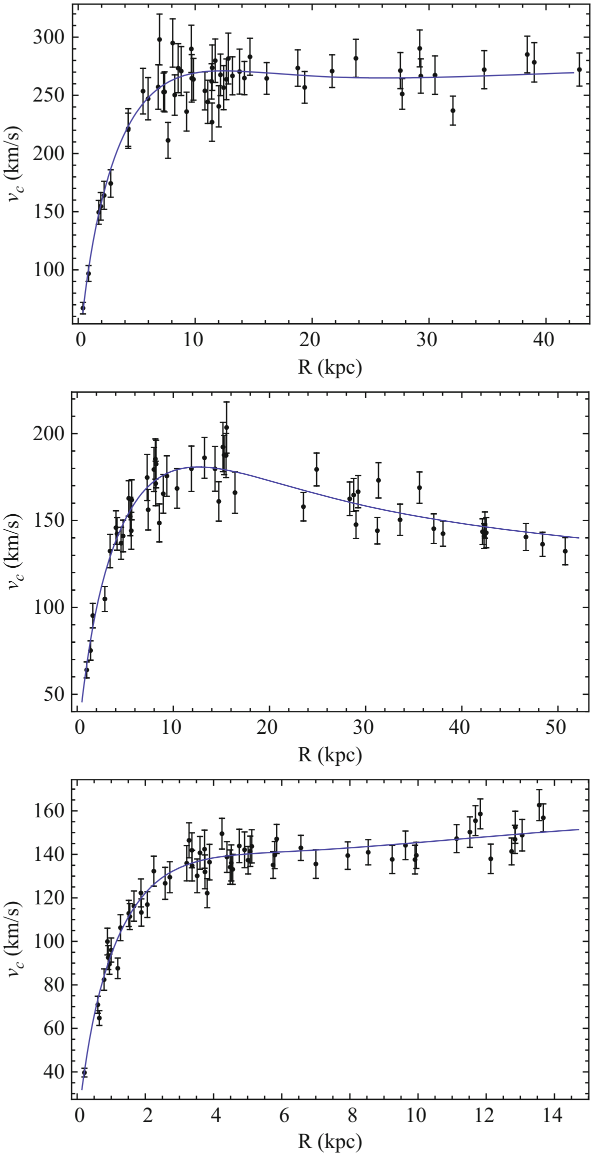

Fig. 1.1 Examples of simulated rotation curves with superimposed theoretical curves. From left to right, model parameters are $(\log M_d, \log M_{vir}, c, f_{DM}, \log \eta_L) = (11.15, 12.90, 10.24, 0.47, 0.36)$, $(10.90, 11.76, 14.77, 0.45, -0.92)$, $(10.04, 12.10, 13.76, 0.54, 1.11)$, while the simulation parameters are set as discussed in the text. Note that, depending on how the model parameters are set, it is possible to get rotation curves which are flat, decreasing or increasing in the outer region [9]

and a measurement of the galaxy surface brightness in a given band. It is then common to set the disc scale-length R_d to the value inferred from photometry, while the disc mass can be inferred from the total luminosity, provided an estimate of the stellar $M/\mathcal{L}$ ratio is somewhat available (e.g. from stellar population synthesis models fitted to the galaxy colors). As a first step, it is assumed that both (R_d, M_d) are known and fit the simulated data for each rotation curve to determine the Navarro-Frenk-White