

Environmental Engineering

Leonardo Di G. Sigalotti  
Jaime Klapp  
Eloy Sira *Editors*

Computational and  
Experimental Fluid  
Mechanics with  
Applications to  
Physics, Engineering  
and the Environment

 Springer

# **Environmental Science and Engineering**

## Environmental Engineering

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Eloy Sira  
Editors

# Computational and Experimental Fluid Mechanics with Applications to Physics, Engineering and the Environment

 Springer

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# Preface

Fluid flows occur everywhere in nature and occupy a relevant place in our technological world as well as in the running of a vast number of industrial processes. They are not only essential to life, but also to understand fundamental physical processes at all measurable scales, from the nanometric world to the cosmological scales. The principles of fluid mechanics are used in almost every form of mechanical and chemical engineering, with far-reaching effects on the technological advances that lead to the multitude of products which determine the high standard of living that nowadays we take for granted. Fluid flows are also known to be at the heart of health, biological, and environmental sciences, including the flows in the human body and its energy supply, the multitude of flows in the entire fauna and flora, and the atmospheric flow processes, which influence the weather and the climate. Thus, fluid flows are vital and their understanding is an essential part of the general education of humans.

This book presents a collection of papers dealing with recent advances in computational and experimental fluid mechanics with applications to physics and engineering. Among these papers, a few ones are reviews outlining the impact of fluid mechanics on important active research areas such as weather prediction and climate change, cancer research, and cosmology. The present collection includes research work presented at the I Workshop of the Venezuelan Society of Fluid Mechanics, held in the *Margarita Island, Venezuela*, on November 5–9, 2012 under the auspices of the Instituto Venezolano de Investigaciones Científicas, IVIC, and the Fondo Nacional de Ciencia, Innovación y Tecnología, FONACIT, of Venezuela. The book begins with invited lectures held during the Workshop by renowned national and international scientists and engineers, covering a wide range of topics, followed by a number of invited seminars presented by young researchers and graduate students working actively in the field of fluid mechanics and related areas.

The I Workshop of the Venezuelan Society of Fluid Mechanics represented a unique opportunity to provide a forum for the presentation of state-of-the-art research in theoretical, experimental, and applied fluid mechanics oriented to engineering technology, where scientists, coming from different universities and research institutions of the country, together with mechanical, chemical, and petroleum engineers from public and private enterprises, with a huge experience in

applied industrial problems, have participated in fruitful discussions on fundamental and technical aspects, paving the way for future collaborations.

The Workshop will be organized every 2 years. The 5 days of oral sessions accommodated 45 talks and had close to 60 attendees with 10 international and 20 national researchers, and more than 30 graduate and undergraduate students. The wide variety of topics presented included free-surface and interface flows, such as drops and bubbles, turbulent flows, multiphase flows with applications to biological and oil extraction systems, shock structure and acoustic waves, opto-fluids, granular fluids, astrophysical and cosmological flows, and computational fluid dynamics. Among the renowned researchers, Joseph J. Niemela, from The Abdus Salam International Centre for Theoretical Physics, ICTP, Trieste, Italy, showed the results of controlled laboratory experiments of turbulent diffusion of heat at high Rayleigh numbers; Dominique Legendre, from the Institut de Mécanique des Fluides de Toulouse, IMFT, Toulouse, France, presented numerical simulations of sliding drops on an inclined solid surface; Catalina Stern-Forgach, from the Department of Physics of the Universidad Nacional Autónoma de México, UNAM, Mexico, described the results of experimental measurements of shock structure and acoustic waves inside a supersonic jet; and José R. Castrejón-Pita, coming from the Department of Engineering of the University of Cambridge, Cambridge, United Kingdom, spoke of the relevance of the breakup of liquid surfaces to industry and discussed current issues faced by researchers working in the field of droplet dynamics. Interesting lectures on bubble growth in viscous liquids were given by Abraham Medina and Abel López-Villa, both from the Instituto Politécnico Nacional (I.P.N.) of Mexico, while Julián Chela-Flores, from The Abdus Salam International Centre for Theoretical Physics, ICTP, Trieste, Italy, gave a magisterial conference on how fluid mechanics is playing a major role in space exploration for understanding the cosmic distribution of life. The theoretical physics of granular fluids and an introductory view of the jamming transition problem were given by Leonardo Trujillo, from the IVIC's Centre of Physics. Other interesting talks were presented by Humberto Cabrera, from the IVIC's Department of Applied Physics, on the Soret effect in binary fluid mixtures; by Luis R. Rojas-Solórzano, from the Department of Energy Conversion and Transport of the Universidad Simón Bolívar, USB, Caracas, Venezuela, who described a multiphase approach to model blood flow in micro-tubes; and Miguel R. Paiva-Rojas, from the Refining and Industrialization Department of the Instituto Tecnológico Venezolano del Petróleo, PDVSA-Intevep, Los Teques, Venezuela, who spoke on the estimation of the gas-liquid-solid phase distribution in a cold slurry bubble column system for hydro-conversion processes. Other local speakers gave short oral presentations on computational and experimental drop dynamics, compositional flows applied to the oil industry, granular and porous media flows, and astrophysical flows.

The short oral presentations were organized by themes: Drops, Particles, and Waves; Multiphase and Multicomponent Flow, Granular and Porous Media Flow; and Astrophysical and Relativistic Flow. The book is aimed to undergraduate and graduate students, as well as to physicists, chemists, and engineers dealing with

fluid mechanics from both the experimental and theoretical point of view. The material is also adequate for both teaching and research. The invited lectures and the other selected contributions are introductory and use a minimum of mathematics.

The editors are deeply indebted to the several institutions that made possible the realization of the I Workshop of the Venezuelan Society of Fluid Mechanics. In particular, we thank the Instituto Venezolano de Investigaciones Científicas, IVIC, and the Fondo Nacional de Ciencia, Innovación y Tecnología, FONACIT, of Venezuela for providing financial support. We are also grateful to the Instituto Tecnológico Venezolano del Petróleo (PDVSA-Intevep), the Centro de Investigaciones de Astronomía Francisco José Duarte, CIDA, the FUNDACITE-Miranda, and the Mexican institutions: Consejo Nacional de Ciencia y Tecnología, CONACYT, Consejo Mexiquense de Ciencia y Tecnología, COMECYT, Instituto Nacional de Investigaciones Nucleares, ININ, and Cinvestav-Abacus of the Instituto Politécnico Nacional, I.P.N.

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Caracas, June 2013

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**Part I**  
**Invited Lectures**

# Environmental Fluid Mechanics: Applications to Weather Forecast and Climate Change

Leonardo Di G. Sigalotti, Eloy Sira, Jaime Klapp and Leonardo Trujillo

**Abstract** Virtually all economic sectors as well as many public and private activities are affected in some measure by changes in weather and climate. Uncertainties in the scope and severity of these changes pose financial and social risks for individuals, businesses, and government agencies, with direct influence on food security and production, transport, health, electricity generation, and water resources. The vulnerability of human settlement to extreme weather and climate episodes is a further aspect that must be emphasized. Hence, achieving accurate weather and climate forecasts has important implications to modern society. In this chapter, we present an overview of the basic fluid-mechanical principles that govern the behaviour of weather and climate. We shall mainly focus on the numerical modelling of weather prediction and climate projections, spanning the range from the very first attempts, based on simple barotropic models, to the development of general circulation models of the atmosphere and ocean to the most recent multi-model ensemble forecasting systems.

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## 1 Introduction

There are many different kinds of naturally occurring fluid flows in the environment. Natural fluid motions are vital, and there is a general strong incentive to study them, particularly those of air in the atmosphere and of water from underground aquifers to surface flows in rivers, lakes, and oceans. Environmental concerns have encouraged interdisciplinarity to a degree that has been increasing in proportion to the acuity of the problems, giving rise to a body of knowledge that comprises several disciplines, including hydrology, meteorology, climatology, and oceanography among others. Whereas the particular objectives of each of these disciplines, such as weather forecasting in meteorology and climate change projections in climatology, encourage disciplinary segregation, environmental concerns compel experts in those disciplines to base their models on the solution of the equations of fluid dynamics.

The threat of climate change is one of the greatest challenges currently facing society. Because of the increased threats imposed by global warming and the increasing severity and occurrence of storms and natural disasters, improving our understanding of the climate system has become an international priority. In simple words, climate refers to the average of weather conditions. Descriptions of the climate generally encompass statistical information concerning the mean and variability of relevant quantities, as temperature, precipitation, and wind, over a multi-year time period. Fluctuations in the Earth system result naturally from interactions between the ocean, the atmosphere, the land, the frozen portion of the Earth's surface (or cryosphere), and the changes in the Earth's energy balance arising from volcanic eruptions and variations in the Sun's intensity. Although global warming has been accepted as incontrovertible, humans continue to alter the composition of the atmosphere, primarily through the burning of fossil fuels. The build up of greenhouse gases and trace constituents is another factor that contributes to changes in the Earth's heat energy balance. Its impact on the planet has been detected and is projected to become increasingly more important in the coming decades and centuries.

Today, a fundamental tool used for predicting weather and climate changes is the use of numerical models, i.e., mathematical models run as computer simulations. However, the basic ideas of weather forecasting and climate modelling were developed about more than a century ago, long before the construction of the first electronic computers (Phillips 1970; Lynch 2008). At these early times, observations were rather sparse and irregular, especially for the upper air and over the oceans, making weather forecasting very imprecise and unreliable. The basic laws of physics, fluid motion, and chemistry played no role and were replaced by the forecaster with crude techniques of extrapolation, knowledge of local climatology, and guesswork based on mere intuition. It was not until the beginning of the last century that meteorologists started to recognize that fluid mechanics and thermodynamics represent the set of fundamental physical principles that govern the flow of the atmosphere (Abbe 1901; Bjerknæs 1904; Willis and Hooke 2006). In particular, Abbe (1901) proposed the first mathematical approach to forecasting, and shortly after Bjerknæs (1904) introduced the idea that rational forecasting should consist of a diagnostic

step, in which the initial state of the atmosphere is determined observationally and represented in charts giving the distribution of the variables at different levels, and a prognostic step, in which the laws of fluid motion are used to calculate the changes of this state over time. Non-linear advection—the transport of fluid properties and characteristics by the motion of the fluid itself—was identified as the primary physical process. However, he employed a graphical approach, rather than numerical methods, for solving the fluid dynamics equations and building up new charts describing the atmosphere some hours later, with the process being repeated iteratively until the desired forecast length was achieved.

The beginning of modern numerical weather prediction (NWP) was pioneered by Richardson (1922), who first attempted a direct solution of the equations of motion using finite difference methods (Lynch 2006). His work impelled profound developments in the theory of meteorology and is the foundation upon which modern forecasting is built. Since then, the advances in numerical analysis, which enabled the design of stable algorithms, the development of the digital computer technology, and the invention of the radiosonde, and its introduction in a global network, providing timely observations of the atmosphere in three-space dimensions (i.e., in latitude, longitude, and height), have completed the task. A definite impulse to modern meteorology was given later on by Charney (1947, 1948, 1950), who developed a set of equations known as the *quasi-geostrophic vorticity* system for calculating the large-scale motions of planetary-scale waves (Charney 1948), giving the first convincing physical explanation for the development of mid-latitude cyclones—his baroclinic instability theory. This theory was capable of producing a quantitatively accurate prediction of the atmospheric flow (Charney et al. 1950; Platzman 1979). In 1979 he led an ad hoc study group on carbon dioxide and climate for the United States National Research Council, with their final written report being one of the earliest modern scientific assessments about global warming (Charney et al. 1979). They estimated that doubling of CO<sub>2</sub> emissions will produce a global warming near 3°C with a probable error of ±1.5°C, which is quite close to the best estimate value of about 3°C for the global temperature increase given by the Intergovernmental Panel on Climate Change (IPCC) Fourth Assessment Report published in 2007.

With the advances in computer technology, numerical weather predictions have achieved breakthrough improvements in many aspects. In the 1960s, operational forecasts started to use models based on numerical solutions of the *primitive* equations—a set of non-linear differential equations, consisting of a form of the familiar Navier-Stokes equations, a continuity equation, and a thermal energy equation (Charney 1955; Hinkelmann 1959; Phillips 1960; Smagorinsky 1963). A six-level primitive equation model was introduced into operations at the National Meteorological Center in Washington in June, 1966, running on a CDC 6600 computer (Shuman and Hovermale 1968). Manipulating the vast datasets and performing the complex calculations necessary to modern weather prediction require some of the most powerful supercomputers in the world. Even with the increasing power of supercomputers, the forecast skill of NWP models extends to about only 6 days. The density and quality of observations used as input to the forecasts and the deficiencies in the models themselves are important factors affecting the accuracy of the predictions.



A more fundamental problem lies in the chaotic nature of the fluid-dynamics equations used to simulate the atmosphere. In addition, these equations need to be supplemented with parameterizations that attempt to capture the phenomenology of small-scale processes, including solar and terrestrial radiation, moisture content (cloudiness and relative humidity), surface hydrology (precipitation, evaporation, snow melt and run-off), heat exchange, soil, vegetation, surface water, and the effects of terrain. On the other hand, the development of regional (limited area) models has facilitated accurate forecasting of the tracks of tropical cyclones and hurricanes as well as of air quality (Shuman 1989; van Dop and Steyn 1991). The inclusion of the interactions of land and vegetation with the atmosphere has led to more realistic forecasts (Xue et al. 1996).

The chaotic nature of the atmospheric flow imposes a limit on predictability, as inherent errors in the initial state grow rapidly and render the forecast useless after some days. A numerical prediction method, known as ensemble forecasting, which is a form of Monte Carlo analysis has been introduced in which multiple numerical predictions, each starting from slightly different initial conditions, are run and the combined outputs are used to deduce probabilistic information about future changes in the atmosphere (Molteni et al. 1996; Toth and Kalnay 1997; Buizza et al. 1999). With this approach, probability forecasts for a wide range of weather events are currently generated and disseminated for use in the operational centres. For instance, seasonal forecasts, with a range of 6 months, are prepared at the European Centre for Medium-Range Weather Forecasts (ECMWF) and at the National Center for Environmental Prediction (NCEP) in Washington. They are made using a coupled atmosphere/ocean model, and a large number of forecasts are combined in an ensemble each month. In particular, these forecast ensembles have demonstrable skills for tropical regions with recent impressive predictions for the onset of El Niño and La Niña events. However, in middle latitudes, as in Europe, no significant skill has yet been achieved by these models. In fact, seasonal forecasting for middle latitudes remains one of the great problems facing us today.

Weather and climate are different in the sense that climate predictions do not need knowledge of weather in detail. A good analogy of the difference between weather and climate is to consider a swimming pool. Suppose that the pool is being slowly filled. If someone dives into it, this will certainly generate waves on the water surface. The waves represent the weather, while the average water level is the climate. A new diver jumping into the pool next day will produce more waves, but the water level will be higher as more water has flowed into the pool. In the atmosphere the ‘water hose’ is increasing the amount of greenhouse gases, which will cause the climate to warm even though we still have a changing weather (waves). Thus, climate scientists use models to forecast the average water level in the pool and not the waves. However, climate modelling derives from efforts first formulated to numerically predict the weather. The first successful long-range simulation of the general circulation of the atmosphere was developed in 1956 (Phillips 1956), which realistically depicted monthly and seasonal patterns in the troposphere (Cox 2002). This work had a galvanizing effect on the meteorological community and thereafter several general circulation models (GCMs) were developed. One early model of

particular interest has been that developed at the National Center for Atmospheric Research (NCAR) (Kasahara and Washington 1967). By the early 1980s, NCAR has developed the Community Climate Model (CCM), which has been continuously refined into the next 20 years (Williamson 1983; Williamson et al. 1987; Williamson and Olson 1994), with the Community Atmosphere Model (CAM 3.0) being the latest version (Collins et al. 2004). On the other hand, coupled atmosphere/ocean climate models such as HadCM3 and HadGEM are used at the Hadley Centre for Climate Prediction and Research in the United Kingdom for a wide range of climate studies (Lynch 2006). Advanced models, such as the atmospheric GCM ECHAM5 developed at the Max Planck Institute for Meteorology (Roeckner et al. 2003), are under continuing refinements and extensions, and are increasing in sophistication and comprehensiveness. Most of them simulate not only the atmosphere and oceans but also a wide range of geophysical, chemical, and biological processes and feedbacks. In particular, these models, now called Earth System Models, are applied to the practical problem of weather prediction and also to the study of climate variability and mankind's impact on it.

## 2 Weather Modelling and Prediction

The atmosphere is a fluid (composed mostly of air) that covers the entire Earth surface. Most of the phenomena which we associate with day-to-day weather occur in its lowest layer, called the troposphere, which ranges in thickness from about 8 km at the poles to 16–20 km over the equator. The troposphere is denser than the layers of the atmosphere above it and contains up to 75 % of the mass of the atmosphere, with approximate composition of 78 % nitrogen, 21 % oxygen, and 1 % small concentrations of other trace gases. Nearly all atmospheric water vapour (or moisture) and aerosols are found in the troposphere. Since temperature decreases with altitude, warm air near the surface of the Earth can readily rise, being less dense than the colder air above it. This induces a vertical movement, or convection, of air which generates clouds and ultimately rain from the moisture within the air, giving rise to much of the weather we experience in our daily lives.

The troposphere is capped by the tropopause, a boundary region of stable temperature, separating the troposphere from the stratosphere, where the air temperature begins to rise. Such a temperature increase prevents much of the air convection beyond the troposphere, and consequently most weather phenomena, including towering cumulonimbus thunderclouds, are confined to the troposphere. For instance, most commercial aircrafts fly in the lower stratosphere, just above the tropopause where clouds are usually absent, as also are significant weather perturbations (Petty 2008). However, vigorous thunderstorms as, for example, those of tropical origin may overshoot into the lower stratosphere and undergo low-frequency vertical oscillations of an hour-order duration, or less (Shenk 1974), which in turn may induce low-frequency atmospheric gravity waves capable of affecting both atmospheric and oceanic currents in the region (Bromirski et al. 2010). Sometimes the temperature

does not decrease with height in the troposphere, but rather increases, which is known as a temperature inversion. In general, temperature inversions limit or prevent the vertical mixing of air, causing a state of atmospheric stability. This can lead to episodes of air pollution, where air becomes stagnant and pollutants emitted at ground level remain trapped underneath the temperature inversion zone (Phalen and Phalen 2012).

Among the most significant scientific advances of the past century is our ability to simulate complex physical systems using numerical methods and predict their evolution. One outstanding example is the development of GCMs of the atmosphere and ocean, which can be used to predict the weather for several days in advance with a high degree of confidence and gain insight into the factors that cause changes in the climate as well as into their likely timing and severity. Here we shall review the most important numerical weather prediction models, which were the precursors to climate prediction systems, viewed as a problem in non-linear fluid mechanics.

## 2.1 Barotropic Models

Barotropic models are short-range prediction models that include only the reversible part of atmospheric physics. That is, the atmosphere is treated as a one-component gas consisting of dry air so that irreversible processes, such as non-adiabatic heating and cloud formation, are not taken into account. The barotropic model was the first kind of NWP model ever successfully implemented (Charney 1948; Charney et al. 1950). It is probably the simplest model that can realistically model atmospheric flow around the Earth. Meteorologists use the word barotropic to describe an atmosphere where *isosteric* surfaces—surfaces of constant specific volume—and *isobaric* surfaces—surfaces of constant pressure—coincide. In other words, the gradient of the specific volume (or density) and the gradient of pressure are parallel and proportional to each other so that the density is a function of pressure (adiabatic atmosphere).

Typical barotropic models are based on a set of equations known as the *quasi-geostrophic* system (Charney et al. 1950). These equations are derived from the Euler equations of motion by assuming that the Coriolis force resulting from horizontal air currents exactly balances the horizontal pressure gradients (geostrophic balance), while in the vertical direction hydrostatic equilibrium is assumed. If the atmosphere is divergence-free, the curl of the Euler equations of motion reduces to the barotropic vorticity equation (Bennett et al. 1993):

$$\frac{D\zeta}{Dt} = 0, \quad (1)$$

where  $D/Dt$  is the substantial time derivative and  $\zeta$  is the absolute vorticity defined by

$$\zeta = m^2 \left[ \frac{\partial}{\partial x} \left( \frac{v}{m} \right) - \frac{\partial}{\partial y} \left( \frac{u}{m} \right) \right] + f, \quad (2)$$

where  $v$  and  $u$  are the horizontal geostrophic wind components in the direction of the map coordinates  $x$  and  $y$ , respectively,  $m$  is the map factor, and  $f = 2\Omega \sin \phi$  is the Coriolis frequency. Here  $\Omega$  is the angular velocity of planetary rotation and  $\phi$  is the latitude. Since the model has non-divergent flow, a streamfunction  $\Psi$  can be defined by

$$v = m \frac{\partial \Psi}{\partial x}, \quad u = -m \frac{\partial \Psi}{\partial y}, \quad (3)$$

so that

$$\zeta = m^2 \nabla^2 \Psi + f. \quad (4)$$

In low-pressure systems, where the Rossby number (Ro) is small, the effects of planetary rotation are large compared to the net wind acceleration, allowing the use of the geostrophic approximation given by Eqs. (1–4) (Marshall and Plumb 2008). Typical barotropic models for operational weather prediction were based on an extended version of Eqs. (1–4) to account for small deviations from strict geostrophic balance—the so-called semi or quasi-geostrophic equations (Phillips 1970; Chynoweth and Sewell 1991). Since the pioneering work of Charney (1948) and (Charney et al. 1950), the quasi-geostrophic equations have become an accepted system of approximate equations for the study of mid-latitude motions of the atmosphere on a synoptic scale, while allowing for the presence of mesoscale phenomena such as the atmospheric fronts.

A barotropic instability is a wave instability associated with shear in a jet-like current and this appears to be of central importance in the tropics. Early attempts of forecasting in the tropics with a barotropic atmospheric model were addressed to predict upper-air flow patterns in the tropical Pacific areas of both the Northern and Southern hemispheres (Jordan 1956; Vederman et al. 1966). A similar model was applied to forecasts of flow patterns at 500 mb level in the Indian region (Shukla and Saha 1970). Barotropic prediction models have also provided the basis for a significant advance of the state of the art of tropical cyclone motion and hurricane track forecasting in the range from one to several days (Bennett et al. 1993; Sanders and Burpee 1968; Sanders et al. 1980; DeMaria 1985). Although there are some situations where tropical cyclone motion can only be modelled using a more general form of the basic equations as, for example, in the case when a vortex interacts with a vertically-sheared basic current, there has been evidence that some aspects of tropical cyclone motion can be described with simple barotropic models. For instance, the SANBAR model (Burpee 2008)—a barotropic tropical cyclone track prediction model designed for the North Atlantic tropical cyclone basin and used operationally during 1973–1984 and 1985–1989, was recognized to be superior to other forecast methods for medium range track forecasts of low-latitude Atlantic tropical cyclones (Neumann and Pelissier 1981). It has also been shown that for the Australian/Southwest Pacific region many aspects of tropical cyclone motion can be explained using a theory based on a barotropic vorticity equation (Holland 1983, 1984). In fact, calculations of the terms in the full form of the vorticity equation, using aircraft and rawinsonde composite data, have shown that the dominant contribution

to the local vorticity change in the regions near the tropical cyclone centre comes from the horizontal advection term (Chan 1984).

Barotropic NWP models have also been used to demonstrate the close coupling existing between the westwards propagating African waves and the broad scale African monsoons on the time scale of 3–5 days (Krishnamurti et al. 1980). It is well-known today that about 80 % of all tropical cyclones on the globe forms near or within the intertropical convergence zone (ITCZ) (Gray 1979). In satellite images, the ITCZ is sometimes observed to undulate, forming cloud patterns. At times, such an undulating ITCZ breaks down into several tropical disturbances within which tropical cyclones may form (Gray 1979; Zehr 1993). The resulting tropical cyclones and typhoons then move into higher latitudes, allowing the ITCZ to reform and perhaps start the cycle over again (Guinn and Schubert 1993). These undulations are a clear signature of easterly waves in the tropical troposphere. Easterly waves have early been recognized to play an important role in tropical cyclogenesis (Riehl 1945). These have since been observed in the Atlantic Ocean and West Africa (Reed et al. 1977; Chen and Ogura 1982), in the Pacific Ocean (Nitta et al. 1985; Nitta and Takayabu 1985; Tai and Ogura 1987; Heta 1991), and in the South China Sea and India (Saha et al. 1981). All these studies concluded that easterly waves occur in the lower tropical troposphere and have typical wavelengths and speeds in the ranges from 2,000 to 4,000 km and  $5\text{--}8\text{ ms}^{-1}$ , respectively. While nearly 60 % of all Atlantic tropical cyclones originates from African easterly waves (Avila and Clark 1989), observational and numerical studies indicate that they result from a convectively modified form of combined barotropic and baroclinic instability of the African easterly jet, which has maximum winds of  $10\text{--}15\text{ ms}^{-1}$  near 700 mb and  $15^\circ\text{N}$  (Norquist et al. 1977; Thorncroft and Hoskins 1994a,b). Barotropic model simulations based on the shallow-water equations have suggested that the ITCZ break-down may play a role in producing the observed tendencies for tropical storms to cluster in time and form polewards of the central latitude of the ITCZ and to the east of existing tropical storms (Nieto Ferreira and Schubert 1997). More recently, barotropic instability calculations have also been employed to investigate the possible importance of barotropic shear variations for explaining the effect of the Madden-Julian oscillation on hurricane formation over the eastern and western North Pacific (Hartmann and Maloney 2001).

In spite of its numerous applications during more than 40 years, the quasi-geostrophic modelling was abandoned because of the development of more efficient ways of integrating the primitive equations (Bengtsson 1999). On the other hand, the incorporation of physical processes, radiation, clouds, precipitation processes, etc. was by far more complicated to implement in the quasi-geostrophic models, and this was an additional reason not to use them any longer in NWP.

## 2.2 Baroclinic Models

The occurrence of large vertical temperature gradients in the troposphere can lead to the formation of convective air currents, which transport the excess energy away from

the surface to higher altitudes where the air is significantly cooler. When this happens we say that the atmosphere is statically unstable. In analogous manner, when the latitudinal temperature distribution is such that a large equator-to-pole temperature gradient exists, the atmosphere will break down into wind flows to move the excess energy from the regions of excess (warm tropics) to regions of deficit (cool poles). In this case, the atmosphere is said to be baroclinically unstable. This imbalance of energy is essentially due to an excess of radiational heating in the tropical latitudes. In a stratified fluid, a source term of the form  $\nabla \rho \times \nabla p / \rho^2$  appears in the vorticity equation whenever *isopycnic* (constant density) surfaces and *isobaric* surfaces are not aligned, which is responsible for the baroclinic contribution to the local vorticity (Marshall and Plumb 2008). In meteorology, a baroclinic atmosphere is one in which the density depends on both the temperature and the pressure.

The most important application of the baroclinic instability is the cyclogenesis process at mid-latitudes, which represents the development of synoptic scale weather disturbances. In other words, it is the leading mechanism shaping the cyclones and anticyclones that influence weather at mid-latitudes. For instance, in the ocean the baroclinic instability is responsible for the generation of mesoscale eddies that play a role in the transport of tracers, which are used in oceanography to deduce flow patterns in the ocean (Davis 1991). In general, vorticity is the curl of the velocity field and its evolution can be broken into contributions from advection (as vortex tubes move with the flow), stretching and twisting (as vortex tubes are pulled or twisted by the flow), and baroclinic vorticity generation (Nadiga and Aurnou 2008). Therefore, the study of the evolution of these baroclinic instabilities is a crucial part of developing theories of mid-latitude weather. The birth of baroclinic NWP models started with the classical work of Charney (1947) and Eady (1949). The energy source for baroclinic instability is the potential energy associated with the environmental flow, and since then meteorologists have become aware that baroclinic instability can develop even in situations of rapid rotation (small  $Ro$ ) and strong stable stratification (large Richardson number,  $Ri$ ) as is typically observed in the atmosphere, where  $Ri$  is a dimensionless number that serves to quantify the ratio of potential to kinetic energy.

Since a tropical cyclone is a huge tropospheric convection cell and the axis of the horizontal wind circulation remains almost vertical during the movement, there was a need to develop baroclinic prediction models capable of simulating the three-dimensional atmospheric motion more closely than single-level barotropic models. After the Electronic Numerical Integrator and Computer (ENIAC) forecast chaired by Charney in the 1950s in Aberdeen, Maryland (Platzman 1979), several baroclinic models were developed in the next few years, which were all based on the quasi-geostrophic system of equations (Phillips 1951, 1954; Charney and Phillips 1953; Matsumoto 1956; Wiin-Nielsen 1959; Kasahara 1960). Most of these models were employed to evaluate the instantaneous movement velocity of tropical cyclones from multi-level data, i.e., the atmosphere is divided into two, or more, levels where prognostic and diagnostic variables are evaluated from known data at these levels. However, it was soon argued that early experiments with baroclinic models capable of generating additional kinetic energy from the store of available potential energy