Scottish Graduate Series

Robert Thomson
Christopher Leburn
Derryck Reid *Editors*

Ultrafast Nonlinear Optics

Springer

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Preface

The Scottish Universities Summer Schools in Physics (SUSSP) was established in 1960, and since then there have been 68 schools (up to the end of 2011). A quick glance at the list of past schools indicates just how wide ranging the SUSSP school topics have been, and represents the breadth of research in Physics which continues to be conducted in Scottish Universities.

The 66th school in the SUSSP series (SUSSP66) was held over 10 days at Heriot-Watt University, Edinburgh, Scotland, between the 11 and 21 of August 2010. The topic of the school was the broad area of "Ultrafast Nonlinear Optics", and it consisted of lectures from 14 renowned international experts in this highly research active area. This book consists of 13 contributed chapters, each of which is either authored or co-authored by one or more of the SUSSP66 lecturers or executive committee members.

The field of Ultrafast Nonlinear Optics is broad and multidisciplinary, and encompasses areas concerned with both the generation and measurement of ultrashort pulses of light, as well as those concerned with the applications of such pulses. Ultrashort pulses are extreme events – both in terms of their durations, and also the high peak powers which their short durations can facilitate. These extreme properties make them powerful experimental tools. On one hand, their ultrashort durations facilitate the probing and manipulation of matter on incredibly short timescales. On the other, their ultrashort durations can facilitate high peak powers which can drive highly nonlinear light-matter interaction processes. The chapters contained within this book cover a complete range of topics, both applied and fundamental in nature, within the area of Ultrafast Nonlinear Optics.


The soliton reenactment team

Including lecturers, guest lecturers, organisers and students, SUSSP66 attracted 133 participants from 28 countries. This included 14 lecturers, 4 guest lecturers and 115 students. Over the 10 working days of the school, there were 42 lectures, 1 computer laboratory based tutorial session, 2 panel discussions (1 industry focused, and 1 on future directions) and 2 lively poster sessions where students presented 82 posters.

In addition to the academic-related activities, the packed social programme also formed an important and highly enjoyable part of the school. Students were invited to take part in various activities, such as a trip to the Edinburgh Military Tattoo, a coach tour to the Scottish Highlands, a guided scientific history walk round Edinburgh which culminated in a well-deserved dram at the top of Arthur's seat, hiking in the Pentland hills, a Scottish Ceilidh and a banquet to finish the school. The students also organised a number of social events themselves – including a commendable attempt by a small band of enthusiastic students to reenact the first observation of a soliton – made by John Scott Russell on the Union Canal nearby the Heriot-Watt University Riccarton Campus (see picture above). The SUSSP66 executive committee sincerely thank Ruth Livingstone and Tobi Lamour for coordinating the social programme – their considerable effort was a key to its success. The executive committee also thank the numerous post-graduate students from the Physics Department at Heriot-Watt University for helping with the social events.

The executive committee are also extremely grateful to the SUSSP66 sponsors: the Scottish Universities Physics Alliance (SUPA), the UK Engineering and Physical Sciences Research Council (EPSRC), the European Physical Society (EPS), the Institute of Physics (IOP) – Quantum Information, Quantum Optics and Quantum Control group, the IOP – Quantum Electronics and Photonics group, the Atomic Weapons Establishment (AWE), Innolume, Venteon, Toptica, Thorlabs, Philips, Coherent, Molecular Machines and Industry (MMI), the James Watt Institute for High Value Manufacturing, Elliot Scientific, Stratton Technologies, Time-Bandwidth, M-Squared Lasers, the Royal Society of Edinburgh, Newport, Spectra-Physics, the European Office of Aerospace Research and Development, the Air Force Office of Scientific Research, the United States Air Force Research Laboratory, the Scotland-Stanford Universities Partnership (SU2P), the Scottish Universities Summer Schools in Physics (SUSSP), the Scottish chapter of the IEEE Photonics Society, Selex-Galileo, Taylor and Francis, Fastlite, Laser Quantum, the Optical Society of America (OSA), Fast-Dot and Alcatel-Thales.

The executive committee hope that this book will act as part of a lasting legacy of an extremely interesting and fulfilling school, where participants not only expanded their knowledge, but also formed lasting friendships and networks.

Edinburgh, January 2012 Robert R. Thomson

Editors' Note

The chapters contained in the book are based on the lectures given by the lecturers at SUSSP66. The chapters are aimed at graduate-student level and are intended to provide the student with an accessible, self-contained and comprehensive gateway into each subject. Chapters 1, 2, 3, and 4 are concerned with the generation and measurement of ultrashort pulses. Chapters 5, 6, and 7 are concerned with fundamental applications of ultrashort pulses in metrology and quantum control. Chapters 8 and 9 are concerned with ultrafast nonlinear optics in optical fibres. Chapters 10, 11, 12, and 13 are concerned with the applications of ultrashort pulses in areas such as particle acceleration, microscopy and micromachining. The editors sincerely thank the authors for their excellent and timely contributions. Matthew Edmonds is acknowledged and thanked by the editors for his help in proofreading a number of chapters.

Contents

Chapter 1
Measuring Ultrashort Optical Pulses

Adam S. Wyatt and Ian A. Walmsley

1.1 Introduction

Modern laser and laser-driven sources can generate light pulses of unprecedented brevity, with durations in the range of picoseconds to attoseconds. Such durations are significantly shorter than any photodetector response time. Further, there is a need for more information about the pulse than the temporal intensity profile obtained from a simple photodetector. Sophisticated applications, such as coherent control of atomic and molecular dynamics demand a detailed knowledge of the electric field of the pulse, and not merely its duration [1].

The need for metrology has increased along with the development of new sources and their application in a wide range of new fields. Of course, the need to determine the pulse duration remains a primary application, both because this parameter is an important specification of the laser output needed for other applications, and because it acts as a diagnostic of the system operation.

Modern mode-locked lasers, for example, generate pulses with spectral bandwidths exceeding one octave and with durations below 10 fs, well beyond anything that can be characterized by means of fast photodetectors. The operation of such lasers relies on a complex combination of linear pulse propagation, influenced by the chromatic dispersion of the laser material, the mirrors and the intra-cavity dispersion compensating devices, together with nonlinear effects, such as self-phase modulation of the pulse in the laser material or by saturation of an intracavity absorption, such as in a semiconductor saturable absorber mirror (SESAM), as well as, in some cases, space-time coupling. The optimization of a mode-locked laser is made practicable by means of a diagnostic providing the electric field as a

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function of time or frequency, or at least providing some temporal information such as the second order intensity autocorrelation. One of the primary limits at present to the generation of few-cycle pulses directly from a laser is the dispersion of the intracavity mirrors and other optical elements. Historically, detailed measurements of the laser output were able to identify this as a major obstacle to generating shorter pulses [2].

Chirped pulse amplification (CPA) operates by lowering the peak power of the pulses in the amplifier gain medium, which would otherwise induce non-linear phase distortion of the pulse [3, 4]. To achieve this, the pulses are stretched in time by means of a dispersive delay line, often based on angular dispersion from diffraction gratings or prisms. After amplification, the pulse is temporally recompressed using an “inverse” dispersive delay line, or compressor, that compensates the dispersion introduced by the stretcher and the propagation through the other amplifier elements. Obtaining peak performance from such a scheme requires a reliable and rapid method to characterize the output. Accurate characterization of the output pulses enables the optimization of the parameters of the system, such as the distance between the two gratings of a compressor and the angle of incidence of the input beam on the gratings. The usual optimization parameters in such an application are the duration of the recompressed pulses, since the peak power scales like the ratio of the energy per pulse to the duration, and the temporal contrast, since pre-pulses can hinder the control or observation of the physical processes of interest, for example the ionisation of a target. Some examples of this application can be found in [5, 6]. The spectral phase of the output pulse from a Ti:sapphire CPA system can be used directly as the basis for a controller to optimize the compressor to minimize the pulse duration, for example. The compressor optimization consists of adjusting the angle of diffraction gratings relative to the input beam and the relative distance between the two gratings. A large cubic spectral phase, for example, gives rise to significant pre-pulses, and the compressor optimization leads to a better pulse shape with a higher intensity.

The bandwidth of an optical pulse can be increased while maintaining a deterministic phase relation between different spectral components by means of various nonlinear optical processes such as self-phase modulation and harmonic generation. All of these require careful compensation of the spectral phase in order to lead to an output pulse with a shorter duration than the input. Further, these processes are dynamically complicated and sensitive to the details of the input pulse shape. Therefore, even characterizing the raw output pulse before recompression can be a difficult task.

Shaped pulses, sometimes of a quite complex temporal structure, are now commonly used to both probe and manipulate fundamental processes in atoms and molecules (see Chap. 5 by Bucksbaum). For instance, the study of primary processes in biologically-relevant systems via ultrafast microscopy is now quite common. The details of the pulse shapes usually contain important information about the dynamical process under study, and this information, residing in both the temporal amplitude and the temporal phase of the field, can only be extracted using modern techniques of metrology. For example, the important phenomenon of the self-action of intense optical pulses in nonlinear media gives rise to a complicated set of dynamics that has analogues in many branches of physics. The study of the changes in the shapes of pulses propagating through such media provides access to these dynamics.

1.2 General Considerations

An electromagnetic pulse may be specified by its electric field alone, at least below intensities that give rise to fields that will accelerate electrons to relativistic energies. Thus a useful notation is that of the *analytic signal*, whose amplitude and phase we seek to determine via measurement. The (real) electric field of the pulse is given in terms of the analytic signal by Eq. (1.1).

$$E(t) = \varepsilon(t) + \varepsilon^*(t) \tag{1.1}$$

where $\varepsilon(t)$ is an analytic function of time (and space, although we suppress other arguments here for clarity). The signal $\varepsilon(t)$ is taken to have compact support in the domain $(-T, T)$, and we shall refer to it henceforth as the "field of the ultrashort pulse".

The spectrum of the pulse is then defined by the Fourier transform (Eq. (1.2))

$$\tilde{\varepsilon}(\omega) = \int_{-T}^{T} dt\ \varepsilon(t)\ e^{i\omega t}, \tag{1.2}$$

so that $\tilde{E}(\omega) = \tilde{\varepsilon}(\omega) + \tilde{\varepsilon}^*(-\omega)$. Note that $\tilde{\varepsilon}(\omega)$ contains only positive frequency components, since $\varepsilon(t) = \int_0^\infty dt\ \tilde{\varepsilon}(\omega)e^{-i\omega t}$. This is therefore a reasonable description for the fields of pulses propagating in charge-free regions of space, for which the pulse area, $\Theta(T) = \int_{-T}^{T} dt\ \varepsilon(t) = \tilde{\varepsilon}(\omega = 0)$ must be zero.

A single pulse is said to be completely characterized if the function $\varepsilon(t)$ is known on the domain $(-T, T)$. In practice one usually adopts the approximation that the pulse is also characterized by the function $\tilde{\varepsilon}(\omega)$ on the domain $(-\Omega, \Omega)$, where $\Omega >> 1/\tau$ with τ the rms pulse duration. The sampling theorem prevents a function from having compact support in both domains, but it is usually a reasonable approximation to truncate the spectral function at large frequencies, where the spectral energy falls below the noise level of the detector.

The analytic signal is complex and therefore can be expressed uniquely in terms of an amplitude and phase

$$E(t) = |E(t)| \exp[i\phi_t(t)] \exp(i\phi_0) \exp(-i\omega_o t), \tag{1.3}$$

Where $|E(t)|$ is referred to as the time-dependent envelope, ω_0 is the carrier frequency (usually chosen near the centre of the pulse spectrum), $\phi_t(t)$ is the time-dependent phase, and ϕ_0 a constant, known as the carrier-envelope offset (CEO) phase. The square of the envelope, $I(t) = |E(t)|^2$, is the time-dependent instantaneous power of the pulse which can be measured if a detector of sufficient electronic bandwidth is available. The derivative of the time-dependent phase accounts for the occurrence of different frequencies at different times, i.e. $\Omega(t) = -\frac{\partial\phi_t}{\partial t}$ is the instantaneous frequency of the pulse that describes the oscillations of the electric field around that time. The frequency representation of the analytic signal

$$\tilde{E}(\omega) = \left|\tilde{E}(\omega)\right| \exp\left[i\phi_\omega(\omega)\right] = \int_{-T}^{T} dt\, E(t)\, e^{i\omega t}, \tag{1.4}$$

can be decomposed similarly, so that $\left|\tilde{E}(\omega)\right|$ is the spectral amplitude and $\phi_\omega(\omega)$ is the spectral phase. The square of the spectral amplitude, $\tilde{I}(\omega) = \left|\tilde{E}(\omega)\right|^2$, is the spectral intensity (strictly speaking this quantity is the spectral density – the quantity measured in the familiar way by means of a spectrometer followed by a photodetector). The spectral phase describes the relative phases of the optical frequencies composing the pulse, and its derivative $\frac{\partial\phi_\omega}{\partial\omega}$ is the group delay $T(\omega)$ at the corresponding frequency, i.e. the time of arrival of a subset of optical frequencies of the pulse around ω.

The necessary and sufficient conditions that must be satisfied by any method that provides a complete specification of an ultrashort pulses field can be found quite generally from a theory based on manipulating the pulses by means of linear filters. The fact that this is possible already implies that apparati based entirely on linear optical elements are capable of pulse characterization, something that was not appreciated until relatively recently [7]. In practice, many of the popular methods make use of nonlinear optical processes, but this is because it has proven difficult to construct linear filters of the correct character or response time, rather than for any fundamental reason.

The inversion protocols for extracting the pulse shape from measured data are also made clear by working with linear transformations, and allows a categorization of different methods, and the development of a catalogue of what is possible in principle. An important feature introduced by the use of nonlinear optics is that the inversion algorithms become more complicated. In some cases they remain deterministic, but in others an iterative search for a solution satisfying the twin constraints of the signal form and the data must be implemented. Thus the two major considerations in pulse characterization are the physical arrangement of the linear and nonlinear components and the inversion procedure [8].

The basic elements required for the complete characterization of optical pulses are quite simple: at least one fast shutter or phase modulator, a spectrometer or an element to temporally stretch the pulse via dispersion, and one or two beamsplitters.

One can think of all elements, except the beamsplitters, as two-port devices: a pulse enters at one port and exits at another. There may be ancillary ports for control signals, such as the timing signal for the shutter opening, for example, but these are essentially linear systems, in that the output pulse field scales linearly with the input pulse field. Thus the input/output relations for these devices are all of the kind

$$\varepsilon_o(t) = \int_{-T}^{T} dt' H\left(t, t'\right) \varepsilon_i\left(t'\right), \tag{1.5}$$

Where $\varepsilon(t)$ is the analytic signal (with subscripts i and o representing the input and output fields respectively), and $H(t, t')$ is the (linear, causal) response function of the device. We will specify the functional forms of the common linear filters given above in subsequent paragraphs.

The beamsplitter is a four-port device, having two input and two output ports. The input-output relations for this device are well known, and the main utility in pulse measurement applications is either in providing a means to generated a replica of a pulse (one input and two outputs) or to combine the unknown pulse with a reference pulse (two inputs and two outputs), or as elements of a interferometer in which phase to amplitude conversion takes place.

We take it that all detectors available have a response that is slow compared to the pulse itself, though they need not be integrating. For pulses with temporal structure of duration less than 100 fs or so, this is usually the case. The measured signal from an integrating detector is related to the incident field, for our purposes, via

$$S(f) = \int_{-T_R}^{T_R} dt' \left|\varepsilon_f\left(t'\right)\right|^2, \tag{1.6}$$

where T_R is the integration time of the detector apparatus.

Combining Eqs. (1.5) and (1.6) implies that the detector signal depends on the two-time correlation function of the field:

$$C\left(t, t''\right) = \varepsilon^*(t)\varepsilon\left(t''\right). \tag{1.7}$$

In general, the signal will be averaged over a train of pulses. If each pulse in the train is not identical then the root quantity characterizing the ensemble of pulses is

$$C\left(t, t''\right) = \left\langle \varepsilon^*(t)\varepsilon\left(t''\right)\right\rangle. \tag{1.8}$$

where the brackets indicate either a time average over the pulse train, or an ensemble average over repeated experiments. Eqs. (1.7) and (1.8) are identical only if each pulse in the train is identical with all others. Note that $C(t, t'')$ is not the same as the correlation function that is derived from the pulse spectral intensity $|\tilde{\varepsilon}(\omega)|^2$. In that case, the Fourier transform yields the reduced correlation

$C'(\tau) = \int_{-\infty}^{\infty} dt\ C(t, t+\tau)$. This obviously contains no more information than the spectrum itself, in contrast to $C(t, t'')$.

A key issue for all methods of pulse characterization is that it is assumed that Eq. (1.7) is the correct form of the correlation function. All inversion algorithms assume this. Of course, for single shot measurements, based on just one pulse from an ensemble, the assumption is necessarily valid. Further, single-shot methods can be used to determine whether the ensemble consists of identical pulses by repeated measurements on single realizations of the ensemble on individual pulses drawn from the train.

It is frequently productive to work with a variation of the correlation function that uses a two dimensional space of time and frequency – the chronocyclic phase space [9]. The intuitive concept of chirp (that is, time-dependent frequency in the pulse) can be most easily seen within this space. A particularly useful function in this regard is the chronocyclic Wigner function, defined as;

$$W(\omega, t) = \int_{-T}^{T} dt'\ C\left(t + \frac{t'}{2}, t - \frac{t'}{2}\right) e^{i\omega t'} \tag{1.9}$$

A particular feature of the Wigner representation is that the marginals of the distribution are the temporal and spectral intensities respectively

$$I(t) = |\varepsilon(t)|^2 = \int d\omega\ W(\omega, t)\,, \tag{1.10}$$

$$\tilde{I}(\omega) = |\tilde{\varepsilon}(\omega)|^2 = \int dt\ W(\omega, t)\,. \tag{1.11}$$

Note also that the Wigner function is sufficient to characterize both individual pulses and partially coherent pulse ensembles. However, the function is not in general positive definite, and cannot therefore be considered a probability distribution of the pulse field. Indeed negative Wigner functions characterize many of the complicated pulse shapes that are in current use in, say, quantum control. For example, a pair of phase-locked Gaussian pulses has a significant region of phase space where its Wigner function is negative. The restrictions on the pulse duration and bandwidth required by Fourier's theorem are inherent in the Wigner function, and there is a minimum area of the chronocyclic phase space that it may occupy.

Example Wigner functions for a number of simple pulse shapes are shown in Fig. 1.1. The concept of a time-dependent frequency or chirp is clearly visible in Figs. 1.1b, d, and the coherence between two separate pulses, that is a well-defined relative phase, is shown in Fig. 1.1c.

This representation sheds some light on the general form of measurements, since Eq. (1.5) may be written in terms of the Wigner representation of the pulse field and that of the measurement apparatus as an overlap integral

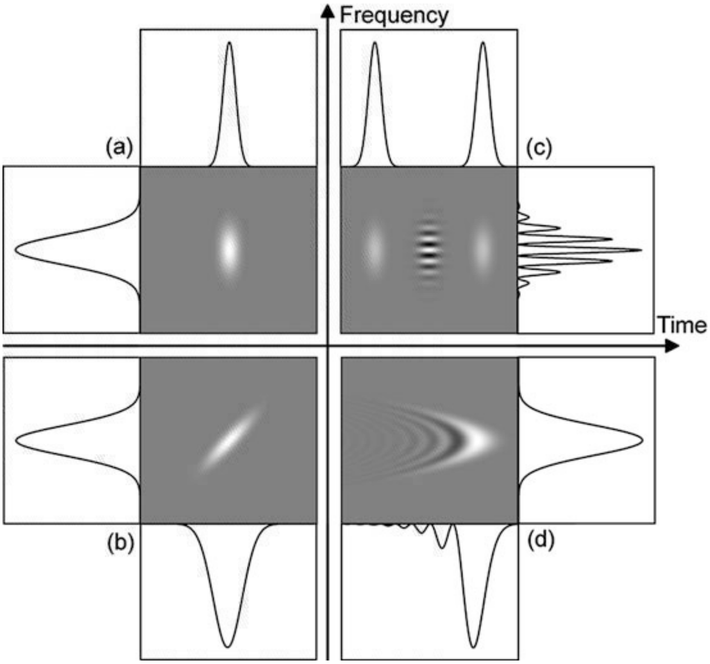

Fig. 1.1 Wigner functions of (**a**) a Fourier-transform limited Gaussian pulse, (**b**) a pulse with Gaussian spectrum and quadratic spectral phase, (**c**) a pair of identical Fourier-transform-limited Gaussian pulses, and (**d**) a pulse with Gaussian spectrum and third-order spectral phase. In each case, the temporal and spectral marginals are plotted

$$S\left(\Omega, T\right) = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} d\omega W\left(\omega, t\right) W_S\left(\omega, t; \Omega, T\right). \quad (1.12)$$

Where $W_S\left(\omega, t; \Omega, T\right)$ is the Wigner chronocyclic representation of the apparatus response function, with Ω and T representing parameters associated with the settings of the apparatus. For example, Ω might be the passband of a spectrometer, and T the time shift introduced by a delay line.

More generally, considering an apparatus characterized by a set of parameters $\{p_i\}$, with phase-space representation $W_S\left(\omega, t; \{p_i\}\right)$ then this function should be able, by suitable choices of the p_i, to explore all of the phase space occupied by the pulse. In this case the data $S\left(\{p_i\}\right)$ contains sufficient information to reconstruct the pulse field. Indeed, this is both a necessary and sufficient condition for characterizing the pulse. The apparatus function can be considered a "window" onto the chronocyclic phase space, through which the pulse itself can be seen.

The chronocyclic phase space may be explored in a number of ways. The three main approaches are: spectrography/sonography, tomography, and interferometry. In the first, the window function is a band-limited function whose representation is similar to that of the pulse shown in Fig. 1.1a. The parameters describing this function are W and T noted in the previous paragraph. The window function moves around the phase space as these parameters are adjusted: parallel to the time axis as T changes, and parallel to the frequency axis as W changes. At each location a "sample" of the field Wigner function is taken. The overlap integral in this case is a convolution, and the inversion algorithm is a deconvolution in two dimensions. Because these dimensions are complementary variables, the inversion is unique.

The second approach makes use of a window function that approximates a line in the phase space. The orientation of this line is adjusted by one parameter and the offset with respect to the origin by another. For example, the window function may be oriented parallel to the time axis, intersecting the frequency axis at Ω. The overlap integral determining the signal is therefore a projection of a "slice" of the pulse Wigner function $W(\omega, t)$ onto the frequency axis. As the orientation is changed about the intersection frequency, a different section of $W(\omega, t)$ is projected onto this frequency, thereby building up a set of spectra parameterised by the window function orientation, so $\{S_i\} \equiv S(\Omega; \{\theta_i\})$. This is called phase-space tomography, by analogy to the process used in medical diagnostics to assemble a 3-dimensional representation of an organ from a set of 2-dimensional projections.

The third approach makes use of an apparatus that shifts the pulse Wigner function in time or frequency (or some combination of the two) and then mixes it with the original, unshifted, version. As can be imagined from Fig. 1.1c, this gives rise to fringes that reveal the relative phase between different components of the pulse. The spectral or temporal phase can be read off from the position of these fringes, and this, together with the direct measurement of the pulse spectral or temporal intensity (the marginals of $W(\omega, t)$) gives the pulse field directly. This approach has the simplifying feature that the shifted pulse replica need not be moved around the phase space, since the interference property of Wigner representations enables the important phase information to be mapped into the amplitude domain.

These operations need to be implemented in laboratory apparatus. We may use the linear filter model to help delineate the necessary and sufficient conditions for such an apparatus. Linear filters are those for which the output field scales linearly with the input field. These filters modulate the pulse being measured and possess a characteristic response time of the order of the pulse itself, though not significantly shorter than it. They change the pulse in a way that is prescribed by an external signal, say the voltage applied to an electroabsorption modulator. Linear filters may be separated into two classes: those with time-stationary response functions and those with time-nonstationary responses. For the former class, which includes the spectrometer and dispersive delay line, the shape of the output pulse does not depend on which time the input pulse arrives. For the latter class, which includes the modulator and the shutter, the output pulse shape clearly depends on the timing of the input pulse with respect to the shutter opening or the modulator drive signal.

Time stationary filters are characterized by response functions of the form $H(t, t') = S(t - t')$, and non-stationary filters by $H(t, t') = N(t)\delta(t - t')$. Equivalently in the frequency domain, stationary filters take the general form

$\tilde{\tilde{H}}(\omega, \omega') = \tilde{S}(\omega)\,\delta(\omega - \omega')$, and nonstationary the form $\tilde{\tilde{H}}(\omega, \omega') = \tilde{N}(\omega - \omega')$, where the tilde represents a Fourier transform.

Representative response functions for the various common elements that facilitate analysis of all pulse measurement apparatuses, are:

$$\text{Shutter}: N^A(t) = e^{-(t-\tau)^2/\tau_g^2}, \tag{1.13a}$$

$$\text{Modulator}: N^P(t) = e^{-i\varphi(t-\tau)^2}, \tag{1.13b}$$

$$\text{Dispersive line}: \tilde{S}^P(\omega) = e^{i\varphi''(\omega-\omega_R)^2}, \tag{1.13c}$$

$$\text{Spectrometer}: \tilde{S}^A(\omega) = e^{-(\omega-\Omega)^2/\Gamma^2}, \tag{1.13d}$$

Spectrographic techniques make use of two sequential filters, one time-stationary (spectral filter) and one time-nonstationary (time gate) followed by a square-law detector. The recorded signal is either a measure of the spectrum of a series of time slices or a measure of the time of arrival of a series of spectral slices depending upon the ordering of the filters. There is no difference in principle between the two possible filter orderings and thus this type of apparatus should be thought of as one that makes simultaneous measurements of the conjugate variables rather than sequential measurements. The success of this approach has been extensively demonstrated in the technique of frequency resolved optical gating (FROG) [10].

Tomographic techniques require in-series time-stationary and time-nonstationary filters so that the entire phase-space can be explored. However, unlike spectrographic techniques, the first filter in a tomographic apparatus is a phase-only filter (either a quadratic temporal phase modulator or a quadratic spectral phase modulator). The inclusion of a quadratic phase-only filter results in a distinctly different interpretation of the measurement, leading to a fundamentally different inversion algorithm. To see this, notice that a phase-only filter does not provide any information on the frequency or the arrival time of a pulse ensemble and hence does not constitute a measurement of either frequency or time. So, a tomographic apparatus does not make a simultaneous measurement of these incompatible variables. Rather, the quadratic phase modulation acts to rotate the phase-space. The square-law detector in combination with the amplitude-only filter records the resulting intensity distribution. A sufficiently large number of phase-space rotations between $-\pi/2$ and $\pi/2$ allows in principle reconstruction of the Wigner function via the inverse Radon transform or of the ambiguity function via a set of inverse Fourier transforms, but such task has not been performed experimentally. However, the assumption that the pulse train is coherent reduces the requirements on the modulator considerably. In that case, a complete rotation of the phase space density is not necessary, and one can use two rotations with small angle (with one of them possibly being zero) to reconstruct the amplitude and phase of the field.

Interferometric techniques require only one slice of the correlation function (or equivalently, the Wigner function) to obtain the electric field amplitude and phase. Roughly speaking, if one wishes to reconstruct the field at N time points, then at least $2N$ independent data points are required. While interferometric techniques are capable of reconstructing the field by recording only the necessary $2N$ points, spectrography and tomography require the measurement of N^2 points. The acquisition of excess data is necessary to obtain a reliable estimate of the pulse shape. Of course, an overcomplete data set is available from direct measurement of the entire correlation function as well.

1.3 A Catalogue of Methods

1.3.1 Intensity Autocorrelation

The simplest technique for gathering at least moderate quantitative information about the temporal structure of an ultrashort pulse is the intensity autocorrelation. In a conventional autocorrelator, two pulse replicas are mixed in a nonlinear material, and the average power of a generated beam (measured with an integrating detector) is recorded as a function of the relative delay between the two test pulse replicas. By assuming a functional form for the temporal shape of the test pulse, one can estimate its duration from the autocorrelation trace. Because of its simplicity, autocorrelation is by far the most common method of "measuring" ultrashort optical pulses. However, the autocorrelation trace by itself provides little more than an estimate of the pulse duration.

The data consist of a one-dimensional array of numbers representing the output pulse energy as a function of the delay, represented here by the function $S_2(\tau)$. This is related to the input field by

$$S_2(\tau) = \frac{1}{T}\int_{-T}^{T} dt\ |\varepsilon(t)\varepsilon(t+\tau)|^2. \tag{1.14}$$

The autocorrelation yields directly a measure of the root-mean-square (rms) pulse duration through the relation:

$$\tau_{rms} = \frac{\int_{-\tau_{\max}}^{\tau_{\max}} d\tau\ \tau^2 S_2(\tau)}{\int_{-\tau_{\max}}^{\tau_{\max}} d\tau\ S_2(\tau)}. \tag{1.15}$$

However, the autocorrelation provides very little information about the temporal phase structure of the pulse, so from it alone, there is no way to determine whether all the frequencies of the pulse arrive at the same time or not. If they do, then the pulse is said to be "transform-limited", and has the shortest possible duration consistent with a given spectrum. The pulse duration obtained from the

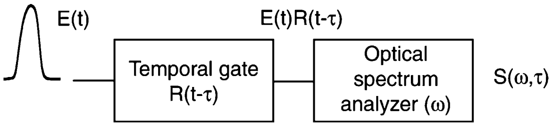

Fig. 1.2 Implementation of a spectrogram

autocorrelation combined with the bandwidth obtained from a measurement of the spectrum thus determines the proximity of the pulse to transform-limited duration. If the pulse is not transform-limited, then these measurements are insufficient to characterize the way in which the pulse is distorted. Thus there are two difficulties with inferring the pulse shape from autocorrelation-related measurements: the temporal intensity profile is not unique and the chirp cannot be determined [11].

1.3.2 Spectrograms

Spectrography is based on the sequential action of a time-stationary and time-nonstationary filter (Fig. 1.2). Under experimentally accessible conditions, the measured trace is exactly a spectrogram or a sonogram of the electric field under test, as can be calculated for signal representation in many other domains [12]. A typical implementation of spectrography uses a temporal gate for the signal under test (for example, the action of the pulse under test with one or several other pulses in a nonlinear optical medium [13], or a "shutter" function provided by a temporal modulator) and a device capable of measuring the optical spectrum (for example, an optical spectrum analyser based on a diffraction grating and imaging optics, or a scanning Fabry-Perot etalon, together with a photodiode whose time response is longer than the inverse bandwidth of the spectrometer itself). The spectrogram of the electric field of the test pulse is obtained by measuring the optical spectrum of the pulse after temporal gating for various relative delays between the pulse and the gate. The experimental trace is therefore:

$$S(\omega,\tau) = \left| \int E(t)R(t-\tau)\exp(i\omega t)dt \right|^2 \tag{1.16}$$

Where ω is the optical frequency and τ the relative delay between the gate and the test pulse. It is important that the resolution of the spectral filter is very high in order to ensure that the measured trace is effectively the spectrogram of the test pulse. A sonogram can be measured by reverting the order of the temporal and spectral gate [13, 14].

The spectrogram is the double convolution of the Wigner function of the pulse with the Wigner function of the gate with a change of sign on the frequency variable [12]:

$$S(\omega,\tau) = \iint W_E(t,\omega'')W_R(t-\tau,\omega-\omega'')dtd\omega'' \tag{1.17}$$

Thus, the pulse field may be estimated from a spectrogram by means of phase retrieval that implements a deconvolution [15]. In fact, this is the only option if the gate is unknown. The spectrogram of Eq. (1.16) is the modulus square of the short-time Fourier transform of the pulse. The trick in phase retrieval is to estimate the phase of the transform, since then, a Fourier transform would directly lead to the recovery of the pulse under test and additionally the gating function.

Phase retrieval is usually ambiguous in one dimension, but is usually unique in two dimensions. The excess data available in the spectrogram enables iterative reconstruction of N complex numbers specifying the field from the N^2 data points, and this can also lead to the simultaneous reconstruction of the gate [16, 17]. Furthermore, in the case of the nonlinear spectrogram, there is often a known functional relation between the pulse and the gate, since the gate is often implemented as a nonlinear interaction with replicas of the pulse under test. Also, other information might be available, such as the spectrum of the pulse or the transfer function of the gate. The recovery can be performed by means of several algorithms. A very robust approach is based on the Principal Component Generalised Projections Algorithm [18]. The protocol works as follows: from sampled representations of the field E_n and gate R_n, one calculates the complex "square root" spectrogram. The modulus of the calculated "square-root" spectrogram is replaced by the measured signal, while the reconstructed phase is kept. A new set of representations (E_{n+1}, R_{n+1}) is calculated by decomposing this constructed function into its singular values. The pair of vectors corresponding to the largest singular value is taken as the set (E_{n+1}, R_{n+1}). The convergence of the algorithm can be monitored by examining the difference between the measured spectrogram and the calculated one using, for example, the rms difference. Also, the consistency of the decomposition into an outer product can be quantified by considering the eigenvalues of the decomposition since, for a perfect decomposition, there is only one non-zero singular value. Moreover, the precision of the estimate of the field can be obtained from the distribution of the eigenvalues [19].

1.3.3 Interferograms

Interferometry is a well-known approach to the characterization of optical fields in the spatial domain. It is a simple method for converting phase information into amplitude information that can then be read using square-law detectors. A similar approach can be taken for the characterization of temporal fields. There are two general classes: test-plus-reference and self-referencing. The former requires a well-characterized reference pulse with spectral support across the entire bandwidth of the test pulse, and with similar temporal support. On the other hand, self-referencing interferometers can do without such an ancilla [20]. This is important, since possessing a well-characterized reference pulse suggests that the problem of measurement has already been solved.

A common means for characterizing spatial wavefronts is by spatial shearing interferometry, in which the spatial phase profile of a beam is determined by

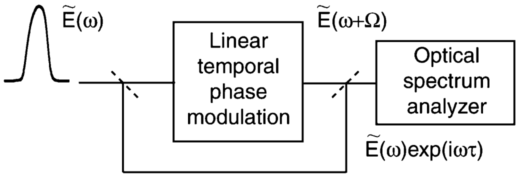

Fig. 1.3 A spectral shearing interferometer

interfering it with a laterally shifted (or sheared) replica. The resulting intensity interferogram can be measured with a square-law detector, and the phase simply extracted. The spectral analogue, in which two spectrally sheared pulses are interfered also allows direct reconstruction of the electric field in the spectral domain using the measured spectral phase and a pulse spectrum [21].

We will focus here on techniques that use the two-frequency correlation function $\tilde{E}(\omega)\tilde{E}^*(\omega-\Omega)$, the phase $\varphi(\omega)-\varphi(\omega-\Omega)$ of which can be concatenated or integrated to get the spectral phase of the initial pulse (note that the spectral intensity can be measured directly with an optical spectrum analyser). The spectral shear Ω is set by the sampling theorem, and it is typically a few percent of the total bandwidth of the pulse under test. Too large a shear would lead to undersampling of the pulse spectrum, while too small a shear could lead to increased sensitivity to noise, and thus reduced precision and in some circumstances reduced accuracy of the reconstruction. The spectral intensity can be obtained either from a separate measurement using the spectrometer, or can be extracted from the correlation function directly.

The quantity $\tilde{E}(\omega)\tilde{E}^*(\omega - \Omega)$ can be obtained by measuring the interference of the pulse under test with its sheared replica with an optical spectrum analyser (Fig. 1.3) The frequency shear Ω can be implemented for example using a linear temporal phase modulation $\exp(i\Omega t)$. The spectral intensity of the two interfering pulses is $\left|\tilde{E}(\omega)\right|^2 + \left|\tilde{E}(\omega - \Omega)\right|^2 + \tilde{E}(\omega)\tilde{E}^*(\omega - \Omega) + \tilde{E}^*(\omega)\tilde{E}(\omega - \Omega)$. The interferometric component of interest can be extracted from several measurements of the spectral density for various relative phases between the two interfering pulses. However, if a delay is introduced between the non-shifted and the shifted replica, this leads to spectral fringes with small spacing, by virtue of the phase $\varphi(\omega)-\varphi(\omega - \Omega) + \omega\tau$. In this case, the interferometric component can be directly extracted using Fourier processing of a single interferogram [22].

1.3.4 Tomograms

The spectrum of a light source is easy to measure experimentally. Mathematically it is the spectral projection of the Wigner function on the frequency axis. This is the key to understanding tomography. If the pulse Wigner function can be rotated in phase space, then a series of spectra measured for different rotation angles constitutes the complete data set needed for tomographic inversion of the pulse itself.