# Stephan Eggersglüß · Rolf Drechsler

# High Quality Test Pattern Generation and Boolean Satisfiability



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#### Preface

The content of the book describes work that has been carried out in the Group of Computer Architecture at the University of Bremen, Germany over the last 5 years. Therefore, we would like to thank all the members of the group for their valuable help. Special thanks go to Daniel Tille and Görschwin Fey for many helpful discussions and support. Both are also co-authors of our previously published book *Test Pattern Generation using Boolean Proof Engines* which describes among other things the basics principles on which the content of this book is based on.

Various chapters of this book are based on scientific papers. Therefore, we would like to acknowledge the work of the co-authors of these papers Görschwin Fey, Hoang M. Le, Juergen Schloeffel and Daniel Tille. Since large parts of the work has been done in collaboration, our special thanks go to the Mentor Graphics Development group in Hamburg, Germany, especially to René Krenz-Bååth (now Hochschule Hamm-Lippstadt). Finally, we would like to thank Lisa Jungmann for her help with the cover design as well as Robert Wille, Judith End and Tom Gmeinder for proof-reading.

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Bremen

Stephan Eggersglüß Rolf Drechsler

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# List of Acronyms

| ARAP   | As-Robust-As-Possible                 |
|--------|---------------------------------------|
| ATE    | Automatic Test Equipment              |
| ATPG   | Automatic Test Pattern Generation     |
| BCP    | Boolean Constraint Propagation        |
| BDD    | Binary Decision Diagram               |
| CNF    | Conjunctive Normal Form               |
| CUT    | Circuit Under Test                    |
| DCA    | Dynamic Clause Activation             |
| DFT    | Design-For-Test                       |
| DTPG   | Deterministic Test Pattern Generation |
| GDFM   | Gate Delay Fault Model                |
| IC     | Integrated Circuit                    |
| IG     | Implication Graph                     |
| ISAT   | Incremental SAT                       |
| LC     | Logic Class                           |
| LWL    | Learned Watch List                    |
| ODC    | Observability Don't Care              |
| PBO    | Pseudo-Boolean Optimization           |
| PB-SAT | Pseudo-Boolean SAT                    |
| PDF    | Path Delay Fault                      |
| PDFM   | Path Delay Fault Model                |
| PI     | Primary Input                         |
| PO     | Primary Output                        |
| PPI    | Pseudo Primary Input                  |
| PPO    | Pseudo Primary Output                 |
| RTPG   | Random Test Pattern Generation        |
| s-a-0  | Stuck-at-0                            |
| s-a-1  | Stuck-at-1                            |
| SAFM   | Stuck-At Fault Model                  |
| SAT    | Boolean Satisfiability, Satisfiable   |
|        |                                       |

| SDD   | Small Delay Defect     |
|-------|------------------------|
| SWL   | Structural Watch List  |
| TF    | Transition Fault       |
| TFM   | Transition Fault Model |
| UNSAT | Unsatisfiable          |

# List of Symbols

| $\downarrow$      | falling transition                                |
|-------------------|---|
| ↑                 | rising transition                                 |
|                   | Boolean AND operator                              |
| +                 | Boolean OR operator                               |
| $\odot$           | resolution operator                               |
| $\oplus$          | Boolean XOR operator                              |
| ÷                 | Boolean NOT of ·                                  |
| $\rightarrow$     | implies   |
| $\leftrightarrow$ | equivalence                                       |
| Φ                 | CNF, set of clauses                               |
| $\Phi_{\rm dyn}$  | dynamically extended CNF                          |
| $\Phi_F$          | fault-specific constraints                        |
| Ψ                 | set of pseudo-Boolean constraints                 |
| Ψ                 | pseudo-Boolean constraint of $\Psi$               |
| η                 | Boolean encoding                                  |
| κ                 | conflict  |
| λ                 | arbitrary literal                                 |
| ω                 | clause of $\Phi$                                  |
| $\omega_{C}$      | conflict clause                                   |
| @x                | at decision level x                               |
| $\mathbb{B}$      | set of Boolean values $\{0,1\}$                   |
| C                 | circuit   |
| Ŧ                 | set of flip-flops                                 |
| F                 | fault   |
| f                 | flip-flop $\in \mathscr{C}$ , faulty line of gate |
| $\mathscr{F}(g)$  | transitive fanin of g                             |
| G                 | set of gates                                      |
| 8                 | gate $\in \mathscr{C}$                            |
| h                 | successor gate of $g$                             |
| I                 | set of primary inputs                             |
| $i_1,\ldots,i_n$  | primary inputs of $\mathscr{C}$                   |

| J                 | J-stack                                    |
|-------------------|--|
| $\mathscr{L}_{x}$ | multiple-valued logic with x values        |
| $\mathscr{O}$     | set of primary outputs                     |
| $o_1,, o_m$       | primary outputs of $\mathscr{C}$           |
| P                 | structural path of $\mathscr{C}$           |
| S                 | set of signal lines or connections         |
| S                 | signal line, connection                    |
| $t_i$             | initial (current) time frame               |
| $t_{i+1}$         | final (next) time frame                    |
| V                 | vector                                     |
| ν                 | (Boolean) value                            |
| X                 | set of Boolean variables, don't care value |
| $x_1,\ldots,x_n$  | Boolean variables                          |
|                   |  |

#### Chapter 1 Introduction

The *Integrated Circuit* (IC) was invented in the 1950s. At the beginning, ICs were mainly used in computers. With the advancing miniaturization of the components, the significance of ICs as part of our daily life grows. Many consumer products such as mobile music players or cell phones use ICs (or "chips") as core engines. A failure of these devices usually results in problems of lesser extent for the owner. But today, ICs also control safety critical applications. For example, chips are responsible for the correct mode of operation in car control systems, avionics or medical equipment. A failure of a chip can be life-threatening in the worst-case. Consequently, the correct mode of operation of a fabricated chip is crucial.

Due to the ever shrinking component sizes of today's designs, the vulnerability of chips to flaws in the manufacturing process increases. The IC manufacturers put much effort in guaranteeing the integrity of their products. A large part of the manufacturing costs is spent for the detection of defects caused by the manufacturing process. Every fabricated chip is subjected to a *post-production test* (or *manufacturing test*) to avoid that defective chips are delivered to customers (and by this could cause failures in operation mode). Thus, the purpose of such a post-production test is to detect any defects caused by the manufacturing process.

Stimuli are applied to the inputs of the *Circuit Under Test* (CUT) during this test. The output responses are monitored. If one or more of the output responses are inconsistent with the specification, the chip will be rejected as erroneous. However, the complexity of modern designs does not allow for a complete test of all possible input stimuli. A complete test for each fabricated chip would be far too time-consuming or costly, since the number of possible tests is exponential in the number of inputs. Instead, a test set is pre-computed that covers a large range of possible defects. Logical fault models are used to abstract from physical defects. The fault model most widespread is the *Stuck-at Fault Model* (SAFM) [Eld59].

This test set is applied to each fabricated chip by *Automatic Test Equipment* (ATE). Because the memory and bandwidth of an ATE is limited, the applied test set has to be as small as possible. A large test set size signifies not only a long test application time but also immense test costs. The computation of the test set, which is known as *Automatic Test Pattern Generation* (ATPG), is the main subject of this book. Due to the large number of potential faults, ATPG is a computationally intensive task and fast algorithms are needed for obtaining a test set in acceptable run time.

Classical ATPG algorithms are mostly based on the D-algorithm proposed by Roth in 1966 [Rot66]. These algorithms work directly on the circuit structure, i.e. a flat gate-level netlist, and typically benefit significantly from their knowledge about the structure of the problem. Several techniques and powerful heuristics have been proposed over the years to improve the effectiveness of ATPG. At the turn of the millennium, the ATPG problem was considered to be solved. The existing algorithms were fast enough and provided sufficient fault coverage. This has changed in the last years.

The size of new designs doubles every 18 months according to Moore's law [Moo65]. Today's circuits consist of multi-million gates and the classical structural ATPG algorithms reach their limits. Tests for a large number of faults can still be generated very quickly. But the size of the set of faults for which no test can be generated in acceptable run time increases significantly. As a result, the high fault coverage demands of the chip manufacturers can barely be met. Thus, the overall quality of the test set decreases due to the lower fault coverage. As a consequence, there is a need for new robust ATPG algorithms especially when considering future design sizes.

Furthermore, another crucial point has been emerged in the field of manufacturing test. Due to the increased operation speed beyond the GHz mark and the small manufacturing technologies, the number of timing-related defects which affects the product quality has increased. This trend is expected to grow with the process technology scaling down towards very deep sub-micron devices. Therefore, *delay testing* has become mandatory to filter out defective devices and to assure that the desired performance specifications are met.

Test generation for delay faults generally needs more computational effort than ATPG for stuck-at faults because test patterns have to be computed over at least two time frames. A large number of delay faults remain unclassified during ATPG for modern designs. Moreover, in contrast to the SAFM, test patterns for delay faults can be classified in different quality levels like robust and non-robust test patterns [KC98]. The quality of a test can be – simply spoken – defined as the probability of fault detection. High quality test patterns are more desirable but usually harder to obtain. As a result, the need for new robust ATPG algorithms is even more urgent in the field of delay test generation. In the last years, *Small Delay Defects* (SDDs) have become more and more serious. Therefore, the ability to detect SDDs has become an important indicator for judging the quality of a delay test. However, ATPG approaches struggle with the generation of tests dedicated to detect SDDs due to the high complexity.

A promising solution to close the gap between test quality requirements and ATPG effectiveness is the application of solvers for *Boolean Satisfiability* (SAT).