The Handbook of Contemporary Semantic Theory

Second Edition

Edited by

Shalom Lappin and Chris Fox

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Contemporary Semantic Theory

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לנכדים זוהר, אלה, גליה, נועם, ועומרי, באהבה

For Ray.

Contents

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Chris Fox Chris Fox's research is located in the intersection of linguistics, computer science, and philosophy. His main interest is in the formal interpretation of language, and foundational issues in semantics. He has authored or coauthored numerous publications in this area, including two books: *The Ontology of Language* (CSLI, 2000) and *Foundations of Intensional Semantics* (Blackwell, 2005). These works explore axiomatic and proof-theoretic accounts of meaning. He also coedited the *Handbook of Natural Language Processing and Computational Linguistics* (Wiley-Blackwell, 2010). His current work is focused on foundational issues in the formal interpretation of language, in addition to an interest in the analysis of imperatives and deontic statements. Before his appointment as Reader at the University of Essex, Fox taught at Goldsmiths College, University of London, and King's College London. He was also a visiting fellow at the Computational Linguistics Institute in Saarbrücken. He holds a B.Sc. in computer science, an M.Sc. in cognitive science, and a Ph.D. from the Cognitive Science Centre, University of Essex.

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Preface

We have been working on the second edition of *The Handbook of Contemporary Semantic Theory* for the past four years. When we started this project we thought that we would produce an update of the first edition. It quickly became apparent to us that we needed a more radical restructuring and revision in order to reflect the very substantial changes that much of the field has experienced in the time since the first edition was published. We think that it is fair to say that the current edition is, in almost all respects, an entirely new book. Most of the authors have changed, the topics have been substantially modified, and much of the research reported employs new methods and approaches.

Editing the *Handbook* has been a highly instructive and enriching experience. It has given us a clear sense of the depth and the vitality of work going on in the field today. We are grateful to the contributors for the enormous amount of thought and effort that they have invested in their chapters. The results are, in our view, of very high quality. We also appreciate their patience and cooperation over the long process of producing and revising the volume. It is their work that has ensured the success of this venture.

We owe a debt of gratitude to our respective families for accepting the distractions of our work on the *Handbook* with understanding and good humor. Their support has made it possible for us to complete this book.

Finally, we are grateful to our editors at Wiley-Blackwell, Danielle Descoteaux and Julia Kirk for their help. We have been privileged to work with them on this and previous projects. We greatly value their professionalism, their support, and their encouragement.

Shalom Lappin and Chris Fox
London and Wivenhoe

Introduction

This second edition of *The Handbook of Contemporary Semantic Theory* is appearing close to 20 years after the first edition was published in 1996. Comparing the two editions offers an interesting perspective on how significantly the field has changed in this time. It also points to elements of continuity that have informed semantic research throughout these years. Many of the issues central to the first edition remain prominent in the second edition. These include, *inter alia*, generalized quantifiers, the nature of semantic and syntactic scope, plurals, ellipsis and anaphora, presupposition, tense, modality, the semantics of questions, the relation between lexical semantics and syntactic argument structure, the role of logic in semantic interpretation, and the interface between semantics and pragmatics.

While many of the problems addressed in the second edition are inherited from the first, the methods with which these problems are formulated and investigated in some areas of the field have changed radically. This is clear from the fact that computational semantics, which took up one chapter in the first edition, has grown into a section of seven chapters in the current edition. Moreover, many of the chapters in other sections apply computational techniques to their respective research questions. As part of this development the investigation of rich-type theories of the kind used in the semantics of programming languages has become a major area of interest in the semantics of natural language. Related to the emergence of such type theories for natural language semantics, we see a renewed interest in proof theory as a way of encoding semantic properties and relations.

Another interesting innovation is the development of probabilistic theories of semantics that model interpretation as a process of reasoning under uncertainty. This approach imports into semantic theory methods that have been widely used in cognitive science and artificial intelligence to account for perception, inference, and concept formation.

The rise of computational approaches and alternative formal methods have facilitated the development of semantic models that admit of rigorous examination through implementation and testing on large corpora. This has allowed researchers to move beyond small fragments that apply to a limited set of constructed examples. In this respect semantics has kept pace with other areas of linguistic theory in which computational modeling, controlled experiments with speakers, and corpus application have become primary tools of research.

The current edition of the *Handbook* is organized thematically into five sections, where each section includes chapters that address related research issues. For some sections the connections among the chapters are fairly loose, bundling together issues that have often been associated with each other in the formal semantics literature. In others, the sections correspond to well defined subfields of research. We have been relaxed about this organizational structure, using it to provide what he hope are useful signpostings to clusters of chapters that deal with a range of connected research problems.

Part I is concerned with generalized quantifiers (GQs), scope, plurals, and ellipsis. In his chapter on generalized quantifiers, Dag Westerståhl provides a comprehensive discussion of the formal

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properties of generalized quantifiers in logic and in natural language. He gives us an overview of research in this area since the late 1980s, with precise definitions of the major classes of GQs, and their relations to the syntactic categories and semantic types of natural language. Particularly useful is his very clear treatment of the expressive power required to characterize different GQ classes. The chapter concludes with a brief discussion of the complexity involved in computing distinct types of GQ.

Chris Barker's chapter analyzes the relationship between semantic scope and syntactic structure. Barker gives us a detailed study of the intricate connections between different sorts of scope interaction and scope ambiguity, and the syntactic environments in which these phenomena occur. He surveys alternative formal and theoretical frameworks for representing the semantic properties of scope taking expressions. He suggests computational models of scope interpretation. This chapter complements the preceding one on GQs, and it provides an illuminating discussion of central questions concerning the nature of the syntax-semantics interface.

Yoad Winter and Remko Scha examine the semantics of plural expressions. A core issue that they address is the distinction between distributive and collective readings of plural noun phrases and verbs. They look at the algebra and the mereology of collective objects, which some plural expressions can be taken to denote. They analyze the relations between different types of quantification and plurality. They consider a variety of theoretical approaches to the problems raised by plural reference. This chapter extends and develops several of the themes raised in the preceding two chapters.

The last chapter in this Part I is devoted to ellipsis. Ruth Kempson *et al*. consider several traditional ellipsis constructions, such as verb phrase ellipsis, bare argument structures, and gapping. They also take up "incomplete" utterances in dialogue. These are constructions that have not generally been handled by the same mechanisms that are proposed for ellipsis resolution. They review the arguments for and against syntactic reconstruction and semantic theories of ellipsis. They consider the application of these theories to dialogue phenomena, and they examine whether a theory of ellipsis can be subsumed under a general theory of anaphora. They propose a unified account of ellipsis within the framework of dynamic syntax, which relies on underspecified linguistic input and informational update procedures for the specification of an incrementally applied "syntax." As in the previous chapters, the role of syntactic mechanisms in determining semantic scope, and the interaction of quantification and scope are important concerns.

Part II consists of chapters on modification, presupposition, tense, and modality. In his chapter on adjectival modification, Dan Lassiter discusses several types of intersective and intensional adjectives, observing that the differences between these classes of modifiers do not constitute a simple binary distinction. An important phenomenon, to which he devotes a considerable amount of attention, is the class of gradable adjectives and the vagueness involved in their application. Lassiter considers leading accounts of gradation, critically discussing theories that posit degrees of modification. In this part of his chapter he describes a probabilistic view of predication, which is further developed in his coauthored chapter with Noah Goodman in Part V.

Chris Potts addresses the nature of presupposition and implicature. He surveys semantic presuppositions, encoded in the meanings of lexical items, and pragmatic presuppositions, which derive from the conditions of successful discourse. He considers the devices for projecting, filtering, and blocking presuppositions through composition of meaning in larger syntactic constructions. Potts gives us a detailed discussion of the relationship between presupposition and pragmatic implicature. He takes up the question of how speakers accommodate both presupposition and implicature in discourse. He critically examines several influential formal theories of the role of presupposition in semantic interpretation.

Tim Fernando's chapter is devoted to tense and aspect. Fernando surveys a variety of temporal logics and semantic theories for representing the structure of time, as it is expressed in natural language. He suggests that this structure corresponds to strings of situations (where situations include the class of events). He proposes the hypothesis that the semantically significant properties and relations that hold among the temporal strings required to interpret tense and aspect can be computed by finite state automata. Fernando offers a detailed discussion of phenomena associated with tense and aspect to motivate his hypothesis.

In the final chapter in Part II, Magdalena and Stefan Kaufmann examine the problems involved in representing different sorts of modal terms. They begin with an overview of modal logic and Kripke frame semantics. Within this framework modal operators are quantifiers over the set of possible worlds, constrained by an accessibility relation. They go on to look at extensions of this system designed to capture the properties of different modal expressions in natural language. A main feature of the system that is subject to revision is the accessibility relation on worlds. It is specified to restrict accessible worlds to those in which the propositions that hold express the common ground of assumptions on which coherent discourse depends. One of the Kaufmanns' central concerns in this chapter is to clarify the relationship between the semantics of modality and the interpretation of conditional sentences.

Part III of the *Handbook* is concerned with the semantics of nondeclarative sentences. In the first chapter in this part, Andrzej Wiśniewski explores the interpretation of questions. A major issue in this area has been the relationship between a question and the set of possible answers in terms of which it is interpreted. Wiśniewski examines this topic in detail. He focusses on the problem of how, given that questions do not have truth values, they can be sound or unsound, and they can sustain inferences and implications. He proposes an account of the semantics of questions within the tradition of erotetic logic, whose historical background he describes.

In the second chapter of this part, Chris Fox discusses the semantics of imperatives. He notes that, like questions, imperatives have logical properties and support entailments, although they lack truth values. He also cites several of the apparent paradoxes that have been generated by previous efforts to model the semantic properties of these sentences. Fox suggests that the logical properties of imperatives are best modelled by a logic in which certain judgement patterns constitute valid inferences, even when their constituent sentences are imperatives rather than propositional assertions. He proposes a fragment of such a logic, which implements an essentially proof-theoretic approach to the task of formalising the semantics of imperatives.

Part IV is devoted to type theory and computational semantics. Aarne Ranta's chapter provides an introduction to the basic concepts of constructive type theory and their applications in logic, mathematics, programming, and linguistics. He demonstrates the power of this framework for natural language semantics with the analysis of donkey anaphora through dependent types. He traces the roots of type theory in earlier work in logic, philosophy, and formal semantics. Ranta illustrates the role of type theory in functional programming through the formalisation of semantically interesting examples in Haskell. He offers an overview of his own system for computational linguistic programming, grammatical framework (GF), in which both the syntactic and semantic properties of expressions are represented in an integrated type theoretical formalism. He goes on to indicate how GF can also be used to capture aspects of linguistic interaction in dialogue.

Robin Cooper and Jonathan Ginzburg present a detailed account of type theory with records (TTR) as a framework for modeling both compositional semantic interpretation and dynamic update in dialogue. They show how TTR achieves the expressive capacity of typed feature structures while sustaining the power of functional application, abstraction, and variable binding in the λ-calculus. A key element of the TTR approach to meaning is the idea that interpretation consists in judging that a situation is of a certain type. Cooper and Ginzburg illustrate how record types and subtyping permit us to capture fine-grained aspects of meaning that elude the classical type theories that have traditionally been used within formal semantics. They also ground TTR in basic types that can be learned through observation as classifiers of situations. In this way TTR builds compositional semantics bottom up from the acquisition of concepts applied in perceptual judgement.

In the third in this part, Shalom Lappin discusses some of the foundational problems that arise with the sparse type theory and Kripke frame semantics of Montague's classical framework.

These include type polymorphism in natural language, fine-grained intensionality, gradience and vagueness, and the absence of an account of semantic learning. Lappin considers property theory with Curry typing (PTCT), which uses rich Curry typing with constrained polymorphism, as an alternative framework of semantic interpretation. He offers a characterization of intensions that relies on the distinction between the denotational and the operational content of computable functions. This provides an explanation of fine-grained intensionality without possible worlds. Lappin concludes the chapter with a brief discussion of probabilistic semantics as an approach that can accommodate gradience and semantic learning.

Ian Pratt-Hartmann addresses the problem of how to determine the complexity of inference in fragments of natural language. He considers various subsets of English exhibiting a range of grammatical constructions: transitive and ditransitive verbs, relative clauses, and determiners expressing several quantifiers. He asks how the expressiveness of these fragments correlates with the complexity of inferences that can be formulated within them. He shows that one can characterize the terms of the tradeoff between the grammatical resources of the fragment on one hand and efficiency of computation on the other, with considerable precision. Following a brief introduction to the basic ideas of complexity theory, Pratt-Hartmann indicates how techniques from computational logic can be used to determine the complexity of the satisfiability problem for the parts of English that he considers. Each of these fragments is identified by a grammar that determines the set of its well formed sentences, and assigns to each of these sentences a model-theoretic interpretation. He then specifies the position of the resulting satisfiability problem with respect to the standard complexity hierarchy. Pratt-Hartmann's chapter introduces a relatively new research program whose objective is to identify the complexity of inference in natural language.

In the fifth chapter in this part, Jan van Eijck considers what is involved in implementing a semantic theory. He compares logic programming and functional programming approaches to this task. He argues for the advantages of using Haskell, a pure functional programming language that realizes a typed λ-calculus as a particularly appropriate framework. Haskell uses flexible, polymorphic typing and lazy evaluation. van Eijck motivates his choice of Haskell, and the project of implementing semantic theories in general, with a detailed set of examples in which he provides Haskell code for computing the representations of central constructions that include, *inter alia*, generalized quantifiers, intransitive, transitive, and ditranstive verbs, passives, relative clauses, and reflexives pronouns. He constructs a model checker to evaluate logical forms, an inference engine for a set of syllogisms, and a system for epistemic update through communication. Each piece of code is clearly discussed and illustrated. Resource programs for the examples are included in an appendix at the end of the chapter.

Stephen Clark provides an in-depth introduction to vector space models of lexical semantics. This approach is motivated by a distributional view of meaning by which one can identify important semantic properties of a term through the linguistic environments in which it occurs. By constructing matrices to encode the distributional values of a lexical item in different contexts and using vector space representations of these patterns, it is possible to apply geometric measures like *cosine* to compute the relative semantic distances and similarities among the elements of a set of words. Clark traces the roots of vector space semantics in information retrieval. He provides worked examples of vector space representations of terms, and cosine relations among them. He devotes the final part of the chapter to the problem of developing a compositional vector space value of a sentence. He describes recent work that uses the types of Joachim Lambek's pregroup grammar as the structural basis for vector composition. The vectors of syntactically complex expressions are computed through tensor products specified in terms of the basis vectors contributed by their constituents.

In the final chapter in this part, Mark Sammons gives us an overview of the Recognizing Textual Entailment (RTE) task. This involves constructing a natural language processing system that correctly identifies cases in which a hypothesis text can be be inferred from a larger piece of text containing a set of assertions that are assumed to hold. As Sammons notes, inference in this task depends upon real-world knowledge, as well as the semantic properties of the sentences in both texts. Recognizing Textual Entailment offers an important test bed for models of interpretation and reasoning. Systems that succeed at this task will have a wide range of applications in the areas of text understanding and dialogue management. Sammons reviews a variety of RTE models ranging from theorem provers to shallow lexical analysis supplemented by statistical machine learning methods. He discusses several state of the art systems, and he gives his outlook for future work in this emerging domain of computational semantics.

Part V of the *Handbook* is devoted to the interfaces between semantics and different parts of the grammar, as well as with other cognitive domains. In his chapter on natural logic Larry Moss considers how much logical entailment can be expressed in natural language. He develops many of the themes introduced in Pratt-Harmann's chapter on semantic complexity, and Sammons' chapter on RTE. Moss formalizes a highly expressive fragment of natural language entailment in an extended syllogistic, which he proves theoretically. He shows that this system is sound and complete, and that a large subclass is decidable. He explores monotonicity properties of quantifiers and polarity features of logical operators. He considers the relationship of Categorial Grammar to the natural logic project. Moss suggests that in selecting a logic to represent natural language entailment we should prefer weaker systems that sustain decidability and tractability. This preference is motivated by the same consideration of cognitive plausibility that guides theory selection in syntax. Lappin applies a similar argument to support an account of intensions that dispenses with possible worlds, in his chapter on type theory.

Malka Rappaport Hovav and Beth Levin approach the syntax-semantics interface from the perspective of the interaction of lexical semantics and syntactic argument structure. They present an overview of the problems involved in identifying the elements of lexical meaning for grammatical heads, specifically verbs, that are relevant to argument realization. They also address the task of specifying principles for projecting the argument patterns of a head from its semantic properties. Rappaport Hovav and Levin look at thematic roles and relations, and the decomposition of lexical meaning into universal features expressing lexical properties and argument relations. They take up the usefulness of thematic role hierarchies in predicting argument patterns, and they critically consider four alternative accounts of argument projection. They illustrate their study of the projection to argument problem with detailed discussion of verb alternation classes.

In his chapter on reference in discourse, Andrew Kehler surveys a range of referring expressions whose referents are underspecified when considered independently of context. These include definite and indefinite noun phrases, demonstratives, and pronouns. He examines a variety of syntactic, semantic, pragmatic, cognitive, and computational factors that play a role in determining reference. Kehler offers a case study of third-person pronouns. He argues that the mechanism that determines the generation of pronouns is distinct from the one that drives interpretation. He presents experimental evidence from psycholinguistic studies on pronoun production and comprehension to support this view. Kehler proposes a Bayesian model of pronominal reference in which the problems of pronominal interpretation and production are to compute the conditional probabilities $p(referent \mid pronoun)$ and $p(pronoun \mid referent)$, respectively, using Bayes' rule.

Noah Goodman and Dan Lassiter propose a probabilistic account of semantics and the role of pragmatic factors in determining meaning in context. On this view, interpretation is a process of reasoning under conditions of uncertainty, which is modeled by Bayesian probability theory. They describe a stochastic λ-calculus and indicate how it is implemented in the programming language, Church. They show how Church functions can be used to assign probabilities to possible worlds, and, in this way, to formalize the meanings of predicates. Compositional procedures of the sort applied in Montague semantics generate probabilistic readings for sentences. Pragmatic factors contribute additional information for updating prior and posterior probabilities through which speakers compute the likelihood of sentences being true in alternative circumstances. Goodman and Lassiter illustrate their approach with detailed examples implemented in Church. They consider several challenging cases, such as quantification and scalar adjectives. Their approach is consonant with ideas suggested in the chapters by Lassiter, Lappin, and Kehler. It applies the methods of mainstream cognitive science to the analysis of linguistic interpretation.

In his chapter on semantics and dialogue, David Schlangen considers the problem of how the interaction between semantics and pragmatics should be captured in an adequate theory of conversation. He points out that, contrary to traditional assumptions, dialogue is not a case of distributed monologue discourse. The interaction of multiple agents is intrinsic to the nature of interpretation in a dialogue. The objects of dialogue are frequently not full sentences. Disfluencies, corrections, repairs, backtracking, and revisions are essential elements of the conversational process. Schlangen studies a variety of phenomena that a good treatment of dialogue must cover. He considers two current theories in detail, and he compares them against the conditions of adequacy that he has identified. He concludes with reflections on the challenges still facing efforts to develop a formal model of dialogue.

Eve Clark discusses the acquisition of lexical meaning in the final chapter of Part V. She provides a guide to the experimental literature on children's learning of words. She describes the processes through which learning is achieved, where these include conversation with adults, specific types of corrective feedback, inference from the meanings of known words to those of new ones, over generalization and restriction, and development of semantic fields and classes. Clark compares two current approaches to word meaning acquisition, considering the comparative strengths and weaknesses of each. She examines different sorts of adult reformulations of child utterances and considers their role in promoting the learning of adult lexical meaning. Clark concludes with the observation that TTR, as described in the chapter by Cooper and Ginzburg, might offer an appropriate formal framework for modelling the update and revision processes through which lexical learning takes place.

Taken together the chapters in the *Handbook* supply a lucid introduction to some of the leading ideas that are propelling cutting-edge work in contemporary semantic theory. They give a vivid sense of the richness of this work and the excitement that surrounds it. Semantics is in a particularly fluid and interesting period of its development. It is absorbing methods and concepts from neighbouring disciplines like computer science and cognitive psychology, while contributing insights and theories to these fields in return. We look forward to the continuation of this flow of research with anticipation.

Part I Quantifiers, Scope, Plurals, and Ellipsis

1 Generalized Quantifiers in Natural Language Semantics*

DAG WESTERSTÅHL

1. Introduction

Generalized quantifiers have been standard tools in natural language semantics since at least the mid-1980s. It is worth briefly recalling how this came about. The starting point was Richard Montague's compositional approach to meaning (Montague, 1974). Frege and Russell had shown how to translate sentences with quantified subjects or objects in first-order logic, but the translation was not compositional. Indeed, Russell made a point of this, concluding that the subject-predicate form of, say, English was *misleading*, since there are no subjects in the logical form. No constituents of the translations

(1) a. $\exists x(\mathit{professor}(x) \wedge \mathit{smoke}(x))$
 b. $\exists x(\forall y(\mathit{king\text{-}of\text{-}F}(y) \leftrightarrow y = x) \wedge \mathit{bald}(x))$

correspond to the subjects "*some professors*" or "*the king of France*" in

(2) a. Some professors smoke
 b. The king of France is bald

respectively. Montague in effect laid this sort of reasoning to rest. He showed that there are compositional translations into *simple type theory*,

(3) a. $((\lambda X \lambda Y \exists x(X(x) \wedge Y(x)))(\mathit{professor}))(\mathit{smoke})$
 b. $((\lambda X \lambda Y \exists x(\forall y(X(y) \leftrightarrow y = x) \wedge Y(x)))(\mathit{king\text{-}of\text{-}F}))(\mathit{bald})$

that, moreover, *β-reduce* precisely to (1a) and (1b). (Montague used an intensional type theory; only the extensional part is relevant here.) The constituent $(\lambda X \lambda Y \exists x(X(x) \wedge Y(x)))(\mathit{professor})$ of (3a), of type $\langle\langle e,t\rangle,t\rangle$, directly translates the DP "*some professors,*" and similarly $(\lambda X \lambda Y \exists x(\forall y(X(y) \leftrightarrow y = x) \wedge Y(x)))(\mathit{king\text{-}of\text{-}F})$ translates "*the king of France.*" Moreover, these English DPs have the form [Det N′], and their determiners are translated by $\lambda X \lambda Y \exists x(X(x) \wedge Y(x))$ and $\lambda X \lambda Y \exists x(\forall y(X(y) \leftrightarrow y = x) \wedge Y(x))$, of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$. Both types of formal expressions denote generalized quantifiers.

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[*] I would like to thank the editors for their patience with this chapter, and an anonymous referee for careful and very helpful remarks.

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Generalized quantifiers had been introduced in logic, for purposes completely unrelated to natural language semantics, by Mostowski (1957) and, in full generality, Lindström (1966). Montague did not appeal to generalized quantifiers, but around 1980 semanticists began to realize that objects of type $\langle\langle e,t\rangle,t\rangle$ and $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$ could interpret arbitrary DPs and Dets, and that logical GQ theory had something to offer; the seminal papers were Barwise and Cooper (1981); Higginbotham and May (1981); Keenan and Stavi (1986). In particular, many common Dets, such as "*most, more than half, an even number of,*" are not definable in first-order logic (*FO*), in contrast with Montague's "*some, every, the.*" But generalized quantifiers are first-order in another sense: they all quantify over individuals. In effect, these authors focused attention on objects of level at most 2 in the type hierarchy. Even when higher types are ignored, a surprising number of linguistic phenomena turn out to be amenable to this setting.

A further step towards classical model theory was taken in van Benthem (1984). Quantifiers of the above-mentioned types are (on each universe) functions from (characteristic functions of) sets to truth values (for DPs), or functions from sets to such functions (for Dets). Van Benthem showed that it was fruitful to construe them as *relations* (unary or binary) *between sets*, and he developed powerful tools for the model-theoretic study of Det denotations. The relational approach ignores the compositional structure that had been the motive to introduce generalized quantifiers into semantics in the first place. But on the other hand it exhibits many of their properties more conspicuously, and makes the applicability of methods from model theory more direct. Besides, for most purposes the functional and the relational approach to generalized quantifiers are essentially notational variants.

In this chapter I will present some highlights of the use of generalized quantifiers in semantics, from the beginning up to the present day. Although many things cannot be covered here, my hope is that the reader will get an impression of the power of these model-theoretic tools in the study of real languages. There are several surveys available where more details concerning particular applications can be found; I will point to them when called for. The reader should *not* leave with the impression, however, that all linguistically interesting issues concerning DPs or determiners (or corresponding means of quantification) can be treated with these tools. Generalized quantifiers are *extensional* objects, and there are subtleties about the meaning of DPs and determiners that they are insensitive to; I will note a few as we go along.[1] This indicates that the tools of GQ theory need to be complemented with other devices, not that they must in the end be abandoned. Indeed, my aim in this chapter is to show that there is a level of semantic analysis for which these tools are just right.

2. Definitions

Quantifiers (from now on I will usually drop "generalized") have a syntactic and a semantic aspect. Syntactically, one constructs a formal language where quantifier symbols are variable-binding operators, like $\forall$ and $\exists$. Unlike $\forall$ and $\exists$, these operators may need to bind the same variable in distinct formulas. For example, a Det interpretation Q concerns two formulas φ and ψ, corresponding to the N′ and the VP in a sentence [[Det N′] VP], and the operator binds the same variable in each. The resulting formula can be written

(4) $Qx(\varphi,\psi)$

as in standard first-order logic with generalized quantifiers, or

(5) $Q(\hat{x}[\varphi])(\hat{x}[\psi])$

as in Barwise and Cooper (1981), or

(6) $[Qx\colon \varphi]\psi$

as in Higginbotham and May (1981).[2] The latter two reflect the constituent structure [[Det N′] VP], whereas (4)—the notation I will use here—fits the relational view of quantifiers. Once a logical language L for quantifiers is fixed, a formal semantics for a corresponding fragment of English can be given via compositional rules *translating* (analyzed) English phrases into L.

However, for this translation to have anything to do with meaning, we need a *semantics* for L. Following a main tradition, this will be a model-theoretic semantics, that is, a specification of a notion of model and a "truth definition"; more accurately, a *satisfaction* relation holding between models, certain L-expressions, and suitable assignments to the variables of corresponding objects in the model. But because our quantifiers are first-order (in the sense explained above), models are just ordinary first-order models, variables range over individuals in universes of such models, and we can help ourselves to the familiar format of the inductive truth definition in first-order logic, with an extra clause for each quantifier besides $\forall$ and $\exists$. To formulate these clauses, we need a precise notion of quantifiers as model-theoretic (not syntactic) objects.

Here it is important to note that quantifiers are *global*: on each non empty set M, a quantifier Q is a relation Q_M between relations over M (i.e. a second-order relation on M), but Q itself is what assigns Q_M to M, that is, it is a function from non empty sets to second-order relations on those sets. (This means that Q is not itself a set but a proper class, a fact without practical consequences in the present context.) The type of Q specifies the number of arguments and the arity of each argument; we use Lindström's simple typing: $\langle n_1, \ldots, n_k \rangle$, where k and each n_i is a positive natural number, stands for a k-ary second-order relation where the i:th argument has arity n_i. So the quantifier in (4) has type $\langle 1, 1 \rangle$ and DP denotations have type $\langle 1 \rangle$; in general, quantifiers of type $\langle 1, \ldots, 1 \rangle$ (relations between *sets*) are called *monadic*, and the others *polyadic*.

Why is it important that quantifiers are global? A reasonable answer is that the meaning of "*every*" or "*at least four*" is independent not only of the nature of the objects quantified over but also the size of the universe (of discourse). "*At least four*" has the *same* meaning in "*at least four cars*," "*at least four thoughts*," and "*at least four real numbers*." These properties are not built into the most general notion of a quantifier. The "topic neutrality" of, for example, "*at least four*" is a familiar model-theoretic property, shared by many (but not all) Det interpretations, but something more is at stake here. A quantifier that meant *at least four* on universes of size less than 100, and *at most ten* on all larger universes would still be "topic-neutral," but it would not mean "the same" on every universe, and presumably no natural language determiner behaves in this way.

We will discuss these properties presently. For now the point is just that the meaning of determiners is such that the universe of discourse is a parameter, not something fixed. This is what makes quantifiers in the model-theoretic sense eminently suitable to interpret them. Indeed, Lindström (1966) defined a quantifier of type τ as a class of models of that type. This is a notational variant of the relational version: for example, for $\tau = \langle 1, 1 \rangle$, writing $(M, A, B) \in Q$ or $Q_M(A, B)$ makes no real difference. But the relational perspective brings out issues that otherwise would be less easily visible, so this is the format we use.

In full generality, then, a (global) *quantifier of type* $\langle n_1, \ldots, n_k \rangle$ is a function Q assigning to each non-empty set M a second-order relation Q_M (if you wish, a local quantifier) on M of that type. Corresponding to Q is a variable-binding operator, also written Q,[3] and $FO(Q)$ is the *logic* obtained from first-order logic FO by adding formulas of the form

(7) $Qx_{11} \ldots x_{1n_1}; \ldots; x_{k1} \ldots x_{kn_k}(\psi_1, \ldots, \psi_k)$

whenever $\psi_1, \ldots, \psi_k$ are formulas. Here all free occurrences of $x_{i1} \ldots x_{in_i}$ (taken to be distinct) are bound in ψ_i by Q. Let $\bar{x}_i$ abbreviate $x_{i1} \ldots x_{in_i}$ and let $\bar{y} = y_1 \ldots y_m$ be the remaining free variables in any of $\psi_1, \ldots, \psi_k$. Then the clause corresponding to Q in the truth (satisfaction) definition for $FO(Q)$ is

$$\mathcal{M} \models Q\bar{x}_1; \ldots ; \bar{x}_k(\psi_1, \ldots, \psi_k)[\bar{b}] \ \Leftrightarrow \ Q_M(R_1, \ldots, R_k)$$

where $\mathcal{M}$ is a model with universe M, $\bar{b} = b_1, \ldots, b_m$ is an assignment to $\bar{y}$, and R_i is the set of n_i-tuples $\bar{a}_i = a_{i1}, \ldots, a_{in_i}$ such that $\mathcal{M} \models \psi_i[\bar{a}_i, \bar{b}]$. As noted, for monadic Q we can simplify and just use one variable:

$$Qx(\psi_1, \ldots, \psi_k)$$

Then, relative to x, and an assignment to the other free variables (if any) in $\psi_1, \ldots, \psi_k$, each ψ_i defines a subset of M.

We will mostly deal with the quantifiers themselves rather than the logical languages obtained by adding them to *FO*. The logical language is, however, useful for displaying scope ambiguities in sentences with nested DPs. And it is indispensable for proving negative expressibility results: To show that Q is *not* definable from certain other quantifiers, you need a precise language for these quantifiers, telling you exactly which the possible defining sentences are.

As noted, a main role for GQ theory in semantics will be played by a certain class of type $\langle 1, 1\rangle$ quantifiers: those interpreting determiners.[4] Here are some examples.

(8) $\mathit{every}_M(A,B) \Leftrightarrow A \subseteq B$
$\mathit{some}_M(A,B) \Leftrightarrow A \cap B \neq \emptyset$
$\mathit{no}_M(A,B) \Leftrightarrow A \cap B = \emptyset$
$\mathit{some\ but\ not\ all}_M(A,B) \Leftrightarrow A \cap B \neq \emptyset$ and $A - B \neq \emptyset$
$\mathit{at\ least\ four}_M(A,B) \Leftrightarrow |A \cap B| \geq 4$ ($|X|$ is the cardinality of X)
$\mathit{between\ six\ and\ nine}_M(A,B) \Leftrightarrow 6 \leq |A \cap B| \leq 9$
$\mathit{most}_M(A,B) \Leftrightarrow |A \cap B| > |A - B|$
$\mathit{more\ than\ a\ third\ of\ the}_M(A,B) \Leftrightarrow |A \cap B| > 1/3 \cdot |A|$
$\mathit{infinitely\ many}_M(A,B) \Leftrightarrow A \cap B$ is infinite
$\mathit{an\ even\ number\ of}_M(A,B) \Leftrightarrow |A \cap B|$ is even
$(\mathit{the}_{sg})_M(A,B) \Leftrightarrow |A| = 1$ and $A \subseteq B$
$(\mathit{the}_{pl})_M(A,B) \Leftrightarrow |A| > 1$ and $A \subseteq B$
$\mathit{the\ ten}_M(A,B) \Leftrightarrow |A| = 10$ and $A \subseteq B$
$\mathit{Mary's}_M(A,B) \Leftrightarrow \emptyset \neq A \cap \{b : \mathit{has}(m,b)\} \subseteq B$
$\mathit{some\ professors'}_M(A,B) \Leftrightarrow \mathit{professor} \cap \{a : A \cap \{b : \mathit{has}(a,b)\} \subseteq B\} \neq \emptyset$
$\mathit{no\ldots except\ Sue}_M(A,B) \Leftrightarrow A \cap B = \{s\}$

The first three are classical Aristotelian quantifiers, except that Aristotle seems to have preferred the universal quantifier with *existential import* (or else he just restricted attention to properties with non-empty extensions):

(9) $(\mathit{all}_{ei})_M(A,B) \Leftrightarrow \emptyset \neq A \subseteq B$

The next three are numerical quantifiers: let us say that Q is *numerical* if it is a Boolean combination of quantifiers of the form *at least* n, for some $n \geq 0$. Note that this makes *every*, *some*, and *no* numerical, as well as the two *trivial* quantifiers **0** and **1**:

(10) $\mathbf{1}_M(A,B) \Leftrightarrow |A \cap B| \geq 0$, i.e. $\mathbf{1}_M(A,B)$ holds for all M and $A, B \subseteq M$
$\mathbf{0} = \neg\mathbf{1}$, i.e. $\mathbf{0}_M(A,B)$ holds for no M, A, B

(This is for type $\langle 1, 1\rangle$; similarly for other types.) Then come two proportional quantifiers: Q is *proportional* if the truth value of $Q_M(A,B)$ depends only on the proportion of Bs among the As:

(11) For $A, A' \neq \emptyset$, if $|A \cap B|/|A| = |A' \cap B'|/|A'|$ then $Q_M(A,B) \Leftrightarrow Q_M(A',B')$.[5]