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Wade D. Cook
Joe Zhu *Editors*

Data Envelopment Analysis

A Handbook on the Modeling
of Internal Structures and Networks



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of Internal Structures and Networks

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In Memory of William Wager Cooper
(July 23, 1914–June 20, 2012)

Preface

Data envelopment analysis (DEA) is a linear programming based approach for measuring relative efficiencies or performances of peer decision making units (DMUs). The performance or efficiency of a DMU is expressed in terms of a set of measures which are classified or coined as DEA inputs and outputs. In conventional DEA, each DMU is treated as a black-box and its internal structures and operations are ignored.

With the publication of the 2nd edition of *Handbook on Data Envelopment Analysis* (eds, Cooper et al. 2011), DEA models for treating DMUs that have internal or network structures have been identified as being on the research frontier (see, for example, Cook and Seiford 2009, and Liu et al. 2013). In fact, there already exists a significant amount of research on both the theory and applications of the network DEA approach. A significant number of researchers and scholars have started to look into the internal structures of DMUs.

Färe and Grosskopf (1996) are the first to propose DEA models when inputs and outputs of DMUs form a network structure. Castelli et al. (2004) study several types of DMU internal structures and develop DEA-type models to measure the overall and component efficiencies. In a different line of research, Kao and Hwang (2008) and Liang et al. (2008) model a specific type of internal structure where DMUs are composed of a two-stage process, namely the output measures from the first stage become input measures to the second stage. Tone and Tsutsui (2009) develop slacks-based network DEA model. There are other variations or extensions to the above earlier work on network DEA models, depending on the particular DMU network structures. Some are based upon the DEA envelopment form and some on the DEA multiplier form.

The current handbook serves as a complement to the *Handbook on Data Envelopment Analysis* (eds, Cooper et al. 2011) in an effort to extend the frontier of DEA research. It provides a comprehensive source for the state-of-the art DEA modeling on internal structures and network DEA.

Chapter 1 by Cook and Zhu provides a survey on two-stage network performance decomposition and modeling techniques. Chapter 2 by Chen et al. discusses the pitfalls in network DEA modeling. The authors point out that caution should be paid when models are developed based upon the envelopment or multiplier forms, because the usual duality (or equivalence) between the DEA envelopment and multiplier linear models is no longer true. Chapter 3 by Kao discusses efficiency decompositions in network DEA under three types of structures, namely series, parallel, and dynamic.

Chapter 4 by Chen, Cook and Zhu studies the determination of the network DEA frontier. In Chap. 5 the same authors then discuss additive efficiency decomposition in network DEA. Kao and Hwang present an approach in scale efficiency measurement in two-stage networks in Chap. 6. Sahoo, Zhu and Tone further discuss the scale efficiency decomposition in two stage networks in Chap. 7.

Chapter 8 by Du et al. offers a bargaining game approach to modeling two-stage networks. Chen et al. in Chap. 9 study shared resources and efficiency decomposition in two-stage networks. Chapter 10 by Chen introduces an approach to computing the technical efficiency scores for a dynamic production network and its sub-processes.

In Chap. 11 Tone and Tsutsui present a slacks-based network DEA. Chapter 12 by Li et al. discusses a DEA modeling technique for a two-stage network process where the inputs of the second stage include both the outputs from the first stage and additional inputs to the second stage.

Chapter 13 by Golany, Hackman and Passy presents an efficiency measurement methodology for multi-stage production systems. Färe, Grosskopf, and Whittaker in Chap. 14 discuss network DEA models, both static and dynamic. The discussion also explores various useful objective functions that can be applied to the models to find the optimal allocation of resources for processes within the black box that are normally invisible to DEA. Chapter 15 by Castelli and Pesenti provides a comprehensive review of various types of network DEA modeling techniques.

In Chap. 16, Cook et al. present shared resources models for deriving aggregate measures of bank-branch performance, with accompanying component measures that make up that aggregate value.

In Chap. 17, Cook et al. examine a set of manufacturing plants operating under a single umbrella, with the objective being to use the component or function measures to decide what might be considered as each plant's core business.

Chapter 18 by Cook et al. considers problem settings where there may be clusters or groups of DMUs that form a hierarchy. The specific case of a set of electric power plants is examined in this context.

Chapter 19 by Fukuyama and Weber models bad outputs in two-stage network DEA. Chapter 20 by Lewis presents an application of network DEA to performance measurement of Major League Baseball (MLB) teams. Lu et al. in Chap. 21 present an application of a two-stage network DEA model for examining the performance of 30 U.S. airline companies. Chapter 22 by Triantis presents two distinct network efficiency models that are applied to engineering systems.

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Chapter 1

DEA for Two-Stage Networks: Efficiency Decompositions and Modeling Techniques

Wade D. Cook and Joe Zhu

Abstract Data envelopment analysis (DEA) is a method for identifying best practices among peer decision making units (DMUs). An important area of development in recent years has been that devoted to applications wherein DMUs represent network processes. One particular subset of such processes is those in which all the outputs from the first stage become inputs to the second stage. We call these types of DMU structures “two-stage networks”. Existing approaches in modeling efficiency of two-stage networks can be categorized as using either Stackelberg (leader-follower), or cooperative game concepts. There are two types of efficiency decomposition; multiplicative and additive. In multiplicative efficiency decomposition, the overall efficiency is defined as a product of the two individual stages’ efficiency scores, whereas in additive efficiency decomposition, the overall efficiency is defined as a weighted average of the two individual stages’ efficiency scores. We discuss modeling techniques used for solving two-stage network DEA models in linear programs.

Keywords Data envelopment analysis (DEA) • Efficiency • Decomposition • Game • Intermediate measure • Network • Cooperative • Two-stage

1.1 Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. (1978), is an approach for identifying best practices among peer decision making units (DMUs) in the presence of multiple inputs and outputs. In many cases DMUs may consist of

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two-stage network structures with intermediate measures. In other words, DMUs under evaluation share a common feature found in many two-stage network structures, namely that outputs from the first stage become the inputs to the second stage. We refer to these as intermediate measures. For example, Seiford and Zhu (1999) use a two-stage network structure to measure the profitability and marketability of US commercial banks. In their study, profitability is measured relative to labor and assets as inputs, and the outputs are profits and revenues. In the second stage, for marketability, the profits and revenue are then used as inputs, while market value, returns and earnings per share constitute the outputs. Zhu (2000) applies the same two-stage network structure to the Fortune Global 500 companies.

Seiford and Zhu (1999) use the standard DEA approach which does not address potential conflicts between the two stages arising from the intermediate measures. Namely, the second stage may have to reduce its inputs (intermediate measures) in order to achieve an 'efficient' status. Such an action would, however, imply a reduction in the first stage outputs, thereby reducing the efficiency of that stage.

Note that these types of DMUs have not only inputs and outputs, but also intermediate measures that flow from one stage to the other. Each stage may also have its own inputs and outputs. Recently, a number of studies have focused on DMUs that appear as two-stage processes. Kao and Hwang (2008) describe a two-stage process where 24 non-life insurance companies use operating and insurance expenses to generate premiums in the first stage, and then underwriting and investment profits in the second stage. Other examples include the impact of information technology use on bank branch performance (Chen and Zhu 2004), two stage Major League Baseball performance (Sexton and Lewis 2003), health care applications (Chilingerian and Sherman 2004), and many others.

Kao and Hwang (2008) define the overall efficiency of the DMU as the product of the efficiencies of the two stages. Such *multiplicative* efficiency decomposition is also studied in Liang et al. (2008), where three DEA models/efficiency decompositions are developed using game theory concepts. More recently, Chen et al. (2009b) present a methodology for representing overall radial efficiency of a DMU as an *additive* weighted average of the radial efficiencies of the individual stages or components that make up the DMU. Cook et al. (2010) extend the additive decomposition approach of Chen et al. (2009b) into more general network structures.

In a review study done by Cook et al. (2010), the authors classify various existing DEA models for measuring efficiency in the aforementioned two-stage network structures or processes. The models fall into four categories: standard DEA approach; efficiency decomposition approach; network DEA approach; and game theoretic approach. Except for the standard DEA approach, all other approaches attempt to correct for the above-referenced conflict issue existing between the two stages.

The rest of the chapter is organized as follows. Section 1.2 presents the generic two-stage process and a general literature review and classification of papers dealing with DMUs having such processes.

In Sects. 1.3 and 1.4, we discuss the efficiency decomposition methodology and game-theoretic approaches. We begin with the work by Liang et al. (2006) where DEA models are developed to measure the performance of supply chains with two members. In their study, because some of the inputs to the second stage are not from the first stage, one of the DEA models is non-linear. However, if we apply their approach to our two-stage processes, and use the overall efficiency definition from Kao and Hwang (2008), we can obtain linear DEA models as in Liang et al. (2008). This establishes the relationships among the works of Liang et al. (2006), Castelli et al. (2004), Kao and Hwang (2008) and Liang et al. (2008). These approaches are then re-categorized as (1.1) the centralized models of Kao and Hwang (2008) and Liang et al. (2008), and (1.2) the non-cooperative (or leader-follower) model. It is shown how to test for uniqueness of the efficiency decomposition.

We then proceed to the network DEA approach in Sect. 1.5. We show that the Kao and Hwang (2008) model and the centralized model of Liang et al. (2008) are equivalent to the network DEA approach of Färe and Grosskopf (1996). Note the fact that, as demonstrated in Chen et al. (2009a), Chen and Zhu's (2004) model under the CRS assumption is equivalent to the Kao and Hwang (2008) model. As a result, we establish the equivalence among these models in dealing with two-stage processes. We discuss as well the determination of the efficient frontier of the two-stage process. Since it is possible that no single DMU is efficient, the standard DEA projections can no longer be used to generate the frontier. See Chen et al. (2010a, 2013) on issues related to DEA frontier identification under network DEA models.

Section 1.6 presents a technique for solving non-linear network DEA models via linear programming problems. Such a technique is often used in additive efficiency decompositions (see, e.g., Liang et al. 2006, 2011, 2013). Section 1.7 discusses a two-stage network structure where outputs from the second stage can be fed back as inputs to the first stage (Liang et al. 2011). Conclusions appear in Sect. 1.8.

1.2 Classification of Network DEA Modeling

Consider a generic two-stage network structure or process as shown in Fig. 1.1. Using the notation of Chen and Zhu (2004), we assume each DMU_j ($j = 1, 2, \dots, n$) has m inputs x_{ij} , ($i = 1, 2, \dots, m$) to the first stage, and D outputs z_{dj} , ($d = 1, 2, \dots, D$)

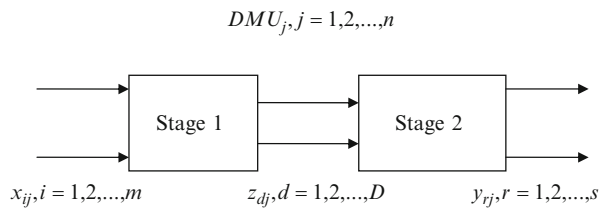


Fig. 1.1 Two-stage process

from that stage. These D outputs then become the inputs to the second stage and will be referred to as intermediate measures. The outputs from the second stage are y_{rj} , ($r = 1, 2, \dots, s$).

We denote the efficiency for the first stage as e_j^1 and second stage as e_j^2 , for each DMU_j . Using the Constant Returns to Scale (CRS) DEA model of Charnes et al. (1978), we define

$$e_j^1 = \frac{\sum_{d=1}^D w_d z_{dj}}{\sum_{i=1}^m v_i x_{ij}} \quad \text{and} \quad e_j^2 = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{d=1}^D \tilde{w}_d z_{dj}} \quad (1.1)$$

where v_i , w_d , $\tilde{w}_d$, and u_r are unknown non-negative weights. Note that w_d can be equal to $\tilde{w}_d$.

There are four types of papers that use various approaches to the modeling of efficiency of DMUs with two-stage processes. Some approaches are equivalent.

1.2.1 Standard DEA Methodology

The first type simply uses the standard DEA model. i.e. two separate DEA runs are applied to the two stages to calculate e_j^1 and e_j^2 , respectively. For example, Chilingirian and Sherman (2004) describe a two-stage process in measuring physician care. Their first stage is a manager-controlled process with inputs including registered nurses, medical supplies, and capital and fixed costs. These inputs generate the outputs or intermediate measures (inputs to the second stage), including patient days, quality of treatment, drugs dispensed, among others. The outputs of the second (physician controlled) stage include research grants, quality of patients, and quantity of individuals trained, by specialty. Other examples include Fortune 500 companies performance (Seiford and Zhu 1999; Zhu 2000). Similar to Seiford and Zhu (1999), Sexton and Lewis (2003) also use the standard DEA approach where in one of their standard DEA models, projected (efficient) intermediate measures are used in the second stage efficiency calculation.

However, as discussed earlier, such an approach does not treat z_{dj} in a coordinated manner. For example, suppose the first stage is DEA efficient and the second stage is not. When the second stage improves its performance, by reducing the inputs z_{dj} via an input-oriented DEA model, the reduced z_{dj} may render the first stage inefficient.

1.2.2 Efficiency Decomposition Methodology

It is useful to point out that given individual efficiency measures e_j^1 and e_j^2 , for stages 1 and 2, respectively, it is reasonable to define the efficiency of the overall two-stage process either as $\frac{1}{2}(e_j^1 + e_j^2)$ or $e_j^1 \bullet e_j^2$. If the input-oriented DEA model is used, then we should as well require that $e_j^1 \leq 1$ and $e_j^2 \leq 1$. The above definition ensures that the two-stage process is efficient if and only if $e_j^1 = e_j^2 = 1$.

If we define $e_j = \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}}$ as the two-stage overall efficiency, then we arrive at

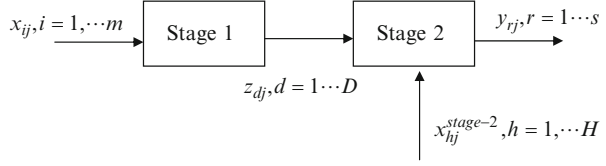
another type of research, as in Kao and Hwang (2008) who describe a two-stage process where 24 non-life insurance companies use operating and insurance expenses to generate premiums in the first stage, and then underwriting and investment profits in the second stage. As in Kao and Hwang (2008), we have $e_j = e_j^1 \bullet e_j^2$ at optimality provided we assume $w_d = \tilde{w}_d$. Note that such a decomposition of efficiency is not available in the standard DEA approach, and the network DEA approaches.

1.2.3 Network DEA

We point out that in these above examples, it is the case that the intermediate measures are the *only* inputs to the second stage, i.e., there are no additional independent inputs to that stage. There are, of course, other types of two-stage processes and even DMUs with network structures that may have inputs to the second stage in addition to the intermediate measures. In a more general situation than two-stage processes, Castelli et al. (2004) discuss DMUs with two-stage and two-layer structures. The network DEA approach of Färe and Whittaker (1995) and Färe and Grosskopf (1996), and the slacks-based network DEA approach of Tone and Tsutsui (2009, 2010) may involve more than two stages. Fukuyama and Weber (2010) considers a slacks-based measure for a two-stage process with bad outputs. More recently, Chen (2009) developed a network DEA model incorporating dynamic effects in production networks. A number of empirical studies have used this type of DEA technique, see, e.g., Avkiran (2009), and Yu and Lin (2008), among others. We call these network DEA approaches.

Similar network DEA approaches are used in two-stage processes described in Fig. 1.1. For example, Chen and Zhu (2004) study the impact of information technology use on bank branches performance (Wang et al. 1997). Under the assumption of variable returns to scale (VRS), Chen and Zhu (2004) and

Fig. 1.2 Two-stage process with additional inputs to the second stage



Chen et al. (2006) develop linear and non-linear models for measuring the impact of information technology on the firm performance via a two-stage process. However, their individual stage efficiency scores do not provide information on the overall performance and best-practices of the two-stage process.

1.2.4 Game-Theoretic Approaches

The fourth type of approach uses game theory concepts. It originates from the work of Liang et al. (2006) who use DEA to measure the performance of supply chains with two members (as in a manufacturer-retailer setting, for example). In Liang et al. (2006), the concepts of the Stackelberg game (or leader-follower) and the cooperative game are used to develop models for measuring performance in supply chain settings. We should point out that in their paper, the second stage(retailer) has not only the inputs from the first stage (manufacturer), but also its own inputs not linked with the first stage, i.e. additional inputs to the second stage are introduced (see, for example, Fig. 1.2 above). As a results,

$$e_j^2 = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{d=1}^D \tilde{w}_d z_{dj} + \sum_{h=1}^H Q_h x_{hj}^2}, \text{ where } x_{hj}^2 \text{ (} h = 1, \dots, H \text{) are inputs to the second stage}$$

that are not related to the first stage. In this case, it may be more convenient and tractable to express the overall efficiency as $\frac{1}{2} \left(e_j^1 + e_j^2 \right)$, since the alternative, namely $e_j^1 \bullet e_j^2$, results in a highly non-linear problem.

We note that their models can actually be directly applied to the two-stage process described in Fig. 1.1, since if there are no additional inputs x_{hj}^2 ($h = 1, \dots, H$), the structure of their two-member supply chain is identical to the two-stage process shown. Liang et al. (2008) provide detailed models for the two-stage process using the same modeling principle as in Liang et al. (2006).

While the current chapter focuses on the two-stage processes that have only the intermediate measures linking the stages, we will discuss the relations among DEA models for specific two-stage processes, and for the more general network structures.

1.3 Centralized Model

Liang et al. (2006) show that using the concept of cooperative game theory, or centralized control, the two stage process can be viewed as one where the stages jointly determine a set of optimal weights on the intermediate factors to maximize their efficiency scores. This would be the case in situations where the manufacturer and retailer jointly determine prices, order quantities, etc., to achieve maximum profit (Huang and Li 2001). In other words, the cooperative or centralized approach is characterized by letting $w_d = \tilde{w}_d$ in (1.1), and the efficiency scores of both stages are optimized simultaneously. The optimization can be based upon maximizing the average of e_o^1 and e_o^2 in a non-linear program as in Liang et al. (2006), Kao and Hwang (2008), and Liang et al. (2008). However, it is noted that because of the

assumption $w_d = \tilde{w}_d$ in (1.1), $e_o^1 \bullet e_o^2$ becomes $\frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}}$. Therefore, instead of maximizing the average of e_o^1 and e_o^2 , we have

$$e_o^{centralized} = \text{Max } e_o^1 \bullet e_o^2 = \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \quad (1.2)$$

$$\text{s.t. } e_j^1 < 1 \quad \text{and} \quad e_j^2 < 1 \quad \text{and} \quad w_d = \tilde{w}_d.$$

Model (1.2) can be converted into the following linear program format:

$$e_o^{centralized} = \text{Max } \sum_{r=1}^s u_r y_{ro}$$

$$\text{s.t. } \sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0 \quad j = 1, 2, \dots, n$$

$$\sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n$$

$$\sum_{i=1}^m v_i x_{io} = 1$$

$$w_d \geq 0, \quad d = 1, 2, \dots, D; \quad v_i \geq 0, \quad i = 1, 2, \dots, m; \quad u_r \geq 0, \quad r = 1, 2, \dots, s \quad (1.3)$$

Model (1.3) is the Kao and Hwang (2008) model and the centralized model developed in Liang et al. (2008). Note that constraints $\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$ are

redundant in Kao and Hwang's (2008) model, since $\sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0$ and $\sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0$ imply $\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$.

Model (1.3) gives the overall efficiency of the two-stage process. Assume the above model (1.3) yields a unique solution. We can then obtain

$$e_o^{1,centralized} = \frac{\sum_{d=1}^D w_d^* z_{do}}{\sum_{i=1}^m v_i^* x_{io}} = \sum_{d=1}^D w_d^* z_{do} \quad \text{and} \quad e_o^{2,centralized} = \frac{\sum_{r=1}^s u_r^* y_{ro}}{\sum_{d=1}^D w_d^* z_{do}} \quad (1.4)$$

as the efficiencies for the first and second stages, respectively. If we denote the optimal value to model (1.3) as $e_o^{centralized}$, then we have $e_o^{centralized} = e_o^{1,centralized} \bullet e_o^{2,centralized}$.

If only one layer is considered in the internal structure of Castelli et al. (2004), then the same above efficiency decomposition can be obtained. Therefore, the approaches of Castelli et al. (2004) and Kao and Hwang (2008) can be viewed as cooperative game models.

As noted in Kao and Hwang (2008), optimal multipliers from model (1.3) may not be unique. They propose deriving the maximum achievable value of $e_o^{1,centralized}$ or $e_o^{2,centralized}$. In fact, as shown in Liang et al. (2008), their models can also be used to test whether $e_o^{1,centralized}$ and $e_o^{2,centralized}$, obtained from model (1.3), are unique. The maximum achievable value of $e_o^{1,centralized}$ can be determined via

$$\begin{aligned} e_o^{1+} &= \text{Max} \sum_{d=1}^D w_d z_{do} \\ \text{s.t.} \quad &\sum_{r=1}^s u_r y_{ro} = e_o^{centralized} \\ &\sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\ &\sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0 \quad j = 1, 2, \dots, n \\ &\sum_{i=1}^m v_i x_{io} = 1 \\ &w_d \geq 0, \quad d = 1, 2, \dots, D; \quad v_i \geq 0, \quad i = 1, 2, \dots, m; \quad u_r \geq 0, \\ &\quad r = 1, 2, \dots, s \end{aligned} \quad (1.5)$$

This yields the minimum of $e_o^{2,centralized}$, namely, $e_o^{2-} = \frac{e_o^{centralized}}{e_o^{1+}}$. The maximum of $e_o^{2,centralized}$ can be calculated via the following linear program,

$$\begin{aligned}
 e_o^{2+} = & \text{Max} \sum_{r=1}^s u_r y_{ro} \\
 \text{s.t.} \quad & \sum_{r=1}^s u_r y_{rj} - e_o^{centralized} \bullet \sum_{i=1}^m v_i x_{io} = 0 \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0 \quad j = 1, 2, \dots, n \\
 & \sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
 & \sum_{d=1}^D w_d z_{do} = 1 \\
 & w_d \geq 0, \quad d = 1, 2, \dots, D; \quad v_i \geq 0, i = 1, 2, \dots, m; \quad u_r \geq 0, \quad r = 1, 2, \dots, s,
 \end{aligned} \tag{1.6}$$

and the minimum of $e_o^{1,centralized}$ is then calculated as $e_k^{1-} = e_o^{centralized} / e_o^{2+}$. Note that $e_o^{1-} = e_o^{1+}$ if and only if $e_o^{2-} = e_o^{2+}$. Note also if $e_o^{1-} = e_o^{1+}$ or $e_o^{2-} = e_o^{2+}$, then $e_o^{1,centralized}$ and $e_o^{2,centralized}$ are uniquely determined via model (1.3). If $e_o^{1-} \neq e_o^{1+}$ or $e_o^{2-} \neq e_o^{2+}$, Liang et al. (2008) develop a procedure to obtain an alternative decomposition of $e_o^{1,centralized}$ and $e_o^{2,centralized}$.

Table 1.1 presents data on 24 non-life insurance companies in Taiwan where there are two intermediate measures (Kao and Hwang 2008). The two inputs to the first stage (premium acquisition) are Operating expenses and Insurance expenses. The intermediate measures (or the outputs from the first stage) are Direct written premiums and Reinsurance premiums. The outputs of the second stage (profit generation) are Underwriting profit and Investment profit.

The efficiency scores for the two individual stages are calculated based upon (1.4) via a set of optimal solutions from model (1.3) (see the 2nd, 3rd and 4th columns of Table 1.2). Note that the efficiency decompositions are identical to those in Kao and Hwang (2008). In fact, the use of models (1.5) and (1.6) indicates that $e_o^{1-} = e_o^{1+}$ and $e_o^{2-} = e_o^{2+}$ for all the DMUs. Therefore, the $e_o^{1,centralized}$ and $e_o^{2,centralized}$ defined in (1.4), or the efficiency decompositions in Kao and Hwang (2008), are uniquely determined via model (1.3).

Table 1.1 Non-life insurance companies in Taiwan

DMU	Company	Operation expenses (x1)	Insurance expenses (x2)	Direct written premiums (z1)	Reinsurance premiums (z2)	Underwriting profit (y1)	Investment profit (y2)
1	Taiwan Fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
5	Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	Shing kong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14	South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	Allianz President	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
19	North America	159,422	182,338	1,141,950	483,291	519,121	46,857
20	Federal	145,442	53,518	316,829	131,920	355,624	26,537
21	Royal & Sunalliance	84,171	26,224	225,888	40,542	51,950	6,491
22	Asia	15,993	10,502	52,063	14,574	82,141	4,181
23	AXA	54,693	28,408	245,910	49,864	0.1	18,980
24	Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

Table 1.2 Results for non-life insurance companies in Taiwan

	Centralized model				Stage 1 as the leader			Stage 2 as the leader			
	$e_o^{I,Cooperative}$	$e_o^{2,Cooperative}$	$e_o^{Cooperative}$	$e_o^{Cooperative}$	$\theta_o^1 (= e_o^{1*})$	e_o^{2*}	$e_o^{1*} \cdot e_o^{2*}$	$e_o^{1^0}$	$\theta_o^2 (= e_o^{2^0})$	$e_o^{1^0} \cdot e_o^{2^0}$	
1	0.99257	0.70447	0.69923	0.69923	0.99257	0.70447	0.69923	0.92622	0.71137	0.66074	
2	0.9985	0.62571	0.62477	0.62477	0.9985	0.62571	0.62477	0.99086	0.62748	0.62175	
3	0.69002	1	0.69002	0.69002	0.69002	1	0.69002	0.69002	1	0.69002	
4	0.72435	0.41999	0.30422	0.30422	0.72435	0.41999	0.30422	0.49792	0.43231	0.21526	
5	0.83066	0.92334	0.76698	0.76698	0.83752	0.8057	0.67478	0.73759	1	0.73759	
6	0.96062	0.40566	0.38968	0.38968	0.96369	0.40101	0.38645	0.96062	0.40566	0.38968	
7	0.67064	0.41241	0.27658	0.27658	0.75208	0.35216	0.26485	0.29991	0.53784	0.1613	
8	0.66302	0.41503	0.27517	0.27517	0.7256	0.37803	0.2743	0.38992	0.51135	0.19939	
9	1	0.22328	0.22328	0.22328	1	0.22328	0.22328	0.43904	0.29196	0.12818	
10	0.86154	0.54084	0.46596	0.46596	0.86154	0.54084	0.46596	0.25868	0.6736	0.17425	
11	0.64679	0.25344	0.16392	0.16392	0.74055	0.16753	0.12407	0.47185	0.32667	0.15414	
12	1	0.75958	0.75958	0.75958	1	0.75958	0.75958	1	0.75958	0.75958	
13	0.67198	0.30925	0.20781	0.20781	0.81068	0.24306	0.19705	0.33839	0.54349	0.18391	
14	0.66992	0.43086	0.28864	0.28864	0.72462	0.37396	0.27098	0.30964	0.51782	0.16034	
15	1	0.61383	0.61383	0.61383	1	0.61383	0.61383	0.71007	0.70473	0.50041	
16	0.88558	0.36152	0.32015	0.32015	0.9072	0.33557	0.30443	0.59872	0.38475	0.23035	
17	0.62761	0.57363	0.36001	0.36001	0.72331	0.45958	0.33242	0.2507	1	0.2507	
18	0.79354	0.32619	0.25884	0.25884	0.79354	0.32619	0.25884	0.65507	0.37366	0.24477	
19	1	0.4112	0.4112	0.4112	1	0.4112	0.4112	0.97884	0.41578	0.40697	
20	0.93322	0.58566	0.54655	0.54655	0.93322	0.58566	0.54655	0.40728	0.90137	0.36711	
21	0.7321	0.27425	0.20078	0.20078	0.75052	0.26232	0.19688	0.69178	0.27951	0.19336	
22	0.58952	1	0.58952	0.58952	0.58952	1	0.58952	0.58952	1	0.58952	
23	0.84256	0.49889	0.42034	0.42034	0.85005	0.45124	0.38358	0.68119	0.55992	0.38141	
24	0.42869	0.31447	0.13481	0.13481	1	0.08703	0.08703	0.39866	0.33509	0.13359	

1.4 Stackelberg Game

In the previous section we examined the cooperative or centralized game approach to the two stage problem. In this section we look at the two-stage process from the perspective of the non-cooperative game. The non-cooperative approach is characterized by the leader-follower, or Stackelberg game. For example, consider a case of a supply chain where there is non-cooperative advertising on the part of the manufacture (leader) and the retailer (follower). The manufacturer determines its optimal brand name investment and local advertising allowance based on an estimation of the local advertisement by the retailer to maximize its profit. The retailer, as a follower on the other hand, based on the information from the manufacturer, determines the optimal local advertisement cost to maximize its profit (Huang and Li 2001).

In a similar manner, if we assume that the first stage is the leader, then the first stage performance is more important, and the efficiency of the second stage is computed subject to the requirement that the efficiency of the first stage is to stay fixed. We first calculate the efficiency for the first stage. Based upon the CRS model, we have for a specific DMU_o

$$\begin{aligned}
 e_o^{1*} &= \text{Max} \sum_{d=1}^D w_d z_{do} \\
 \text{s.t.} \quad & \sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
 & \sum_{i=1}^m v_i x_{io} = 1 \\
 & w_d \geq 0, \quad d = 1, 2, \dots, D; \quad v_i \geq 0, \quad i = 1, 2, \dots, m.
 \end{aligned} \tag{1.7}$$

Note that model (1.7) is in fact the standard (CCR) DEA model. i.e., e_o^{1*} is the regular DEA efficiency score.

Once we obtain the efficiency for the first stage, the second stage will only consider w_d that maintains $e_o^1 = e_o^{1*}$. Or, in other words, the second stage now treats $\sum_{d=1}^D w_d z_{dj}$ as the “single” input subject to the restriction that the efficiency score of the first stage remains at e_o^{1*} . The model for computing e_o^2 , the second stage’s efficiency, can be calculated as (Liang et al. 2008)

$$\begin{aligned}
e_o^{2*} &= \text{Max} \frac{\sum_{r=1}^s U_r y_{ro}}{Q \sum_{d=1}^D w_d z_{do}} \\
s.t. \quad &\frac{\sum_{r=1}^s U_r y_{rj}}{Q \sum_{d=1}^D w_d z_{dj}} \leq 1 \quad j = 1, 2, \dots, n \\
&\sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
&\sum_{i=1}^m v_i x_{io} = 1 \\
&\sum_{d=1}^D w_d z_{do} = e_o^{1*} \\
&U_r, Q, w_d, v_i \geq 0, \quad r = 1, 2, \dots, s; \quad d = 1, 2, \dots, D; \quad i = 1, 2, \dots, m
\end{aligned} \tag{1.8}$$

Note that in model (1.8), the efficiency of the first stage is set equal to e_o^{1*} . Let $u_r = \frac{U_r}{Q}$, $r = 1, 2, \dots, s$. Model (1.8) is then equivalent to the following linear model

$$\begin{aligned}
e_o^{2*} &= \text{Max} \left(\sum_{r=1}^s u_r y_{ro} \right) / e_o^{1*} \\
s.t. \quad &\sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0 \quad j = 1, 2, \dots, n \\
&\sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
&\sum_{i=1}^m v_i x_{io} = 1 \\
&\sum_{d=1}^D w_d z_{do} = e_o^{1*} \\
&w_d \geq 0, \quad d = 1, 2, \dots, D; \quad v_i \geq 0, \quad i = 1, 2, \dots, m; \quad u_r \geq 0, \quad r = 1, 2, \dots, s
\end{aligned} \tag{1.9}$$

In a similar manner, if we take the second stage as the leader, we then calculate the regular DEA efficiency ($e_o^{2^o}$) for the second stage first using the CCR model.

Once we obtain the second stage efficiency, the efficiency for the first stage, namely $e_o^{1^o}$, is calculated via the following linear program (see Liang et al. 2008)

$$\begin{aligned}
 \frac{1}{e_o^{1^o}} = & \text{Min} \sum_{i=1}^m v_i x_{io} \\
 \text{s.t. } & \sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, 2, \dots, n \\
 & \sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0 \quad j = 1, 2, \dots, n \\
 & \sum_{d=1}^D w_d z_{do} = 1 \\
 & \sum_{r=1}^s u_r y_{ro} = e_o^{2^o} \\
 & w_d \geq 0, \quad d = 1, 2, \dots, D; \quad v_i \geq 0, \quad i = 1, 2, \dots, m; \quad u_r \geq 0, \quad r = 1, 2, \dots, s
 \end{aligned} \tag{1.10}$$

We note that in (1.9), $e_o^{1^*} \bullet e_o^{2^*} = \sum_{r=1}^s u_r^* y_{ro}$ at optimality, with $\sum_{i=1}^m v_i^* x_{io} = 1$. i.e.,

$$e_o^{1^*} \bullet e_o^{2^*} = \frac{\sum_{r=1}^s u_r^* y_{ro}}{\sum_{i=1}^m v_i^* x_{io}}. \text{ Note also that at optimality, } \frac{\sum_{r=1}^s u_r^* y_{ro}}{\sum_{i=1}^m v_i^* x_{io}} = e_o^{1^o} \bullet e_o^{2^o} \text{ in model}$$

(1.10). This indicates that the leader-follower approach also implies an efficiency decomposition for the two-stage process. i.e., the overall efficiency is a product of efficiencies of individual stages. Further, note that in the first-stage leader case, $e_o^{1^*}$ and $e_o^{2^*}$, and in the second-stage leader case, $e_o^{1^o}$ and $e_o^{2^o}$, are optimal values to linear programs. Therefore, such efficiency decomposition is unique, and is not affected by possible multiple optimal solutions. However, the two approaches may not yield the same efficiency decomposition.

Note that ultimately, a common set of weights is used at both stages in both centralized and Stackelberg game approaches. However, in the Stackelberg game approach, the efficiency scores of two stages, e_o^1 and e_o^2 , are not optimized simultaneously.

Liang et al. (2008) also study the relationships among non-cooperative and centralized models and the standard DEA approach. We here summarize their findings.

Let θ_o^1 and θ_o^2 be the standard CRS efficiency scores for the two stages.

Theorem 1 *If there is only one intermediate measure, then $e_o^{1*} = \theta_o^1$ and $e_o^{2*} = \theta_o^2$ regardless of the assumption of whether the first stage is a leader or follower, where e_o^{1*} and e_o^{2*} are obtained via the non-cooperative approach.*

Theorem 1 indicates that when there is only one intermediate measure, the non-cooperative approach yields the same result as applying the standard DEA model to each stage.

Under the condition of multiple intermediate measures, we have

Theorem 2 *For a specific DMU_o, $e_o^{centralized} \geq e_o^{1*} \bullet e_o^{2*}$, where $e_o^{centralized}$ is the optimal value to model (1.3), and e_o^{1*} and e_o^{2*} are obtained via the non-cooperative (leader-follower) approach.*

Based upon Theorems 1 and 2, we must have

Theorem 3 *If there is only one intermediate measure, then $e_o^{centralized} = \theta_o^1 \bullet \theta_o^2$ with $\theta_o^1 = e_o^{1,centralized}$ and $\theta_o^2 = e_o^{2,centralized}$, where θ_o^1 and θ_o^2 are the CRS efficiency scores for the two stages, respectively, and $e_o^{1,centralized}$ and $e_o^{2,centralized}$ are defined in (1.4).*

When there is only one intermediate measure, Theorem 3 indicates that (i) both the non-cooperative and centralized models yield the same result as applying the standard DEA model to each stage, and (ii) the efficiency decomposition is unique.

We finally note that the following is true with respect to the relations between the non-cooperative and centralized approaches.

Theorem 4

- (i) $e_o^{1,centralized} \geq e_o^{1*}$ and $\theta_o^2(=e_o^{2*}) \geq e_o^{2,centralized}$ when the second stage is the leader,
- (ii) $e_o^{2,centralized} \geq e_o^{2*}$ and $\theta_o^1(=e_o^{1*}) > e_o^{1,centralized}$ when the first stage is the leader.

The results in Table 1.2 also verify Theorems 2 and 4. We finally note that $e_o^{centralized} = e_o^{1*} \bullet e_o^{2*}$ holds for 12 DMUs (50 % of the companies), where e_o^{1*} and e_o^{2*} represent the efficiency scores for the two stages when the first stage is treated as the leader. Note also that $e_o^{centralized} = e_o^{1^o} \bullet e_o^{2^o}$ holds for only one DMU, namely DMU 6, where $e_o^{1^o}$ and $e_o^{2^o}$ represent the efficiency scores for the two stages when the second stage is treated as the leader. This may indicate that the first stage or the premium-generating stage is more important.

1.5 Network DEA

If we model the two-stage process shown in Fig. 1.1 using the network approach of Färe and Grosskopf (1996), we have

$$\begin{aligned}
& \min_{\Theta, \lambda_j, \mu_j, \tilde{z}} \Theta \\
& \text{subject to} \\
& \text{(stage 1)} \\
& \sum_{j=1}^n \lambda_j x_{ij} \leq \Theta x_{ij_o} \quad i = 1, \dots, m \\
& \sum_{j=1}^n \lambda_j z_{dj} \geq \tilde{z}_{dj_o} \quad d = 1, \dots, D \\
& \lambda_j \geq 0, \quad j = 1, \dots, n \\
& \text{(stage 2)} \\
& \sum_{j=1}^n \mu_j z_{dj} \leq \tilde{z}_{dj_o} \quad d = 1, \dots, D \\
& \sum_{j=1}^n \mu_j y_{rj} \geq y_{rj_o} \quad r = 1, \dots, s \\
& \mu_j \geq 0, \quad j = 1, \dots, n
\end{aligned} \tag{1.11}$$

where $\tilde{z}_{dj_o}$ are set as decision variables related to the intermediate measures.

Model (1.11) is equivalent to the following model

$$\begin{aligned}
& \min_{\Theta, \lambda_j, \mu_j, \tilde{z}} \Theta \\
& \text{subject to} \\
& \sum_{j=1}^n \lambda_j x_{ij} \leq \Theta x_{ij_o} \quad i = 1, \dots, m \\
& \sum_{j=1}^n (\lambda_j - \mu_j) z_{dj} \geq 0 \quad d = 1, \dots, D \\
& \sum_{j=1}^n \mu_j y_{rj} \geq y_{rj_o} \quad r = 1, \dots, s \\
& \lambda_j, \mu_j \geq 0, \quad j = 1, \dots, n
\end{aligned} \tag{1.12}$$

Model (1.12) is the dual to the centralized model (1.3). Therefore, the network DEA approach of Färe and Grosskopf (1996) yields results equivalent to the centralized model (1.3) of Liang et al. (2008) and Kao and Hwang (2008).

Chen et al. (2009a) show that the following CRS version of the Chen and Zhu's (2004) model is equivalent to model (1.3). (If we add the convexity constraints $\sum \lambda_j = \sum \mu_j = 1$ into model (1.13), then model (1.13) becomes the original Chen and Zhu (2004) model under the variable returns to scale assumption.)

$$\begin{aligned}
& \min_{\alpha, \beta, \lambda_j, \mu_j, \tilde{z}} \alpha - \beta \\
& \text{subject to} \\
& \text{(stage 1)} \\
& \sum_{j=1}^n \lambda_j x_{ij} \leq \alpha x_{ij_o} \quad i = 1, \dots, m \\
& \sum_{j=1}^n \lambda_j z_{dj} \geq \tilde{z}_{dj_o} \quad d = 1, \dots, D \\
& \lambda_j \geq 0, \quad j = 1, \dots, n \\
& \alpha \leq 1 \\
& \text{(stage 2)} \\
& \sum_{j=1}^n \mu_j z_{dj} \leq \tilde{z}_{dj_o} \quad d = 1, \dots, D \\
& \sum_{j=1}^n \mu_j y_{rj} \geq \beta y_{rj_o} \quad r = 1, \dots, s \\
& \mu_j \geq 0, \quad j = 1, \dots, n \\
& \beta \geq 1
\end{aligned} \tag{1.13}$$

Thus, since both the network DEA model (1.11) and model (1.13) are equivalent to model (1.3), they ((1.11) and (1.13)) must then be equivalent to each other. This implies that $\beta = 1$ at optimality in model (1.13).

Chen et al. (2010a) demonstrate that the centralized model (1.3) may not yield information on the efficient frontier of the two-stage process in Fig. 1.1. In other words, due to the existence of intermediate measures, the usual procedure of adjusting the inputs or outputs by the efficiency scores obtained from model (1.3), as in the standard DEA approach, does not necessarily yield a frontier projection.

We note that the network DEA approach only provides information on the overall efficiency of the two-stages, and does not yield information on the individual stages. However, the equivalence between models (1.11) and (1.13) indicates that the network DEA approach generates an efficient frontier point, since model (1.13) ensures that a frontier point is obtained if $\alpha < 1$ in optimality. See Chen et al. (2010a).

1.6 Searching for the Global Optimal Solution

While in the previous sections, the DEA models can be converted into linear programs due to the specific nature of two-stage network processes depicted in Fig. 1.1. A slight modification to Fig. 1.1, for example, by introducing additional (independent) inputs to the second stage, the resulting models are not necessarily linear.