Kurt Fischer

Relativity for Everyone

How Space-Time Bends

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To Yukiko:
You edited the text to become readable.

Preface

Dear Reader,

I encountered the theory of relativity at first while in my teens, in the public library of my hometown. There were two kinds of books: The easy ones did not really explain, but displayed large, colored pictures of the science-fiction kind. The serious books seem to explain something, but that was hidden in a mass of mathematical symbols, and *those* they did not explain, so I was back to square one. Nevertheless, they aroused my interest in physics. It also incited me to fill the gap. The result is this book.

This book is about light, energy, mass, space, time, and gravity: I explain to you how the theory of special relativity and the theory of general relativity work out.

We will use many **thought experiments**, and show you how physicists create and solve models. This method is the one used by Einstein himself: He understood the theory in *physical* and *geometrical* pictures. We will follow Einstein's original train of thought as closely as possible, using even some of his own thought experiments.

I will present some involved arguments, so you need your imagination, but no complicated mathematics to understand the essence of it all. However, the beauty of physics is, that we *can* calculate how the numbers work out. Therefore I prepared the equations of the theory of relativity together with the most important *exact* solutions, using only elementary mathematics. Even the Einstein equation of gravity, showing the bending of space and time, we present *in detail*, in common language. We will see why it is the *simplest theory* of gravity. We will not be content with analogies like "space bends like the surface of a sphere!"

In the first four chapters, I explain what is called the theory of special relativity: I describe the relation between light, matter, space, and time.

1. In Chap. 1, I describe after introducing the basics, that mass and energy are the opposite site of the same coin.
2. In Chap. 2, we will see why time and length are "relative".
3. In Chap. 3, we will see why any electric wire shows relativity in everyday life.
4. In Chap. 4, we learn that while riding a merry-go-round, school-geometry ceases to be true.

In the Chaps. 5 to 9 I describe gravity, that is the theory of general relativity.

5. In Chap. 5, I show you that earth's gravity does not pull at you at all, and that it bends space *and* time.
6. In Chap. 6, we will see in detail which effects the bending space and time are causing.
7. In Chap. 7, we explain thoroughly the meaning of the famous Einstein equation of gravity, and why it is the simplest possible way of describing gravity.
8. In Chap. 8, we introduce the most famous *exact* solution of the Einstein equation, that is the Schwarzschild solution, in simple terms.
9. In Chap. 9, we use the solutions to the Einstein equation, and explore the famous predictions of the theory of general relativity, as for example how much a light beam bends while passing at the sun, how large and heavy black holes are, why the orbits of the planets around the sun turn slowly around the sun, and why the universe had a big bang, but why its future is unclear to us.

One word about highlighted text: Indexed words appear in **boldface**. Such you can easily find them on their page, searching from the index. Before going on with describing the strange properties of light, we better tell in what units we measure and compare the size of things.

Units and Symbols

In physics, we use certain **units** to measure things. Lengths we measure only in meters, time only in seconds, *not* in minutes, hours, or days. Mass we measure only in kilograms. Any *other* unit we use in this book is a combination of these units. For example, speed we measure in meters per second. Other units like pounds or inches or such like we never use. The merit of this is that we can *leave the units away* in all calculations, because we know anyway what units to add *afterwards*, just because we *fixed* them at the beginning.

We will encounter often really large or very small numbers. For example, while numbers like "one thousand" we can write down as 1000, the number of one billion two hundred fifty-two million seven hundred eighty-four 1,250,000,784 is much harder to read. Mostly we are only interested in the first three or so digits, for a rough estimate of how large things are. Here physicists count the number of digits after the first one, that is nine in this case, and write

$$1.25 \times 10^9$$

In the same way, a very small number like 0.000145 we write as 1.45×10^{-4} by counting the number of leading zeros. Then we can easily multiply such numbers: We multiply $1.25 \times 10^9 \times 1.45 \times 10^{-4}$ by multiplying at first 1.25 and 1.45 which is roughly 1.81, and adding the exponents $9 - 4 = 5$, so that the result is $\approx 1.81 \times 10^5$. Here the symbol $\approx$ means "**is roughly equal to**".

For example, we use for the speed of light mostly the rough value

$$\text{speed of light} \approx 3.00 \times 10^8 \text{ meters per second}$$

We will encounter other important numbers of nature in the text. We collected them in Table A.1 for reference.

Special Expressions

At last, one word about how we use phrases like "far enough away from something", "fast enough", and the like. For example, we say

> "if the astronaut is far enough away from earth, the astronaut can neglect earth's gravity."

We are aware of the fact, that gravity is *never* exactly zero, even *very* far away from earth. The point is here, that the astronaut wants to measure some effect, with gravity spoiling this effect, to, say, one percent of the result that the astronaut would get in *really* empty space. So if the astronaut finds that gravity is still spoiling his experiment too much, he is *free to move* to a place which is *so far away* from earth that there, gravity really *does* spoil his experiment only up to the desired one percent at the most. Of course, if he wants to measure even more accurately, he must move even farther away from earth. That is why we say, in short, that if he moves "far enough away" from earth, he always can neglect earth's gravity *to the extent that he wants to* neglect it.

Tokuyama, Japan Kurt Fischer

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Chapter 1
Light, Matter, and Energy

1.1 Light Beams

An astronaut floating in space switches on a torch or a laser beamer. The emerging light beam travels at the speed

$$299{,}792{,}458 \text{ meters per second} \quad (1.1)$$

This is the **speed of light in vacuum**. *Exactly*.

How can we verify this? First of all, we need an apparatus to send a light beam, such as the box on the left in Fig. 1.1. It is open to the right. This box symbolizes the torch, or the laser, or the *sender* for short. The black horizontal arrow stands for the light beam. It travels through the gray box on the right, which is some speedometer, *measuring* the speed of the light beam in meters per second. We will *not* discuss what constitutes such an apparatus: We just assume that there *are* such devices.

1.2 First Law of Relativity: Straight, Steady Speed Is Relative

Does the speed of light change if we move the torch while sending a light beam? This prompts the question: Moving *relative* to what?

If we are moving in a fast train, then we do *not* feel the steady speed of the train, but only feel a slight tremble, that is the *non-steady* part of the speed. For example, in Fig. 1.2 a table is standing inside a train. The train is moving straightly and steadily. Then the black ball will not begin to move on the table. Do you feel the tremendous speed while you are sitting at home at a table, feeling yourself at rest? Which speed? The tremendous speed with which the earth is moving around the sun, for example. This speed is at least during a few minutes, say, nearly straight and steady. Or the speed with which the whole solar system is moving along the galaxy. And not to forget the speed with which the galaxy is moving—to where?

Since the outgoing middle ages we know that we *cannot* detect if we are moving steadily, straight ahead in some direction, by *no means whatsoever*. We *always* can

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Fig. 1.1 Light beam from *left* to *right*, sketched as *black horizontal arrow*. The *box* on the *left* stands for some sender like a torch or laser, the *gray box* on the *right is* the light-speedometer, displaying the speed of light

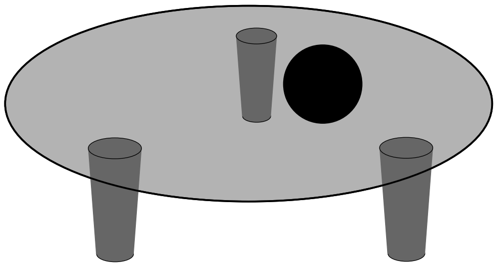

Fig. 1.2 The ball on the glass table in a train will not begin to move on the table, if the train moves straightly and steadily

take the point of view, that *we are at rest*, in the same way as we do when we "sit at a table".

Then even while sitting at a table in a steadily, straight ahead moving train, we can say that *we* are at rest, and the whole station plus the surrounding area we pass through is moving *towards* us. At the same time, a friend standing on the platform of the station, objects: Of course the train is moving towards the station, and the station together with the earth around it are at rest!

Then who is correct? Answer: *Both* we and our friend are correct in insisting to be at rest: The train is moving only *relative* to the train station and the earth around it. This is the **first law of the theory of relativity**, formulated by **Galilei** a few hundred years ago:

The speed of straight, steady motion of a body we can only measure *relative* to other bodies. The laws of nature do *not* depend on a straight and steady speed at which we may move relative to other bodies.

1.3 Measuring the Speed of Light

We measure the speed of the bodies in Fig. 1.3 *relative* to the ground. While sending the light beam, we move the torch towards the speedometer with, say, 10000 meters per second, *relative* to the ground, while the speedometer rests relative to the ground. In order that nothing should disturb the light beam, we prepared the situation such that there is no air above the ground.

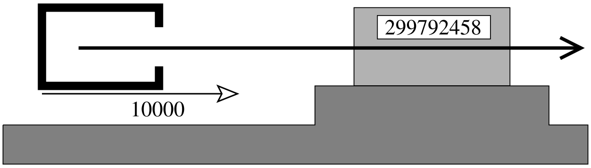

Fig. 1.3 Now the sender moves towards the speedometer with 10000 meters per second relative to the ground. The ground we indicated in *dark gray*

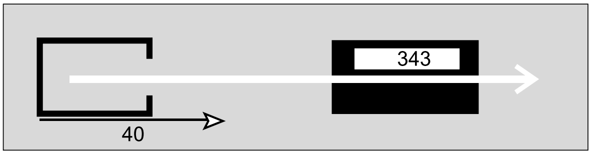

Fig. 1.4 A loudspeaker moves towards the speedometer with 40 meters per second. The speed of sound remains the same as if the loudspeaker would rest

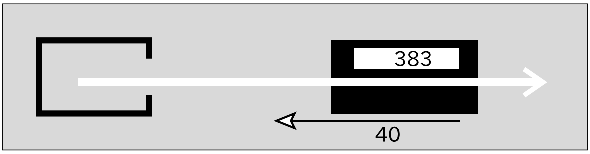

Fig. 1.5 If the speedometer is moving towards the loudspeaker which is resting in the air, we measure a larger speed of sound

Nevertheless, the speedometer shows the very same speed of light! Now, this does not sound so astonishing. Here is an analogy: Replace the torch with a loudspeaker, and the speedometer for light with a speedometer for sound, as in Fig. 1.4. The sound we drew as white arrow. The calm air we depicted as light gray background. The sound travels through the calm air with a speed of about 343 meters per second.

The sound travels through calm air, and so its movement does not feel the moving loudspeaker: We will measure the same 343 meters per second, even if the loudspeaker travels towards the speedometer with 40 meters per second. So is this not a very similar situation to the light beam?

No, because next we will put the loudspeaker at rest inside calm air, and move the *speedometer* towards the loudspeaker, with the same 40 meters per second.

Because the sound moves with 343 meters per second *relative* to calm air to the right, and the detector moves with 40 meters per second *relative* to the calm air as well, to the left, we will measure a sound speed of

$$343 + 40 = 383 \text{ meters per second.}$$

This we sketched in Fig. 1.5.

1.4 Second Law of Relativity: Speed of Light Is Absolute

Next we repeat that thought experiment with light, as in Fig. 1.6.

The astonishing result is, that for light the speedometer *still* shows the same speed of light! That means that light does *not* need any medium like "air" to travel: It