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Per Olsson

Transport Phenomena in Newtonian Fluids - A Concise Primer



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Per Olsson
Göteborg
Sweden

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*The original version of the book was revised.
For detailed information please see erratum.
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Introduction

This book describes transport phenomena in Newtonian fluids such as momentum transport, energy transport and mass transport. The book contains detailed derivations of the transport equations for these transport phenomena. The book also contains analytical solutions to the transport equations in some simple geometries.

[Chapter 1](#) is a description of the basic mathematics used in the book. The chapter is not intended to be a textbook of mathematics, but contains only such information which is necessary for the reader to be able to read and understand the book's other content.

[Chapter 2](#), which deals with momentum transport, contains a derivation of the Navier-Stokes-Duhem equation describing flow in a Newtonian fluid. [Chapter 2](#) also contains the derivations of the Bernoulli equation, the pressure equation and the wave equation for sound waves. Further, the chapter contains analytical solutions to the flow equation in some simple geometries. The chapter also describes the boundary layer, turbulent flow and flow separation.

[Chapter 3](#), which deals with energy transport, contains a derivation of the heat transport equation describing heat transport in a flowing Newtonian fluid. Heat transport in a flowing fluid is caused by thermal conduction and convection. The chapter also contains a definition of the heat transfer coefficient and analytical solutions for the heat transfer coefficient in some simple geometries. [Chapter 2](#) contains the solutions to the Navier-Stokes equation in these geometries.

[Chapter 4](#), which deals with mass transport, contains a derivation of the mass transport equation describing mass transport in a flowing Newtonian fluid. Mass transport in a flowing fluid is caused by diffusion and convection. The chapter also contains a definition of the mass transfer coefficient and analytical solutions for the mass transfer coefficient in some simple geometries. [Chapter 2](#) contains the solutions to the Navier-Stokes equation in these geometries.

Chapter 1

Elementary Mathematics

1.1 Introduction

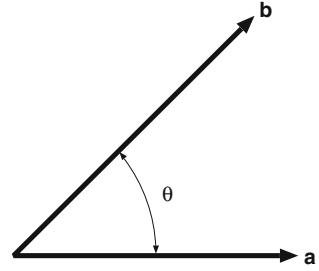
This chapter is intended for readers who are not familiar with the vector and tensor notation appearing in the book. The transport equations become much more compact if they are written with vector or tensor notation. This is especially true when the flow equation is written with tensor notation. The chapter is not intended to be a textbook of mathematics, but contains only such information which is necessary for the reader to be able to read and understand the book's other content. There is something improper to speak about vector and tensor notation. It is more proper to speak about *symbolic* and *indicial* notation but in this book *symbolic notation* will be called *vector notation* and *indicial notation* will be called *tensor notation*. Section. 1.3 contains only very basic information about tensors. The most important in Sect. 1.3 is the *Einstein summation convention* and the way to write partial derivatives with respect to the space coordinates with tensor notation. All quantities which are written with tensor notation in this book are Cartesian tensors. Equations written in other coordinate systems are not written with tensor notation. The chapter also contains descriptions of line integrals, surface integrals, volume integrals and some mathematical theorems such as the Stokes theorem and the Gauss theorem.

1.2 Vector Notation

An example of a vector is the the space vector \mathbf{x} in a Cartesian coordinate system. Vectors are denoted by bold straight style in this book. The space vector \mathbf{x} can be written

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (1.1)$$

An erratum to this chapter is available at https://doi.org/10.1007/978-3-319-01309-1_5.

Fig. 1.1 The vectors **a** and **b**

where x , y and z are space coordinates in a Cartesian coordinate system.

1.2.1 Scalar Product

The scalar product between the vectors **a** and **b** is denoted $\mathbf{a} \cdot \mathbf{b}$ and is defined (see Fig. 1.1)

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= \mathbf{b} \cdot \mathbf{a} = [a_1 \ a_2 \ a_3] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \\ &= a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta \end{aligned} \quad (1.2)$$

1.2.2 Cross Product

The cross product between the vectors **a** and **b** is denoted $\mathbf{a} \times \mathbf{b}$ and is defined (see Fig. 1.2)

Fig. 1.2 The vectors **a**, **b** and $\mathbf{a} \times \mathbf{b}$ 