

Environmental Engineering

Jaime Klapp
Abraham Medina *Editors*

Experimental and Computational Fluid Mechanics

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Experimental and Computational Fluid Mechanics

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Preface

The understanding of fluid flow has become a very important field of research because of its relevance in many human activities. The study of fluid flow is essential in many fields such as physics, biology, medicine, engineering, environmental sciences, energy, atmospheric flow processes that influence the weather and the climate, and so on. Many industrial processes are governed by the equations of fluid dynamics and so its study is valuable. Numerical simulations of fluid flow is reducing the high cost of very expensive experimental tests. Dynamical fluid dynamics are also very important for understanding fundamental physical processes at all scales.

This book presents recent experimental and theoretical advances in fluid dynamics in physics and engineering. It begins with invited lectures given during the International Enzo Levi Spring School 2012 held at the Universidad Autónoma Metropolitana-Azcapotzalco, May 17–18, 2012, and invited seminars presented in the 18th National Congress of the Fluid Dynamics Division of the Mexican Physical Society, held at the Centro de Investigación Científica y Educación Superior de Ensenada (CICESE), Ensenada, Baja California, Mexico, November 21–23, 2012.

The Spring School is organized every year in honor of Prof. Enzo Levi, a well-known Mexican scientist that dedicated his research to the study of fluids. He was one of the founders of the Instituto de Ingeniería (Engineering Institute) of the Universidad Nacional Autónoma de México (UNAM), and of the Instituto Mexicano de Tecnología del Agua (Mexican Institute for Water Technology) of the National Water Commission. He was the mentor of several generations of Mexican Engineers.

During the 2 days school, lectures were given by well-known national and international scientists. In 2012, many people attended the meeting with 50 researchers and more than a hundred graduate and undergraduate students. A wide variety of topics were presented by young national researchers that included asymptotic methods in fluids, convection, computational methods for biological systems, interfacial fluid dynamics, and problems related to colloidal dispersion and fractured porous media. Among the lectures we want to mention a very interesting description of the Alya Cardiac Computational Model given by Mariano Vazquez of the Barcelona Supercomputing Center in Spain, two lectures by Rouslan Kretchetnikov of the University of California in Santa Barbara entitled Walking with coffee: why it splits and A few puzzles in interfacial fluid dynamics,

two lectures by Cesar Treviño (UNAM) on asymptotic method in fluids and linear stability of mixed convection flows. Other interesting lectures were given on the Turbulence of marine currents by Angel Ruíz (UNAM), on the hydrodynamic interactions of heterogeneous colloidal dispersions by Catalina Haro (UAM-A), on the Boundary element method in Fluid Mechanics by Abel López Villa (IPN), and on An Experimental Analogy of Pedestrians under Panic Situations by Abraham Medina (IPN). Several of these lectures were included in Part I of the Book.

The Annual Fluid Dynamics Congress has a different format. In 2012, it lasted 3 days and was composed of five plenary lectures and many short oral presentations of students and researchers.

In part I, we also included the plenary lectures given during the congress by international well-known invited speakers and some of the most interesting short oral contributions. Among the plenary lectures, we can mention those given on the basic concepts of ignition and fuel through diffusion flames by Amable Liñan of the Universidad Politécnica de Madrid, on the Transition to turbulence in stratified wakes by Patrice Meunier of the Aix-Marseille University, France, on flow driven by harmonic forcing in planetary atmospheres and cores by Michael Le Bars, also of the Aix-Marseille University, France, on the role of symmetry in biomimetic wakes for propulsion by Verónica Raspa of the University of Paris Diderot, France, on toroidal vortex with chaotic stream lines by Oscar Velasco of CICESE, Baja California, México, on the symmetry instability in a mixed convection problem by Cesar Treviño of UNAM, and on The boundary element method for bubble formation by Abel López of the IPN, México.

The other short presentations are organized in topics: Multiphase Flow and Granular Media (Part II), Convection and Diffusion (Part III), Vortex, Oceanography and Meteorology (Part IV), and General Fluid Dynamics and Applications (Part V).

In Part II, Multiphase Flow and Granular Media, we have focused on petroleum-related applications, you can find interesting contributions on the nitrogen dispersion in the neighborhood of a well, on a phenomenological description of heavy oil, on an experimental study of the growth of bubbles in corrugated tubes, which has applications for foam injection in fractured porous media, on surface tension and interfacial tension measurements in water-surfactant-oil systems, and other related problems. Other contributions in this section are in granular media and mass flow in a silo.

Convection and Diffusion can be found in Part III, with interesting contributions on conjugate convection in an open cavity, and on transport of particles in a periodically forced flow. We also found two applications to porous media flow: experimental studies of a vaporization front, and the mass and mass transfer during steam injection. There is also an interesting numerical study with SPH of the Kelvin–Helmholtz instability.

In Part IV, Vortex, Oceanography, and Meteorology, one can find three contributions on atmospheric fluid dynamics, the first on forced dynamics by normal wind to the boundary, the second on numerical experiments of wind circulation off the Baja California coast, and the third one is a review on environmental fluid

dynamics. Two further contributions are on the effect of the inlet flow angle on the vortex induced vibration of a collinear array of flexible cylinders and the wake patterns behind a flapping foil.

Finally, in Part V, General Fluid Dynamics and Applications, we find several contributions of fluid dynamics to various fields such as magnetohydrodynamics, evaporation of a liquid layer, a study of the drag coefficient in the Navier–Stokes fractional equation, interactions of fluids, numerical simulation of biological systems with the DPD method, and soil transport of contaminants.

The book is aimed to fourth-year undergraduate and graduate students, and to scientists in the field of physics, engineering, and chemistry that have interest in fluid dynamics from the experimental and theoretical point of view. The material includes recent advances in experimental and theoretical fluid dynamics and is adequate for both teaching and research. The invited lectures are introductory and avoid the use of complicated mathematics. The other selected contributions are also adequate to fourth year undergraduate and graduate students.

The editors are very grateful to the institutions who made possible the realization of the International Enzo Levi Spring School 2012 and the 18th National Congress of the Fluid Dynamics Division of the Mexican Physical Society, especially the Consejo Nacional de Ciencia y Tecnología (CONACYT), the Sociedad Mexicana de Física, the Universidad Autónoma Metropolitana-Azcapotzalco, the Centro de Investigación Científica y Educación Superior de Ensenada (CICESE), the Universidad Autónoma de México (UNAM), the Instituto Politécnico Nacional (IPN), Cinvestav-Abacus, and the Instituto Nacional de Investigaciones Nucleares (ININ).

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Part I
Invited Lectures

Symmetry Breaking Instability in a Mixed Convection Problem

J. C. Cajas, L. Martínez-Suástegui and C. Treviño

Abstract In this work, the stability of a strongly non-parallel symmetrical counterflow mixed convection problem is studied, using numerically generated eigenfunctions. The base flow is numerically obtained for each value of the buoyancy parameter (Richardson number), and the stability of this flow is analyzed by increasing its value while all the others remained fixed. The perturbed linear functions are numerically generated by introducing a transient modulated asymmetrical buoyancy, relaxing at later times to ‘numerical eigenfunctions’. The time evolution of the amplitude of these perturbations is used to obtain the stability characteristics. Symmetry breaking instability occurs, for fixed geometry, Reynolds and Prandtl numbers, for values of the buoyancy parameter larger than a critical one. However, there is also a window for the buoyancy parameter below this critical value, where the system shows instability, producing a slightly asymmetric thermal and flow response.

1 Introduction

Mixed convection is defined as heat transfer situations where both natural and forced convection mechanisms interact. In particular, the oscillatory behavior in mixed convection flows is of great interest because of its rich dynamical features and useful

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results to applied problems. Chang and Lin (1993) studied the steady laminar and transient oscillatory mixed convection in a symmetrically heated vertical plane channel, subjected to an opposing buoyancy assuming a fully developed velocity profile at the inlet and discrete heat sources that are maintained at uniform and equal heat fluxes. The authors pointed out that an oscillatory flow with a single fundamental frequency is found when the buoyancy parameter, or Richardson number, exceeds a critical value. Lin et al. (1993) investigated numerically the detailed flow and thermal characteristics in transient laminar opposing mixed convection in a vertical plane channel subjected to a symmetrical heat input. Their results show that at high opposing buoyancy, sudden flow asymmetry and oscillation occur simultaneously in an early steady flow after the initial transient. Evans and Greif (1997) showed the strong effects of buoyancy, even for small temperature differences, on the downward flow of nitrogen in a partially heated tall vertical channel and reported time-dependent oscillations, including periodic flow reversals along the channel walls. Martínez-Suástegui and Treviño (2007, 2008) investigated the transient laminar mixed convection in an asymmetrically and differentially heated vertical channel of finite length subjected to an opposing buoyancy. Their results show that a final steady or oscillatory flow response is obtained depending on the value of the Reynolds and Richardson numbers, and that the critical value of the buoyancy strength between the two regimes strongly depends on the value of the Reynolds number.

Stability analyses in natural and mixed convection flows have been developed through the years, since they provide further insight of the instability mechanisms present in such situations and give quantitative information about the defining and critical parameters involved. Ever since the pioneering work of Lorenz (1963), who studied the instability of finite systems of deterministic ordinary non-linear differential equations representing forced dissipative hydrodynamic flows, stability analyses of fluid flow phenomena experienced a rapid growth and became the starting point of the chaos theory. Carey and Gebhart (1983) studied theoretically and experimentally the stability and disturbance amplification produced by the combination of a natural convection flow induced by a vertical uniform heat flux surface and a forced convection flow given by a uniform free stream by assuming a weak forced convection flow with strong buoyancy effects. The authors presented stability planes and constant amplification contours, and found very good agreement between the two approaches. Daniels (1989) studied the stationary instability of the convective flow between differentially heated vertical planes and determined the subsequent structure of the neutral curve for stationary disturbances. Later, Chait and Korpela (1989) studied numerically the multicellular flow between two vertical parallel plates using a time-splitting pseudo spectral method for a steady flow of air and a time-periodic flow of oil. In the case of air, the authors analyzed parametrically the three-dimensional linear stability of the flow and found that the domain of stable two-dimensional cellular motion is constrained by the Eckhaus instability and by two types of monotone instabilities. Their results show that the two-dimensional multicellular flow is unstable above a Grashof number of about 8,550. Hence, the authors conclude that the flow of air in a sufficiently tall enclosure should be considered three-dimensional for most practical applications. Two decades ago, Rogers et al. (1993) studied the finite amplitude

instability of mixed convection of air in a vertical concentric annulus with each cylinder maintained at a different temperature by use of weakly non-linear instability theory and direct numerical simulation. They found three different instabilities in the parameter space of Grashof and Reynolds numbers: one due to shear and the two others induced by thermal effects. Suslov and Paolucci (1995) studied the stability of mixed convection flow in a tall vertical channel under non-Boussinesq conditions and showed that the stability characteristics, such as the critical Grashof number and the disturbance wave speed, depend strongly on the temperature difference when fluid properties are allowed to vary. Chen and Chung (1996; 1998) studied the stability of a differentially heated vertical channel for various Prandtl numbers and showed that both the Prandtl and Reynolds numbers hold very important effects on the instability mechanism for high Prandtl number fluids. More recently, van Putten et al. (2001) presented experimental results of heat transfer processes in mixed convection from a ducted vertical hot-plate thermal flow sensor for aiding and opposing flows for three different values of the Grashof numbers, $Gr = 289, 411$ and 456 for a range or Reynolds number from 0 to 120. The authors found, in the transition from free to mixed and to essentially forced convection, distinct sequences of instabilities in the flow that lead to several local minima and maxima in the heat transfer from the plate. Benoit Cushman-Roisin (2005) proposed a non-usual approach to the Kelvin-Helmholtz instability by considering the latter as a boundary-value problem instead of using the traditional approach of an initial-value problem where wave perturbations of a two-layer shear flow grow over time into billows and eventually generate vertical mixing. His results show that although the wavelength and period of the critical wave differ from the classical problem, the outcome of the boundary-value problem is the same as for the initial-value configuration. In the last years, Guillet et al. (2007) considered the case of laminar mixed convection flow between vertical parallel plates heated uniformly, they used a method based on the center manifold theorem to reduce the Navier-Stokes equations to ordinary differential equations in the vicinity of a trivial stationary solution and pointed out that when the forcing parameter or Rayleigh number increases beyond a critical value, the stationary solution is a pitchfork bifurcation point of the system. In the same year, Bera and Khalili (2007) used the linear theory of stability analysis to study numerically the impact of permeability on the stability of a buoyancy-opposed mixed convection in a vertical channel. They found that two main instability modes (Rayleigh-Taylor and buoyant instability) appear, and that for Darcy numbers $\leq 10^{-9}$, the Rayleigh-Taylor instability dominates within the entire Reynolds number range considered. In addition, they also found that for the same Re , the fully developed base flow is highly unstable (stable) for porous media with high (low) permeability, whilst the introduction of a one order of reduction in the permeability in the main flow direction made the system approximately twenty times more stable.

Although the oscillatory behavior of Navier-Stokes type systems in mixed convection has received relatively little attention, it is known that these flows can exhibit interesting dynamical phenomena. In many cases, the transition to turbulence is a chain of oscillatory states separated by bifurcations of different types. Examples where the bifurcation structure of such systems has been studied follow.

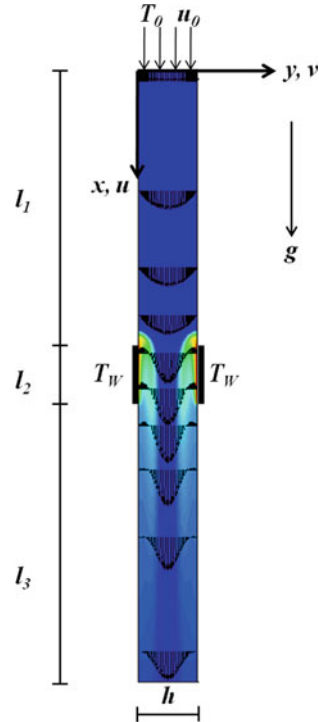
Meron and Procaccia (1987) showed that in dynamical systems described by critical flows, the onset of chaos is via gluing bifurcations, and that these systems can be analyzed using discontinuous maps of the interval. The gluing bifurcation is a class of global bifurcations where, as an external control parameter is varied, two symmetrically related time-periodic states simultaneously become homoclinic to an unstable saddle state and result in a single symmetric time-periodic state (Epstein and Pojman 1998; Ambruster et al. 1996; Abshagen et al. 2001). Arneodo et al. (1981) performed a study for one route to chaos via a cascade of bifurcations involving homoclinic orbits. Rucklidge (1993) described the transition to chaos through gluing processes in a three-dimensional magnetoconvection model. Marques et al. (2001) studied numerically a one-dimensional route in parameter space of a periodically forced flow with symmetry and provided a comprehensive analysis of the route to chaos, which involves a new and convoluted symmetry breaking that includes heteroclinic, homoclinic and gluing bifurcations. Lopez and Marques (2000) obtained three-tori solutions of the Navier-Stokes equations and their dynamics by use of a global Poincaré map. Their results show that these solutions undergo global bifurcations that include a new gluing bifurcation associated with homoclinic and heteroclinic connections to unstable solutions (two-tori), that act as organizing centers for the three-tori dynamics.

The foregoing survey of literature reveals that there are relatively few studies that address the thermal and flow stability in internal mixed convection for opposing flow in situations where flow reversal occurs. In the present study, a detailed numerical investigation is done of the symmetry breaking instability for laminar opposing mixed convection flow in a vertical channel of finite length subjected to isothermal and discrete heat inputs. For fixed Reynolds, Prandtl and geometry, the stability characteristics are obtained for increasing values of the buoyancy parameter.

2 Problem Description

The loss of stability is studied for a symmetrical counter-current mixed convection problem inside a vertical channel of finite length with a flat velocity distribution at the channel entrance, with the channel walls heated discretely and symmetrically. The schematic view of the geometry considered is shown in Fig. 1. The forced flow is driven by gravitational force acting vertically downward, entering the duct with a uniform velocity u_0 and ambient temperature T_0 . Axial distances from the entrance section are measured by the x coordinate (positive downward), while transverse distances are measured by y ($y = 0$ at the left wall). Both walls, separated by a distance h have discrete heat sources of length l_2 located at $x = l_1$, with uniform wall temperature T_w , where $T_w > T_0$. All other surfaces of the channel walls are assumed adiabatic insulators. Flow rectifiers are placed at the channel entrance and exit, thus producing a parallel flow at $x = 0$ and $x = l_1 + l_2 + l_3$. The viscous dissipation in the energy equation is neglected and the thermophysical properties of the fluid are assumed to be constant except for the density in the buoyancy term, which

Fig. 1 Schematic representation of the channel



is treated according to the Boussinesq approximation. By the use of the vorticity ($\Omega = \partial V/\partial X - \partial U/\partial Y$) and stream function formulation ($U = \partial\psi/\partial Y$, $V = -\partial\psi/\partial X$), the flow is described by the nondimensional equations

$$\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} = -\Omega, \quad (1)$$

$$\frac{\partial\Omega}{\partial\tau} + \frac{\partial\psi}{\partial Y} \frac{\partial\Omega}{\partial X} - \frac{\partial\psi}{\partial X} \frac{\partial\Omega}{\partial Y} = \frac{1}{Re} \left(\frac{\partial^2\Omega}{\partial X^2} + \frac{\partial^2\Omega}{\partial Y^2} \right) - Ri \frac{\partial\theta}{\partial Y}, \quad (2)$$

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\psi}{\partial Y} \frac{\partial\theta}{\partial X} - \frac{\partial\psi}{\partial X} \frac{\partial\theta}{\partial Y} = \frac{1}{RePr} \left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} \right). \quad (3)$$

where $\mathbf{V} = (U, V)$ is the dimensionless velocity vector and θ is the dimensionless temperature. In the above equations, all velocity components (U in the X -direction and V in the Y -direction) are scaled with the inflow velocity, u_0 , $U = u/u_0$ and $V = v/u_0$; the longitudinal coordinates are scaled with the channel width h , $X = x/h$ and $Y = y/h$; the time is scaled with the residence time h/u_0 , $\tau = tu_0/h$; the temperature is normalized as $\theta = (T - T_0)/(T_w - T_0)$. The non-dimensional parameters

appearing in the above equations are the Reynolds number, $Re = u_0 h / \nu$ (inertial to viscous forces), the Prandtl number, $Pr = \nu / \alpha$ (viscous to thermal diffusivities) and the Richardson number, $Ri = g \beta (T_w - T_0) h / u_0^2$ (buoyancy induced potential to forced kinetic energy). Here, g is the gravity acceleration and β is the thermal expansion coefficient. Another buoyancy parameter employed frequently is the Grashof number, $Gr = Ri Re^2$, which relates buoyancy to viscous forces. Additional non-dimensional geometrical parameters arise through the boundary conditions, $L_i = l_i / h$, with $i = 1, 2, 3$. Equations (1–3) have to be solved with the following boundary conditions. Uniform flow at the channel entrance: $\psi(0, Y) - Y = \Omega(0, Y) = 0$, no slip at the walls, $\psi(X, 0) = \Omega(X, 0) - 2(\psi(X, 0) - \psi(X, \Delta Y)) / \Delta Y^2 = 0$, $\psi(X, 1) - 1 = \Omega(X, 1) - 2(\psi(X, 1) - \psi(X, 1 - \Delta Y)) / \Delta Y^2 = 0$, and relaxed parallel flow conditions at the channel exit: $\partial\psi / \partial X|_{X=L} = \partial^2\psi / \partial X \partial Y|_{X=L} = 0$, where L is the total length of the channel, $L = L_1 + L_2 + L_3$. The boundary conditions for temperature are the following. Fixed temperature at the heated sections $\theta = 1$ at $Y = Y_w$ at $L_1 \leq X \leq L_1 + L_2$. Adiabatic channel walls are considered, $\partial\theta / \partial Y = 0$ except at $L_1 < X < L_1 + L_2$ and $Y = Y_w$. The dynamical properties of the system are described using the average non-dimensional heat fluxes or Nusselt numbers at both heated plates, $Nu_{L,R} = |\bar{q}_{L,R}| h / (k(T_w - T_0))$ and the non-dimensional first moment of the longitudinal velocity, $Y_p = (1/h^2 u_0) \int_0^h y u dy$. Here k is the thermal conductivity of the fluid. $Nu_{L,R}$ depend only on time and Y_p is a function of the longitudinal position and time. Due to baroclinicity -last term of Eq. (2)-, vorticity is produced and vortices (large recirculation bubbles) are generated. The position of the recirculation zones are represented by a stagnation point at $X = X_s(\tau)$, defined by the maximum value of X , where the longitudinal velocity component is non-negative in the vortex region. Eqs. (1–3) are numerically solved using a strongly non-uniform staggered grid system with a denser clustering near the heated plate. The technique employed has been described elsewhere Mart?nez-Su?stegui et al. (2011). In this work the Reynolds number is $Re = 100$, with $l = 12h$. The length of the heated slabs is $l_2 = h$, and they are located at $x = l_1 = 5.5h$, that is $l_1 = l_3 = 5.5h$. The non-dimensional time step $\Delta\tau = u_0 \Delta t / h$ has been set to 5×10^{-4} , and computation is terminated when the time evolution of the system state reaches steady-state or a final self-sustained oscillatory state.

3 Numerical Solution

After switching buoyancy on and for relatively small values of the Richardson number, the flow reverses close to the heated slabs and a pair of symmetric vortices develop due to baroclinicity. Both vortices reach a maximum position represented by the stagnation point, which decreases (higher positions) as the buoyancy parameter increases. There is an equilibrium between buoyancy in the hot recirculation bubble and the dynamical pressure and drag from the cold downward fluid. For a range of values of the Richardson number of $Ri < 5.2$, the transient response leads to a

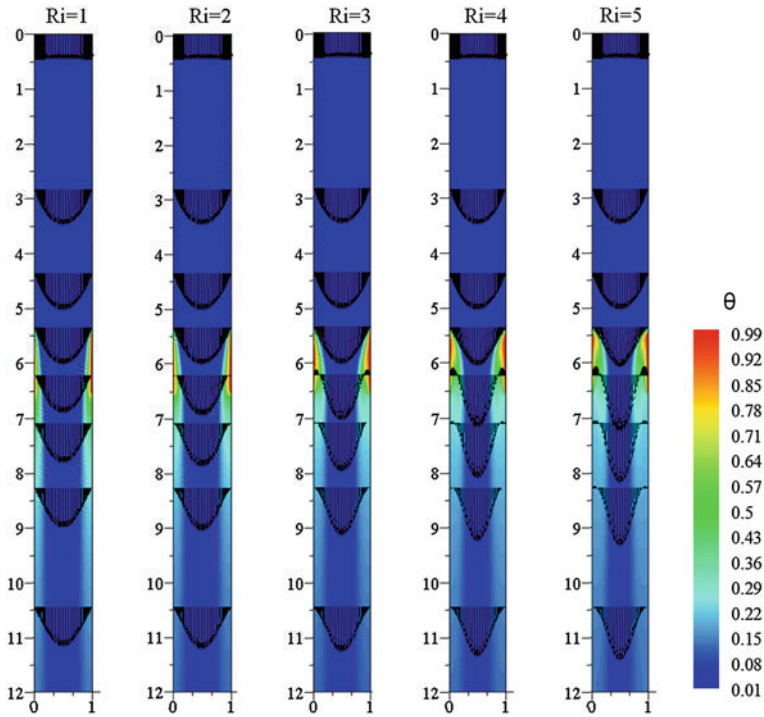


Fig. 2 Final states for a steady and symmetrical dynamical response of the system

symmetrical steady state solution. For these relatively low Richardson numbers there is only a weak interaction in the dynamics of both recirculation bubbles. Figure 2 shows the velocity and temperature profiles after the dynamical system has reached a symmetrical steady state. For this range of the buoyancy parameter, the final states of the system correspond to steady symmetric flow. As the Richardson number increases, small amplitude flow oscillations appear in the downstream region of the channel and their amplitude increase for increasing values of the buoyancy parameter. These flow oscillations are a manifestation of the Kelvin-Helmholtz instability due to the strong shear between the ascending hot fluid and the descending cold fluid located in the middle of the channel. This triggers a symmetry-breaking bifurcation and one of the two vortices (assumed to be close to the left heated surface, for simplicity) climbs while the other is pushed down by the downward flow with increased longitudinal momentum. If one vortex climbs, the cold fluid deflects to the other side of the channel increasing the longitudinal velocity and thus the momentum, pushing down the other vortex. As the fluid reaches the top of the lower vortex, the cold downward fluid switches sides again, thus supporting the upper recirculation bubble. Crude numerical simulations show that the symmetry breaking bifurcation occurs for a Richardson number of $Ri = 5.2$. As a result, a stable non-symmetric pattern develops after a relatively short transient. Figure 3 shows the resulting evolution of

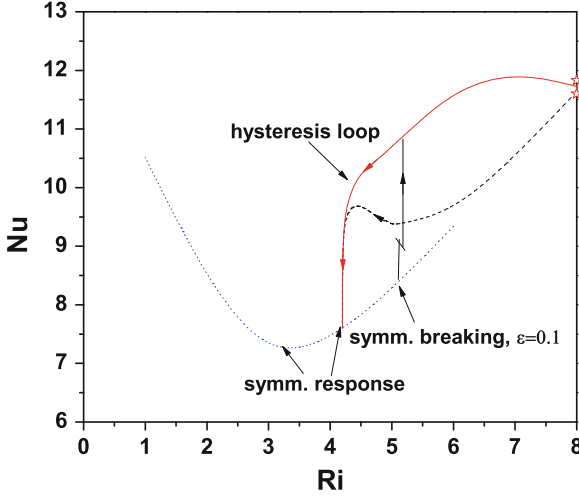


Fig. 3 Hysteresis behavior for $Re = 100$ and $Pr = 7$

the overall Nusselt number for the specific case of a Reynolds number of 100 and a Prandtl number of 7. As the Richardson number increases, the Nusselt number first decreases and later increases. This behavior is due to the change of the flow direction close to the wall. For a Richardson number close to 5.2 symmetry breaks and one of the recirculation bubble climbs while the other is pushed down, producing two solution branches for both hot surfaces. If now, the Richardson number decreases, the response continues to be asymmetric for values down to 4.1, where suddenly the symmetric behavior is recovered. This hysteresis loop is a clear indication of a subcritical bifurcation process to be analyzed below.

4 Linear Stability Analysis

For a given fixed Reynolds number and small values of the Richardson number, a symmetrical flow response is obtained. In order to study the stability of the symmetrical flow, a symmetrical strongly non-parallel base flow with variables denoted by ψ_0 , Ω_0 and θ_0 as functions of the Richardson number, is numerically generated, given by Eqs. (1–3), but with the following symmetrical boundary conditions: Uniform flow at the channel entrance: $\psi_0(0, Y) - Y = \Omega_0(0, Y) = \theta_0(0, Y) = 0$; no slip at the wall, $\psi_0(X, 0) = \Omega_0(X, 0) - 2(\psi_0(X, 0) - \psi_0(X, \Delta Y))/\Delta Y^2 = 0$; symmetry conditions at the symmetry plane ($Y = 0.5$), $\psi_0(X, 0.5) - 0.5 = \Omega_0(X, 0.5) = \partial\theta_0/\partial Y = 0$ and relaxed parallel flow conditions at the channel exit: $\partial\psi_0/\partial X|_{X=L} = \partial^2\psi_0/\partial X\partial Y|_{X=L} = 0$. A solution of the form $\Omega = \sum_{i=0}^{\infty} \delta^i \Omega_i$, $\psi = \sum_{i=0}^{\infty} \delta^i \psi_i$ and $\theta = \sum_{i=0}^{\infty} \delta^i \theta_i$, is assumed, with $\delta \rightarrow 0$. The linearized first order equations are