ENVIRONMENTAL HYDRAULICS SERIES

Modeling Software

Edited by Jean-Michel Tanguy

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Table of Contents

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volume 5

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Introduction [1]

This fifth volume of the environmental hydraulics series completes the series. Volume 1 described hydrological and fluvial processes; while Volume 2 covered estuarine and littoral processes. Volume 2 also described the mathematical modeling of these processes, emphasizing the consistency between models. Volume 3 lists and describes the numerical methods used to solve systems of partial differential equations in hydrological contexts. Following these physical and theoretical considerations, Volume 4 describes a wide range of real-world studies carried out using commercial computer models.

This final volume thus develops the theme of the earlier installments, discussing a range of commercial modeling tools that can be used to treat examples such as those described earlier in the series.

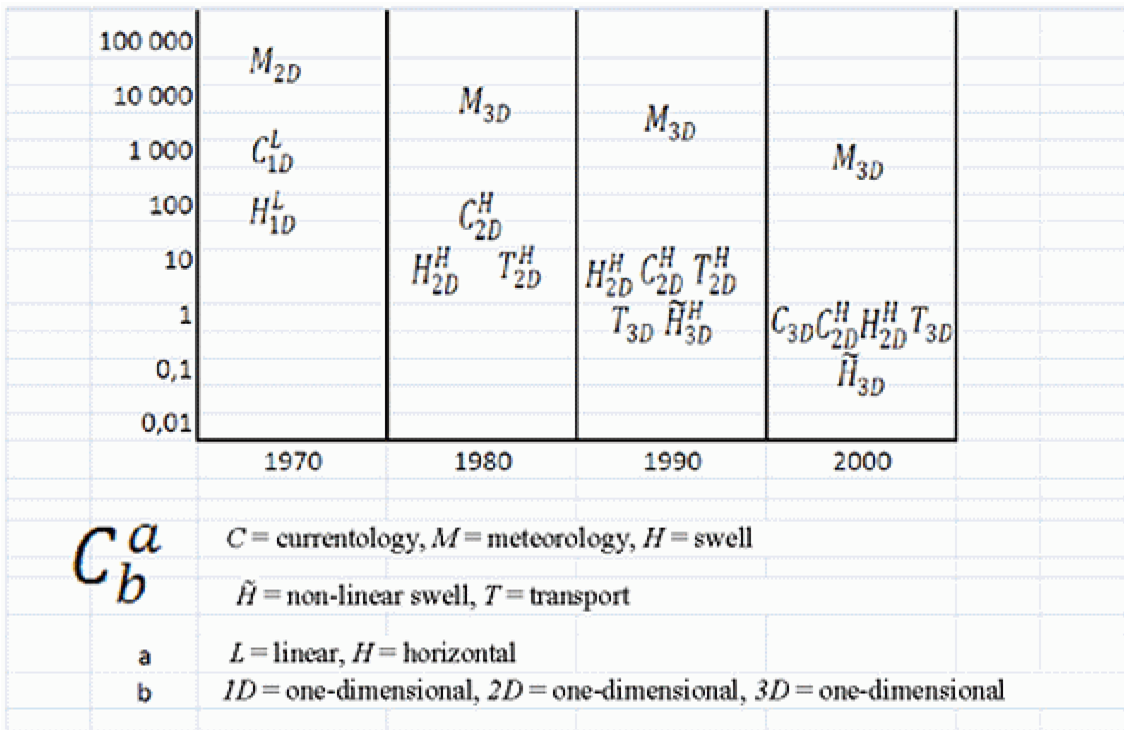
In order to remain faithful to the theme of the series we will start with hydrological modeling tools and go on to discuss tools treating maritime morphodynamics.

This volume is divided into three main sections: 3D models, which represent the most recent advances in numerical modeling, and which are currently beginning to emerge onto the commercial stage, 2D models which are seeing more and more widespread use in engineering applications, and 1D models which remain the most widely used tools for engineers.

It is worth recalling a brief history of the evolution of spatial discretization within modeling tools, as shown in Figure i.1.

The first models appeared in the 1960s in the field of meteorology, a field with strong scientific and strategic interest. Over the next decade the first computer models emerged to replace manual calculations. These were largely developed by consultant engineers. These models mostly consisted of Fortran calculation loops solving the 1D Saint-Venant model, based on extremely simplified geometries such as trapezoidally structured river models.

Figure i.1. *Developments in discretization of models over recent decades (showing mesh cell size in meters)*



The field was still being pioneered, and there were as many models as there were research centers. Pre- and post-processing was extremely primitive: the alphanumeric consoles of the time did not have graphical capabilities. These first computer codes were nevertheless the state of the art for their creators. Originating in the USA, the first codes developed with federal funding started to be distributed for free (e.g. the HEC-RAS[1] family).

This was the catalyst for the commercialization of simulation codes, followed by the appearance of the first graphical pre- and post-processors. The invention of inkjet printers then made it possible for engineers to produce graphical representations of their simulation results.

This paved the way for subsequent innovation. Two main directions were taken by the specialists within the field:

– the first retained a 1D approach, but complemented hydraulic models with models describing associated processes such as inert substances transport, sediment transport and bed evolution. In terms of couplings between processes, the challenges lie in the reconciliation of time scales which may be extremely different, in the quantification of coupling terms that are not easy to estimate, and in the couplings themselves. For example, suspended sediment transport takes place on much smaller time scales than the hydrodynamics, which may in some cases be considered to be steady state in comparison;

– the second focused on extending the models to more dimensions: this resulted in the appearance of 2D and subsequently 3D models. Research into numerical methods led to higher performance algorithms: for example determination of the free surface using a 1D model is much easier than determining the same free surface in two dimensions.

It is worth noting that these two approaches were taken by different communities: the first approach was mostly taken by physicists, while the second was more the domain of applied mathematicians. We will see later on, however, that these two communities are now converging on a common goal: families of coupled 3D codes.

Figure i.1 also shows that the discretization of models, as represented by the mesh density, has become much finer over the decades: for meteorology there has been an improvement by a factor of 100, and for hydraulics a factor of 1,000. In hydraulics, and for related disciplines such as transport and wave mechanics, there has been an expansion from 1D models to 3D models. As for wave mechanics, which demand an extremely fine mesh that is a function of the wavelength, 3D nonlinear models are now available. This development has of course been made possible by advances in the capabilities of computers, but has also been driven forwards by research into modeling techniques.

The diagram implies that a factor of 10 gain should be expected for most types of models over the next decade.

Data availability

Models must be fed with data, and this should be available at suitable intervals and be of a suitable quality and spatial density to suit the modeling tools to be used. Thus 1D models, which use transverse cross-sections of the river, do not require a high degree of precision, whereas a 2D model requires a higher accuracy of spatial data, sampled at a higher density, particularly over the flood plain.

Thus the more dimensions to the model, the more expensive it will be to fund, and the greater the need for large quantities of accurate data that is complex to use. Such a model will be more complex to implement and use. On the other hand it will be much more accurate, and will make it possible to treat more local phenomena.

The situation is yet more complex with real-time models. In addition to the implementation data described earlier, these must be fed with measurements collected in real-time, which must be integrated using assimilation procedures — a process described in the chapters discussing real-time models.

Model coupling

The final stage involves coupling of 1D, 2D and 3D models using a “toolbox” approach. This makes it possible to rationalize the performance of these tools. It is possible to work on the propagation of a flash flood along hundreds of kilometers of a river, based on boundary conditions recorded at limnimetric or tidal stations, and then use 2D models to simulate the distribution of water heights around the confluence of two rivers, in order to analyze the effects of the flood on the vulnerability of a specific area.

The same is true in maritime environments: 2D models can be used on large scales, and the details of longshore currents can be described using 3D models, which is crucial in the determination of deposition and erosion regions around coastal engineering structures.

Research directions

A range of areas are the subject of research interest at present. Here we will restrict ourselves to general considerations, referring the reader to earlier volumes for further information on areas of current research. We will however mention a number of points that we feel are important:

– the quantification of physical processes is very dependent on the metrology used (radar, satellite, non-intrusive systems). Certain processes may appear very simple, but be challenging to quantify: an example in hydrology is the fact that there is still not a clear understanding of how rainfall water accumulates, infiltrates and enters water courses. A homogeneous film of water is not a concept that exists in nature. Other processes are by nature highly complex to study and quantify, as is the case in fluvial morphodynamics where there is no clear understanding of how to model helicoïdal currents and their effects on the deformation of meanders;

– improvements in numerical methods, which enable more precise and more reliable calculations to be carried out and results to be obtained more rapidly;

– the question of uncertainties is also a key area of current research. It is important when results are presented from a model that they are accompanied by an indication of the associated uncertainties. These may have a number of different origins: measurements, models, etc.;

– the appearance of new, very promising types of models which by nature have the potential to herald a departure from earlier generations of models.

There is another area which is often overlooked, which is rather more technical than scientific: this is the question of the user interface. So much time has been lost in implementing such models: for example the data conditioning, development of boundary conditions, introduction of manmade structures with appropriate behavior, and inclusion of local topographical detail, but also processing of the results and presentation of the results in an appropriate form along with an indication of their associated uncertainties.

New platforms are appearing which can be used to optimize the conditioning and production of results, but more work is still required. Indeed, if we consider the advancements in science and technology it may seem astonishing that engineering models are still so awkward to use!

It seems reasonable to ask why, given all the technologies currently available, no model is available that allows the user to see everything at any point in time, and to interact fully with the model. For example, in flood wave propagation, it is of primary importance to determine settled areas that are susceptible to flooding. In the majority of cases, a number of complementary actions are possible to reduce the impact of the hazard. This requires the simulation of a range of different scenarios in parallel in order to determine the best way to protect these at-risk areas: modifying a structure to simulate the breaching of levées or opening a dam sluice to divert part of the flood, and immediately analyze the consequences on the lowering of the level in urban areas. All this is performed in real time, incorporating many different types of ground-truth data.

Such features are crucial to those entrusted with making the decisions.

It is possible to go further still, and introduce knowledge into the models that may enable the tools themselves to recommend certain courses of action and analyze their consequences. All this is currently feasible; we have the technical abilities to develop tools of this nature.

Unfortunately, the scientific community is mired in the challenge of optimizing numerical methods, and is not investing enough effort in the ergonomics of simulation tools. The question may also be asked as to whether it is import useful to gain 1 mm of accuracy in the results from the model, or to continue to use less refined models but to devote a great deal more effort on data assimilation. Evidently there are not enough economic incentives in the risk analysis or engineering to justify such considerations.

Even present-day models are heavily encumbered: they currently take far too much time to set in motion and often must be launched “manually”. How is it possible that we still rely on a workflow as archaic as pre-processing/ computation/ post-processing? Nowadays fully object-oriented languages[2] make it possible to interact in real time with the simulation procedures, but the previous generation of models has not yet incorporated these innovative approaches that transfer significant responsibility to the modeler. By putting the user in the driving seat, the software enables the modeler to respond to events occurring on the ground by adjusting water management installations, constructing barriers or demolishing obstacles.

The lack of a more intuitive approach is a real shame, because NTMs nowadays have extremely high precision, thanks to the easy-to-use IGN Geoportal and Google Earth and the availability of extremely high quality databases such as NTMs based on aerial laser-based measurements.

Finally, while Météo-France is able to simulate the weather over the entire planet, in other fields, we are barely able to describe the development of flash flood waves within a river catchment area.

This rather blunt observation constitutes a call to arms for the pooling of resources between cognitive scientists, ergonomics experts, artificial intelligence experts, software engineers specializing in graphical visualization, physicists and applied mathematicians with the aim of defining the tools of the future for the realtime simulation of physical phenomena.

Structure of Volume 5

Volume 5 consists of three parts, each consisting of a number of chapters grouped together, based on the number of dimensions of the computer codes they discuss. We start with 3D models (Part 1) and then step down to 2D (Part 2) and finish with 1D (Part 3). Each chapter not only presents the simulation modules but also discusses the pre- and post-processing, along with example applications illustrating the abilities of the tool.

Part 1: 3D models

In Chapter 1, we begin by describing a non-linear wave propagation code: REFLUX3D. This is a highly sophisticated tool for wave simulation. Its complexity is a direct result of the effect it models, which are extremely difficult to model in the vicinity of the coast, near complicated coastal structures and underwater man-made structures. Near these underwater structures, the wave behavior may be altered dramatically, and this requires specific treatment within the model.

Next, in Chapter 2, the TELEMAC 3D family of codes is discussed. In addition to a hydrodynamics module, this incorporates other modules handling effects such as the transport of dilute suspended tracers, bed loading transport and bed evolution, and transport of cohesive sediments in suspension.

Chapter 3 makes a foray into the world of meteorology, presenting the range of codes used operationally by Météo-France: a hierarchy of models dedicated to atmospherical modeling (ARPEGE) feeding into models covering more restricted areas (ALADIN) and ones treating yet smaller regions (AROME), with resolutions close to a kilometer and giving a very detailed description of the processes that take place on these scales.

Chapter 4 demonstrates the crucial role of 3D hydrogeological tools such as MARTHE in modeling flows within soils. The sub-soil water currents that transport pollutants traverse layers with a wide range of different characteristics, and the pollutants respond very differently depending on whether these are saturated or unsaturated regions.

Part 2: 2D models

Chapter 5 concerns the SIM model. This is a hydrometeorological tool used to estimate surface runoff as a function of position over a mesh with 8 km long sides. This toolchain couples atmospheric forcing with water cycle and energy cycle models, along with a hydrological model that can be used to determine the discharges within water courses.

Within the field of hydrology, new models such as MARINE, which is showcased in Chapter 6, are taking the form of integrated platforms that mix preprocessing, calculation and post-processing, and modeling hydrological and hydraulic processes. Calculations are based on kinematic wave modules for hydrology and models of river hydraulics. This tool is currently being adapted to real-time operation.

Chapter 7 presents another hydrological platform, ATHIS, which incorporates several different types of hydrological and hydraulic modules. It also operates as an integrated platform, which is also in the course of being adapted for real-time use for rapid-response forecasting of flash flooding within river catchment areas.

Chapter 8 descibes LARSIM, which is a conceptual type of flood forecasting system able to reproduce flood discharges through a continuous simulation of the water cycle. It is used in flood forecasting.

Still within the domain of hydrology, the TOPMODEL code discussed in Chapter 9 is an example of an original approach in that it combines a Hortonian infiltration approach with an approach based on contributions from different areas of surface water, two current complementary explanations for the generation of surface runoff.

In Chapter 10, we return to the TELEMAC2D family of flow models, which consists of a range of models analogous to the 3D family of the same name.

Thanks to the use of 2D models such as RUBAR 20TS, described in Chapter 11, the processes of suspended sediment transport and river bed evolution can now be modeled. RUBAR is a code which couples a Saint-Venant type of flow model with a suspended sediment transport model.

Chapter 12 focuses on NAVMER. This is a ship course simulator that uses a 2D flow code to determine the current fields and a trajectory model that uses this information and the characteristics of the vessel to determine its course. This code is also used to determine the implications for ship maneuverability of man-made structures or sections of watercourses with complex current patterns.

Part 3: 1D models

Chapter 13 describes VAG, a very simple wave propagation code that is not only widely used in simple configurations, but is also used for the determination of boundary conditions to the sea for more complex models.

The SOPHIE real-time hydrological platform, discussed in Chapter 14, is a modular application able to host a range of different types of flood prediction models. It is used operationally in the flood prediction network within France.

Chapter 15 is dedicated to a comparison of two 1D hydraulic flow codes: MASCARET and RUBAR3.

Chapter 16 discusses the generic characteristics of “cell based” 1D flow models.

Chapter 17 focuses on CANOE, a multi-use tool for urban hydrology. Built on a detailed description of both the sewerage network and the roadways, it determines the distribution of flows over the course of a hydrometerorological event.

The Prose 1D model, studied in Chapter 18, simulates the impact on a hydrographical network of pollution from diffuse rather than point sources.

Chapter 19 gives a generic presentation of substance transport models.

Chapter 20 makes a comparison between two morphodynamic simulation codes, RUBARBE and TSAR, which simulate the interactions between hydraulics, sediment transport and bed evolution.

Chapter 21 describes the PAMHIR hydraulic modeling environment for the design and use of 1D numerical models for river hydraulics.

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[1] Introduction written by Jean-Michel TANGUY.

[1] HEC: *Hydraulic Engineering Center — River Analysis System*: *US Corps of Engineers*.

[2] Adobe Flash

PART 1

3D Models

Chapter 1

Non-Linear Waves with REFLUX 3D [1]

1.1. Context

The Boussinesq equations [BOU 72] are the result of vertical integration of the conservation of momentum and conservation of mass equations for an incompressible fluid. The vertical component of velocity is then assumed to vary linearly as a function of depth, in order to reduce the three-dimensional problem down to a two-dimensional one. The Boussinesq equations take into account energy transfer between multiple frequency components, the changes in shape of individual waves and the evolution of a group of random waves.

Boussinesq wrote his equations in 1872 [BOU 72] for the propagation of waves on flat beds. It took another century for the first formulation to be developed for non-flat beds, by Peregrine in 1967 [PER 67]. The main limitation of this most common form of the Boussinesq equations is that these equations are only valid for relatively shallow depths of water. It was only very recently, in the 1990s, that a range of models derived from the original Boussinesq equations were developed in order to extend their validity to deeper water, and in most cases also improving the dispersion equation for the waves in the process.

The three-dimensional Reflux 3D hydrodynamic model (or the Reflux 2DV two-dimensional equivalent) developed by

Meftah [MEF 98] takes an innovative approach known as the *h – s* approach [MEF 99], which leads implicitly to a very good approximation of the dispersion relation. In hydraulics, the horizontal dimensions are large compared to the vertical dimension, and this new approach involves a specific treatment of variables in the vertical direction: the model uses a finite element approximation in the horizontal plane (O*xy*) and an analytic type of approximation (function series) in the vertical direction (O*z*).

This approach is similar to that taken by Nadaoka *et al*. [NAD 94] and Massel [MAS 93], who chose to use a basis of hyperbolic functions in the vertical direction. In this approach, there is no longer a three-dimensional grid, but rather each vertical column is replaced by a single node with a larger number of degrees of freedom. If Legendre polynomials are selected as the basis, such a model can then be classified as an extended Boussinesq type of model.

1.1.1. *System of equations to be solved*

In order to simplify and abbreviate the notation of our equations, we will write them here in a 2D vertical basis. In the O*xz* plane, the conservation of mass and conservation of momentum equations can be written:

$$\text{(1.1)}\quad \begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + w\dfrac{\partial u}{\partial z} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = 0, \\ \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + w\dfrac{\partial w}{\partial z} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial z} = 0, \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0, \end{cases}$$

where *p, u, w, g* and *ρ* are the pressure, the horizontal and vertical velocity components, the gravitational constant and the density of the fluid, respectively. The pressure within the fluid can be broken down in the following manner:

$$\text{(1.2)}\quad p = \rho g(h - z) + p_{atm} + \rho p'$$

where $\rho g(h–z)$ is the hydrostatic component of pressure, p_{atm} the atmospherical pressure at the level of the free surface, $\rho p'$ is the non-hydrostaticcomponent and $h(x,t)$ is the height of the free surface. The boundary condition on the pressure at the free surface is expressed by:

(1.3) $p(x,h) = p_{atm}, \quad \text{where} \quad p'(x,h) = 0.$

The kinematic continuity equation at the free surface is written as:

(1.4) $\frac{\partial h}{\partial t} + u_h \frac{\partial h}{\partial x} - w_h = 0,$

where (u_h, w_h) are the velocity components at the free surface. The kinematic continuity equation on the bed is written as:

(1.5) $u_b \frac{\mathrm{d} z_b}{\mathrm{d} x} - w_b = 0,$

where $z_b\,(x)$ is the time-independent height of the bed and (u_b, w_b) are the velocity components on the bed.

1.1.2. *h–s method*

The horizontal velocity component is written in the form of a series as a function of height:

(1.6) $u(x,z,t) = \varphi_i(z, z_b, h)\, u_i(x,t),$

where the ϕ_i functions, which we will initially take to be the Legendre polynomials, form a basis of appropriate orthogonal functions. N is the discretization order of the model.

Legendre polynomials

Our default choice for our basis is the Legendre polynomials. These polynomials are defined as follows:

$$\varphi_1(\xi) = 1, \quad \varphi_i(\xi) = \frac{1}{(i-1)\,!\,2^{i-1}} \frac{\mathrm{d}^{i-1}}{\mathrm{d}\xi^{i-1}} \left(\xi^2 - 1\right)^{i-1}, \quad \text{where} \quad \xi = 2\frac{z - z_b}{h - z_b}.$$

The Legendre polynomials form an orthogonal basis, which offers a number of advantages: simpler scalar products, a mass matrix and simpler boundary conditions. In general terms, their scalar products are faster to compute than for other mathematical functions (hyperbolic, trigonometric, logarithmic, etc.).

Mixed polynomial-logarithmic basis

It is known that the vertical velocity profile for fluvial flow takes a logarithmic form. For this reason a non-orthogonal basis may also be used. This basis is similar to the Legendre polynomial basis, but a logarithmic function of the form $\varphi(z)=b\log(a \cdot z)$ is added, where the a and b coefficients depend on the properties of the flow as well as the characteristics of the bed.

Hyperbolic – propagating mode basis

In order to model wave propagation, it seems natural to use a series of hyperbolic functions:

$$\varphi_i(z, z_b) = \frac{\cosh\left[k_i\left(z - z_b\right)\right]}{\cosh\left[k_i\left(h - z_b\right)\right]},$$

where the wave numbers k_i are real and depend on the characteristics of the wave in question. This functional basis, described by Nadaoka *et al*. [NAD 94] possesses excellent linear dispersion properties, but has the drawback that it is not orthogonal.

Hyperbolic basis - evanescent modes

The same hyperbolic basis can be used, but with wavenumbers k_i selected to be solutions to the linear dispersion relation $\omega^2 = gk \tanh(kH)$ with $H=h - z_b$.

The wave number k_1 is the only real solution to this equation, with the other solutions k_i being pure imaginary. Massel [MAS 93] described this series of functions, which gives a generalized non-linear model of the non-stationary mild-slope equation. This basis has the advantage of orthogonality, and the drawback of being valid only over a relatively narrow frequency band around k_1.

Integration of the conservation of mass equation from the bed at z_b to the free surface h leads to the following equation:

(1.7) $$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0, \quad \text{with } q = \int_{z_f}^{h} u\,dz.$$

Integration of the conservation of mass equation from the bed at z_b to height z gives the expression for the vertical component of velocity w as a function of a number of variables which only depend on z. Similarly, integration of the equation giving the conservation of the vertical momentum component (using the formula for w derived earlier) from height z to the free surface h enables us to decompose the pressure p into functions that only depend on z. The system to be solved for our model is then

(1.8) $$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + w\dfrac{\partial u}{\partial z} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = 0, \\ \dfrac{\partial h}{\partial t} + \dfrac{\partial q}{\partial x} = 0, \end{cases}$$

where u, w and p are three functions that can be expressed in the form of a series of functions of the components u_i of the horizontal velocity, and functions depending only on z.

1.1.3. *Linear dispersion*

A new equation system is established based on (1.8) by considering propagation over a flat bed of a wave that is sufficiently small to permit the equations to be linearized:

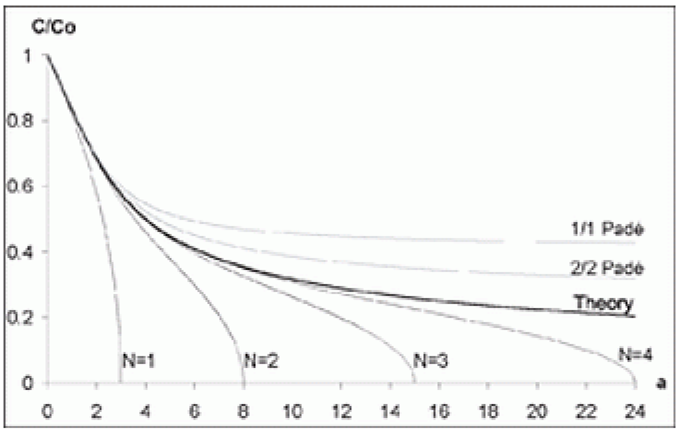
(1.9) $$\begin{cases} \varphi_i \dfrac{\partial u_i}{\partial t} + g\dfrac{\partial h}{\partial x} = \theta_i \dfrac{\partial^3 u_i}{\partial t\,\partial x^2}, \\ \dfrac{\partial h}{\partial t} + H\dfrac{\partial u_1}{\partial x} = 0, \end{cases}$$

with:

$$\theta_i\left(z, z_b, h\right) = \int_z^h \int_{z_b}^{z_1} \varphi_i\left(z_2, z_b, h\right) dz_1\,dz_2.$$

The first equation is projected onto the basis of the functions ϕ_i , and we will look for solutions of the form $u_i = \mathbf{u}_i \, e^{j(kx - \omega t)}$ and $h = \mathbf{h} \, e^{j(kx - \omega t)}$. We will then look for non-trivial solutions of this linear system with $N + 1$ unknowns. Details of the calculation are given by Meftah *et al*. [MEF 03] for numbers of functions N ranging from 1 to 3, using Legendre polynomials. This is generalized to other bases in Meftah *et al*. [MEF 04].

Figure 1.1. *Linear dispersion using Legendre polynomials*



Taking c_0 to be the speed for long wavelengths, equal to $\sqrt{gH}$, and **a** to be a dimensionless number equal to $\frac{H\omega^2}{g}$, Figure 1.1 plots the speeds obtained using various extended Boussinesq models as a function of the pulsation ω of the wave.

Saint-Venant type models exhibit very poor dispersive properties, with the wave velocity being equal to c_0 regardless of the nature of the incident wave.

The *Boussinesq standard* model of Peregrine [PER 67] corresponds to the *h–s* approximation with a single polynomial (*N=1*). It exhibits dispersive characteristics that are better than those of a Saint-Venant model, but is valid only for relatively long wavelengths.

The h–s approach generalizes the Peregrine model to short wavelengths while retaining its properties: it approximates