

Research in Mathematics Education  
*Series Editors: Jinfa Cai · James Middleton*

Florence Mihaela Singer  
Nerida F. Ellerton  
Jinfa Cai *Editors*

# Mathematical Problem Posing

From Research to Effective Practice

 Springer

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Series editors  
Jinfa Cai  
James Middleton

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Florence Mihaela Singer • Nerida F. Ellerton  
Jinfa Cai  
Editors

# Mathematical Problem Posing

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Research in Mathematics Education

ISBN 978-1-4614-6257-6

ISBN 978-1-4614-6258-3 (eBook)

DOI 10.1007/978-1-4614-6258-3

Library of Congress Control Number: 2015938695

Springer New York Heidelberg Dordrecht London

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Printed on acid-free paper

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# Foreword

This is the second book in the *Research in Mathematics Education* series. Since the publication of the first edition in 1983 of *The Art of Problem Posing* by Brown and Walter, there has been increased effort to incorporate problem posing into school mathematics at different educational levels around the world. In the field of mathematics education, problem posing has been viewed not only as a means to understand students' mathematical thinking but also as a means to teach mathematics with understanding. This volume has at least the following three features. First, it presents the state of the art of research in mathematical problem posing. Readers will be well informed about problem-posing research as a line of scientific inquiry. The 52 authors of the 26 chapters pay careful attention to both past accomplishment and future directions of studies. Thus, this book should be useful for graduate courses related to mathematical problem posing and problem solving or as a foundation upon which to propose lines of inquiry into problem posing. Second, this book includes many great ideas to assist those implementing problem-posing tasks into classrooms; many of these ideas have already been tested in classrooms. Thus, this book can be used by mathematics teacher educators for designing and implementing teacher professional development sessions for practicing teachers. Third, this book truly has an international scope. Authors from 16 different countries have not only used diverse conceptualizations of problem posing but also presented a wide range of approaches for investigating issues related to problem posing.

As we indicated in the Foreword of the first book of the series, *Research Trends in Mathematics Teacher Education*, we have designed the solicitation, review, and revision process of volumes in the series to produce thematic volumes, allowing researchers to access numerous studies on a theme in a single, peer-reviewed source. Our intent for this series is to publish the latest research in the field in a timely fashion. This design is particularly geared towards highlighting the work of promising graduate students and junior faculty working in conjunction with senior scholars. The audience for this monograph series consists of those in the intersection between researchers and mathematics education leaders—people who need the highest

quality research, methodological rigor, and potentially transformative implications ready at hand to help them make decisions regarding the improvement of teaching, learning, policy, and practice. With this vision, our mission of this book series is:

1. To support the sharing of critical research findings among members of the mathematics education community
2. To support graduate students and junior faculty and induct them into the research community by pairing them with senior faculty in the production of the highest quality peer-reviewed, research papers
3. To support the usefulness, and widespread adoption, of research-based innovation

We are grateful for the support of Melissa James from Springer in developing and publishing this book series, as well as the support for the publication of this volume.

We thank the editors (Singer, Ellerton, and Cai) and all of the authors who have contributed to this comprehensive and insightful book!

Jinfa Cai  
James Middleton

# Preface

## Mathematical Problem Posing Today: A Cross-Cultural View

The era of information and communication technology creates new social environments and needs. Living in a world where interdependency and dynamics become main features of the global society, young generations have to face unpredictable changes they should learn coping with. Consequently, education systems all over the world support (or at least should pay attention to) a very fast process of changing priorities. Inherently, teaching and learning strategies are influenced by this context.

As a practice of learning and thinking, problem posing may play an essential role in this change. Since 1970, when Paulo Freire introduced the term problem-posing education in his book *Pedagogy of the Oppressed* as a metaphor for emphasizing critical thinking, the problem-posing methodology extended to various domains of knowledge. Within learning environments that offer a range of activities, sources for study, opportunities for interaction, and an emphasis on exploration and application, students can actively construct meaning in both the natural and simulated worlds, in the classroom. Teachers and students might create knowledge together in a variety of contexts and generate and address critical questions about the knowledge they produce. In Freire's vision, all these could help to develop more democratic, diverse, critically thinking members of society.

Mathematics as a tool for rational thinking can play an important role in preparing the fluent thinkers needed in the dynamic world of today (and tomorrow). For a long time, both the mathematics community and school practice have ranked *problem solving* as the top component of the mathematical domain. However, arguments in favor of problem posing come from at least two directions: from the past, where history shows that problem posing is the agent of change within scientific paradigms, and from the future, where the knowledge economy and the knowledge society trigger unprecedented demands and put enormous pressure on educational systems all over the world. We started this book envisioning that a fresh look at

problem posing is by all means a necessary step nowadays. The aim of *Mathematical Problem Posing: From Research to Effective Practice* is thus threefold: to present an updated overview of contemporary research on problem posing; to draw attention to successfully applied experiences; and to identify main directions for further research and new teaching and learning practices.

In a structured way, the book starts with multiple perspectives for defining the field of problem posing in the context of mathematics education, continues with the place problem posing holds in the school curriculum, and concludes with problem posing in teacher education programs and teacher professional development.

The book is multidimensional from a range of perspectives. From a conceptual view, the papers included in this collection present different epistemological, philosophical, and pedagogical approaches to problem posing. Concerning methodology, the studies of the volume range from qualitative research to quantitative meta-analysis. They range, with respect to the target population of students, from primary graders to intermediate and upper secondary grades. They also range, with respect to the target population of teachers, from preservice teachers (for all grades) to in-service teachers working at various levels of education. However, maybe the most important dimension of the book is its multicultural coverage. The authors come from different geographical areas: 16 countries are listed with the authors' affiliations (Australia, Belgium, Canada, China, Czech Republic, Israel, Italy, Japan, Malaysia, Norway, Romania, Serbia, Singapore, Sweden, the Netherlands, and the United States of America), from 4 continents, to which we can even add the diversity of backgrounds and experiences in a variety of cultural environments of many of the authors. This cultural diversity brings into the book various representations, expressions, knowledge, skills, and attitudes towards approaches to problem posing. The cultural diversity of authors' backgrounds makes the multiplicity of perspectives presented in the book deeply authentic. It also shows that problem posing is becoming more and more a global phenomenon.

The collection of articles in this book covers the way from research to effective practice by offering a large gamut of ideas, critical analyses, and successful experiences. The book starts with defining the field of problem posing in the context of mathematics education. In this first part, Jinfa Cai and his colleagues come up with a vision of problem posing as lenses for understanding and improving students' learning of mathematics. In a more specific approach, Ragnhild Hansen and Gert Hana show how problem posing can be emphasized in a modelling perspective, while Jasmina Milinković conceptualizes problem posing via transformation, and Sergei Abramovich and Eun Kyeong Cho explain how to use digital technology for mathematical problem posing. Further, Cinzia Bonotto and Lisa Dal Santo look at the connection between problem posing and creativity in relation to problem solving, while Vincent Matsko and Jerald Thomas explore ways to foster creativity in mathematics classrooms. Florence Mihaela Singer and Cristian Voica develop a framework for using problem posing as a tool for identifying and developing mathematical creativity.

The second part of the book provides practical examples of using problem posing in school mathematics teaching. Here, Victor Cifarelli and Volkan Sevim relate

reformulation and sense-making within the problem-solving process to problem posing; Sharada Gade and Charlotta Blomqvist discuss the role of explicit mediation for developing problem-posing capacity of fourth and fifth graders, while Kees Klaassen and Michiel Doorman find in problem posing good opportunities for providing students with content-specific motives. A content like statistical literacy is seen by Lyn English and Jane Watson as a relevant opportunity for problem posing in the elementary school. Further, Mitsunori Imaoka, Tetsu Shimomura, and Eikoh Kanno describe effective ways of using computers for problem posing in upper grades, a topic that is rarely addressed. From Singapore, Kwek Meek Lin proposes a research experiment in which problem posing is used as an assessment tool in the lower secondary school. In the final two chapters of this part of the book, from a multicultural perspective, Xianwei Van Harpen and Norma Presmeg analyze the outcomes of a comparative investigation of high school students' mathematical problem posing in the United States and China, while Limin Chen, Wim Van Dooren, and Lieven Verschaffel come up with a design experiment for enhancing the development of Chinese fifth graders' problem-posing and problem-solving abilities, beliefs, and attitudes.

From a focus on students who are involved in problem posing in the classroom, as we have seen in second part of the book, the authors move to the (future) teacher who is to orchestrate such activities and discuss, in the third part of the book, mathematics problem posing in teacher education programs and teacher professional development. More specifically, Roslinda Rosli, Mary Margaret Capraro, and their colleagues address the relationship between problem solving and problem posing in a study with middle grade preservice teachers. The same relationship is addressed by Vrunda Prabhu and Bronislaw Czarnocha in the context of an integrated teaching/research methodology that has become known as Teaching-Research/New York City (TR/NYCity) methodology. Further, Rosa Leikin explains how to teach in a dynamic geometry environment and to use it as a tool for mathematical problem posing and geometry investigations by using examples from a course with prospective mathematics teachers. Ilana Lavy describes studies conducted in dynamic geometry environments that adopted "what if not" strategies. Todd Grundmeier provides details of the results of an exploratory study that incorporates problem posing in a mathematics course for prospective elementary and middle school teachers, where the content coverage included problem solving, data analysis and probability, discrete mathematics, and algebraic thinking. From a different perspective, problem posing was used as a motivational tool; this aspect is addressed by Alena Hošpesová and Marie Tichá in a study investigating primary school teacher training. Two other studies also explore ways in which problem posing has been investigated in preservice teacher training. Thus, Michal Klinshtern, Boris Koichu, and Avi Berman find unexpected perceptions of teachers as problem posers, while Helena Osana and Ildiko Pelczer succeed in classifying problem posing in mathematics professional development in a few distinct categories, despite the continuing paucity of empirical studies. Giving prospective elementary teachers the opportunity to pose personally and socially relevant mathematics problems is an important focus for Sandra Crespo's chapter. Finally, in a study with prospective and practicing middle school

teachers, Nerida Ellerton shows how problem posing can become an integral component of the mathematics curriculum and introduces the concept of a *Pedagogy of Problem Posing*.

In the final part, we provide an overview of this book's special contributions to the field. We comment there how the book takes into consideration past literature, energizes present practices, and looks towards future learning, teaching, and research endeavors. Beyond the diversity of approaches and cultural spaces reflected into this collection, the book brings together the visions of experienced contemporary personalities who have researched and written on problem posing as well as those of some remarkable young professionals who have embarked on promoting this new and emerging field.

Bucharest, Romania  
Normal, IL, USA  
Newark, DE, USA  
March, 2015

Florence Mihaela Singer  
Nerida F. Ellerton  
Jinfa Cai

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problem posing because it redistributes power dynamics in the classroom with regard to who produces and who consumes mathematical knowledge. Problem posing is associated with the production of disciplinary knowledge and with critical pedagogy. This makes problem posing a key practice for teachers of mathematics to develop as learners of mathematics and of mathematics teaching. She is currently involved in a longitudinal study of prospective teachers documenting how they develop their problem posing practices over time. Her work has been published in *Educational Studies in Mathematics*, *Journal of Mathematics Teacher Education*, *For the Learning of Mathematics*, and *Teaching Children Mathematics*.

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She is an active member of the Czech and international research communities (e.g., she was a member of the International Committee of The International Group for the Psychology of Mathematics Education (PME); she was one of the organizers of Thematic Working Groups on teacher education on several recent conferences of ERME; she is a member of informal community Learners' Perspective Study). She is a member of several Councils of Ph.D. Studies in didactics of mathematics in the Czech Republic, and she supervises Ph.D. students.

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**Mitsunori Imaoka** was a Professor in the Department of Mathematics Education at Hiroshima University until 2012 and a member of meetings organized by MEXT of Japan to study and discuss improvements to the national curricula for high school mathematics during 1998–1999 and 2004–2009. He is now teaching mathematics in Hiroshima Institute of Technology. In his own teaching in the 1990s, he found that student problem posing produced authentic mathematical thinking. He therefore became motivated to research effective ways of involving upper-grade students in problem-posing. His first report on his problem-posing work was published in the *Journal of JASME*, 2001. Since then, in collaboration with Tetsu Shimomura and Eikoh Kanno, he has continued practical computer studies on problem posing by prospective teachers, and on various types of problem-posing activities by high school students.

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Meek Lin holds a Master of Education in Mathematics Education from the National Institute of Education, Nanyang Technological University. Her research interests include assessment, problem solving, and teacher knowledge. Her research studies often culminated in conference presentations including, for example, "Dual

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**Vincent J. Matsko** studied at Carnegie Mellon University. After teaching at the university for several years, he began teaching at secondary schools for students especially talented in mathematics and science. It was then that he became interested in fostering students’ creativity in mathematics, especially when it came to problem posing. He is an avid problem writer himself and enjoys the challenge of creating novel problems. Please also see [www.vincematsko.com](http://www.vincematsko.com).

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**Ildiko Pelczer** is a Ph.D. student in Mathematics Education at Concordia University in Montreal, Canada. Her research focuses on preservice teachers' mathematical knowledge for teaching with a special interest on their specialized content knowledge. She has collaborated on studies concerning pedagogical interventions to enhance teachers' problem posing abilities and studies on identifying features of mathematical creativity in students' thinking.

**Vrunda Prabhu** whose Ph.D. in Point Set topology addressed fine points of Čech compactification, was a born educator from her early childhood. She was a Montessori mother who incorporated the philosophy embodied in Maria Montessori's book *The Discovery of the Child* into her son's upbringing. Later,

while working with a Dalit community in Tamil Nadu, India, she had formulated the new program “Montessori for Mothers” as the tool to “discover the child” and as the foundations for a new literacy campaign. Vrunda had acquired her initial teaching-research skills in the RUMEC community studying and applying APOS, Piaget-based learning theory into calculus. A co-PI on two NSF grants and several internal CUNY grants, she had focused her attention on the design of creative learning environment in remedial mathematics classes, culminating in the discovery of Koestler’s bisociation theory of the Aha! Moment for teaching mathematics. She passed away prematurely in the spring of 2013 leaving behind a wealth of education materials as well as several notebooks of her poetry, both being published posthumously.

**Norma Presmeg** is Professor Emerita in the Mathematics Department of Illinois State University. Her research interests include a long-standing interest in creativity, going back to her thesis on Albert Einstein’s creative thought, which led to her research interest in visualization in mathematics education. Mathematical problem solving, problem posing, and the role of metaphors in mathematics education were themes that emerged from this research, along with the importance of affective issues. Concurrently, the role of culture in learning and teaching mathematics was a theme that led to interest in ethnomathematics. All of these threads are supported by a theoretical interest in semiotics, which is ongoing, and more recently by the potential of transcendental phenomenology to introduce new questions in research in mathematics education.

**Roslinda Rosli** recently completed her doctoral studies in the Department of Teaching, Learning, and Culture at Texas A&M University with an emphasis area in mathematics education. She is a senior lecturer in the Department of Teaching and Learning Innovation at the National University of Malaysia. She teaches statistics, research methodology, and mathematics education courses. Dr. Roslinda presented at over a dozen local, national, and international conferences and is currently engaged in continuing to grow her resume of published works. Her research areas include mathematical knowledge for teaching, problem posing, problem solving, and teacher education. This book chapter was part of the scholarly work from her Ph.D. dissertation research, and she is looking forward to publishing additional works in the near future.

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