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Aranya Chakrabortty
Marija D. Ilić *Editors*

Control and Optimization Methods for Electric Smart Grids

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Aranya Chakrabortty • Marija D. Ilić
Editors

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Editors
Aranya Chakrabortty
Electrical and Computer Engineering
FREEDM Systems Center
North Carolina State University
Raleigh, NC 27695, USA
aranya.chakrabortty@ncsu.edu

Marija D. Ilić
Electrical and Computer Engineering
Engineering and Public Policy
Carnegie Mellon University
Pittsburgh, PA 15213-3890, USA
milic@ece.cmu.edu

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To Joe,
A true pioneer in rethinking sensing, communications and control of electric power systems

Preface

In the 21st century, electric power engineering is going green and smart. Triggered by several recent catastrophes such as the major blackout in the Northeastern USA in 2003 and Hurricane Katrina in New Orleans in 2005 together with the Energy Act of 2007, the term *smart grid* has become almost ubiquitous across the world not only as a political concept but also as an entirely new technology requiring a tremendous amount of inter-disciplinary research. The initiatives taken by the smart grid research community in the United States have so far been successful in bringing researchers from power system engineering, signal processing, computer science, communications, business, and finance as well as chemical and wind engineering among other disciplines under the same roof in order to cater to the diverse research needs of this technology. As a part of this enterprise power engineers, for example, are investigating efficient and intelligent ways of energy distribution and load management, computer scientists are researching cyber security issues for reliable sharing of information across the grid, the signals community is looking into advancing instrumentation facilities for detailed grid monitoring, wind engineers are studying renewable energy integration, while business administrators are reframing power system market policies to adapt to these new changes in the system.

Concomitant with these advances, researchers have also come to recognize the urgent need for new systems-level knowledge of power and energy systems for sustaining the advancement of this emerging field. This, in turn, has led to a natural demand for the two lifelines of system theory in smart grid research, namely – control and optimization. Almost every facet of making a power system *smart* or self-regulated boils down to using control theory in some form or other. Relevant examples include modeling, identification, estimation, robustness, optimal control, and decision-making over networks. Over the past two years, for instance, several research workshops, conference tutorial sessions, and national meetings have been organized to discuss the strong potential of control and optimization in smart grid applications starting from small residential-level energy management, smart metering, and power markets to much broader-scale problems such as wide-area monitoring and control. The editors of this book have been involved in the organization of many of these workshops. Several of the contributing authors too have given invited presentations in these gatherings, three of the most notable ones being a special session in the IEEE Conference on Decision and Control (Atlanta, GA, 2010), a tutorial session in the American Control Conference (San Francisco, CA, 2011) and a Workshop on Cyber-Physical Applications in Smart Power Systems (North Carolina State University, Raleigh, NC, 2011). The initial idea for publishing this book arose from these meetings with the objective of consolidating some of the most promising and transformative recent research in smart grid control in hopes of laying the foundation for future advances in this critical field of study.

The book contains eighteen chapters written by leading researchers in power, control, and communication systems. The essays are organized into three broad sections, namely Architectures and Integration, Modeling and Analysis, and Communication and Control. As is apparent from their titles, the main perspective of these sections is to capture in a holistic way how tomorrow's grid will need to be an enormously complex system in order to solve the problems that we are facing today. Literally, with every passing day, our national grid is becoming integrated with new generation in the form of renewable energy resources, new loads in the form of smart vehicles, new sensors such as smart meters and Phasor Measurement Units, and newer mechanisms of decision-making guided by complex power market dynamics. Our goal is to capture the spectrum of this exponential transformation, and at the same time present the plethora of open problems that this transformation poses for our control theory colleagues. Many of these problems may sound like routine questions in control and optimization, but they often lead to challenging, interesting, and ultimately highly rewarding directions for theoretical research. To the best of our knowledge, this is the first comprehensive book on this topic.

The Architectures and Integration section opens the book with visionary ideas on sustainable architectures for power system operation and control under significant penetration of highly variable renewable energy resources presented in Chap. 1. This is followed by a discussion on the economics of electricity markets and their impacts on demand response in Chap. 2. Chapter 3 furthers the demand response concept for enabling random energy integration. Chapter 4 illustrates several practical constraints in smart grid sensing and communications that may destabilize real-time power market operations and proposes new communication topologies that can bypass such problems. Chapter 5 presents a fresh control perspective to demand-side energy management in residential and commercial units using convex optimization-based model predictive control. Chapter 6 highlights the architectural challenges needed for integration of plug-in-hybrid vehicles into the grid focusing on problems related to demand response and communications necessary to accomplish the smart features of these smart vehicles.

The Modeling and Analysis section presents the upcoming research directions on mathematical modeling, data analysis, and information processing in power systems. Chapter 7 opens this section with a modeling framework that can be highly useful for analyzing the impacts of wind power penetration on the dynamics of the conventional grid. Chapter 8 delves into novel data analysis techniques for wide-area oscillation tracking in large-scale power systems and highlights the importance of signal processing as a major tool for wide-area monitoring research. Chapter 9 models the dynamic mechanisms of cascading failures in geographically dispersed grids, while Chap. 10 presents a reliability modeling framework for tomorrow's phasor-integrated power system using ideas of real-time fault diagnosis and Markovian models of measurement networks. The discussion switches gears towards the computational aspects of the smart grid in Chap. 11, which presents a modeling and control strategy for data centers that are becoming essential parts of today's grid operations. Chapter 12 closes this section with a discussion on how various geometrical properties of power system topologies can influence disturbance propagation in the system, thereby highlighting the importance of combinatorics in smart grid research.

The Communication and Control section unites some of the most critical control design challenges for tomorrow's grid with a focus on how communications and security will play integral roles in the execution of such controllers. The section opens with Chap. 13, which presents game-theoretic optimization problems for charging coordination of plug-in electric vehicles, and, thereby addresses the intersection of two emerging trends in the modernization of power systems, namely vehicle electrification and flexible loading. Chapter 14 addresses the seminal problem of cyber-security of smart grids and presents a vulnerability assessment framework to quantify risk due to intelligent coordinated attacks on grid assets. Chapter 15 discusses the applications of wide-area phasor measurement technology in distribution-level power systems, initiating a new line of thinking on monitoring and control. Chapter 16 fuses different ideas of cooperative control theory to develop *distributed* algorithms for economic dispatch, thereby enabling the future grid to be independent of centralized decision-making strategies. Chapters 17 and 18 address wide-area control problems for oscillation damping in power transmission systems. The former presents a new adaptive control approach for delay compensation in Synchrophasor-based feedback, while the latter using model reference control and clustering methods for adding damping to inter-area oscillations.

As can be seen, the chapters in each section maintain their own thematic continuity and at the same time have significant overlaps with chapters in other sections as well. Therefore, one may read the book in its entirety or focus on individual chapters. Due to its broad scope, this will be an ideal resource for students in advanced graduate-level courses and special topics in both power and control systems. It will also interest utility engineers who seek an intuitive understanding of the emerging applications of control and optimization methods in smart grids. Until now there has been very little literature concerning the formulation of a comprehensive control problem for the smart grid enterprise, and on relating different models and approaches to its overall solution methodology. The chapters in this book represent work in progress by the community on the way towards such solutions.

We would like to express our gratitude to all the contributing authors for providing their valuable input toward the development of this book. We also thank our colleagues Murat Arcak, John Wen, Tariq Samad, Ning Lu, Steven Elliot, M. A. Pai, Massoud Amin, Manu Parashar, Sumit Roy, and Yufeng Xin for many

interesting discussions that have motivated us to undertake the idea of publishing this book. Sincere thanks also go to Merry Stuber from Springer for proofreading different versions of the manuscript and guiding the editorial work.

Last but not least, this book is dedicated to Joe Chow on the occasion of his 60th birthday. Dr. Chow is one of the most distinguished researchers and educators in the field of power systems and control theory. His research career spans nearly 40 years, and includes pioneering contributions to singular perturbation theory in the late 1970s, multivariable control of power systems in the 1980s, FACTS controller designs in the 1990s, and Wide-area Phasor Measurements over the past two decades. His ground-breaking work has earned him the highest acclaim from both power and control research communities all around the world. We take great pleasure in offering this book as a small token of appreciation in honor of Joe on this very special occasion.

Raleigh, NC, USA Aranya Chakrabortty
Pittsburgh, PA, USA Marija D. Ilić

Contents

Contributors

Anuradha Annaswamy Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, USA

I. Safak Bayram Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA

Subhashish Bhattacharya Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA

Duncan Callaway Energy and Resources Group, University of California, Berkeley, CA, USA

Aranya Chakrabortty Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA

Lijun Chen Engineering and Applied Science, California Institute of Technology, Pasadena, CA, USA

Joe H. Chow Electrical, Computer and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY, USA

Mo-Yuen Chow Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA

Michael Devetsikiotis Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA

Alejandro D. Domínguez García Department of Electrical Engineering, University of Illinois, Urbana Champaign, IL, USA

S. Ghiocel Electrical, Computer and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY, USA

Manimaran Govindarasu Department of Electrical and Computer Engineering, Iowa State University, Ames, IA, USA

Fabrizio Granelli Department of Information Engineering and Computer Science, University of Trento, Trento, Italy

Daniel Greene Palo Alto Research Center, Palo Alto, CA, USA

Daniel A. Haughton School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ, USA

Gerald Heydt School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ, USA

T. Hikihara Department of Electrical Engineering, Kyoto University, Kyoto, Japan

Haitham Hindi Palo Alto Research Center, Palo Alto, CA, USA

Ian Hiskens Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA

Marija D. Ilić Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA

Libin Jiang Engineering and Applied Science, California Institute of Technology, Pasadena, CA, USA

Arman Kiani Institute of Automatic Control Engineering, Technische Universität München, Munich, Germany

Anupama Kowli CSL and the ECE Department, University of Illinois, Urbana-Champaign, IL, USA

Bruce Krogh Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA

Caitlin Laventall AOL.com, Palo Alto, CA, USA

Na Li Engineering and Applied Science, California Institute of Technology, Pasadena, CA, USA

Chen-Ching Liu School of Electrical, Electronic and Mechanical Engineering, University College Dublin, Dublin, Ireland

Qixing Liu Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA

Steven H. Low Engineering and Applied Science, California Institute of Technology, Pasadena, CA, USA

Zhongjing Ma School of Automation, Beijing Institute of Technology, and the Key Laboratory of Complex System Intelligent Control and Decision (Beijing Institute of Technology), Ministry of Education, Beijing, China

Sean Meyn CSL and the ECE Department, University of Illinois, Urbana-Champaign, IL, USA

I. Mezic Department of Mechanical Engineering, University of California, Santa Barbara, CA, USA

George Michailidis Department of Statistics, University of Michigan, Ann Arbor, MI, USA

Matias Negrete-Pincetic CSL and the ECE Department, University of Illinois, Urbana-Champaign, IL, USA

Xueping Pan School of Energy and Electrical Engineering, Hohai University, Nanjing, China

Luca Parolini Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA

Matthew C. Ruschmann Department of Electrical and Computer Engineering, Binghamton University, Binghamton, NY, USA

Anna Scaglione University of California, Davis, CA, USA

Ehsan Shafieepoorfard CSL and the ECE Department, University of Illinois, Urbana-Champaign, IL, USA

Uday V. Shanbhag Department of Industrial and Enterprise Systems Engineering, University of Illinois, Urbana Champaign, IL, USA

Bruno Sinopoli Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, USA

Siddharth Sridhar Department of Electrical and Computer Engineering, Iowa State University, Ames, IA, USA

Y. Susuki Department of Electrical Engineering, Kyoto University, Kyoto, Japan

Robert J. Thomas Cornell University, Ithaca, NY, USA

Vaithianathan "Mani" Venkatasubramanian School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA, USA

Gui Wang CSL and the ECE Department, University of Illinois, Urbana-Champaign, IL, USA

Zhifang Wang University of California, Davis, CA, USA

N. Eva Wu Department of Electrical and Computer Engineering, Binghamton University, Binghamton, NY, USA

Ziang Zhang Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, NC, USA

Part I
Architectures and Integration

Toward Sensing, Communications and Control Architectures for Frequency Regulation in Systems with Highly Variable Resources

Marija D. Ilić and Qixing Liu

Abstract The basic objective of this chapter is to rethink frequency regulation in electric power systems as a problem of cyber system design for a particular class of complex dynamical systems. It is suggested that the measurements, communications, and control architectures must be designed with a clear understanding of the temporal and spatial characteristics of the power grid as well as of its generation and load dynamics. The problem of Automatic Generation Control (AGC) and frequency regulation design lends itself well to supporting this somewhat general observation because its current implementation draws on unique structures and assumptions common to model aggregation in typical large-scale dynamic network systems. We describe how these assumptions are changing as a result of both organizational and technological industry changes. We propose the interactions variable-based modeling framework necessary for deriving models, which relax conventional assumptions when that is needed. Using this framework, we show that the measurements, communications, and control architectures key to ensuring acceptable frequency response depend on the types of disturbances, the electrical characteristics of the interconnected system and the desired technical and economic performance. The simulations illustrate several qualitatively different electric energy systems. This approach is by and large motivated by today's AGC and its measurement, communications and control architectures. It is with this in mind that we refer to our interactions variable-based frequency regulation framework as "enhanced AGC" (E-AGC). The enhancements come from accounting for temporal and spatial characteristics of the system which require a more advanced frequency regulation design than the one presently in place. Our proposed interactions variable-based aggregation modeling could form the basis

M.D. Ilić (✉) • Q. Liu
Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA 15213, USA
e-mail: milic@ece.cmu.edu; lqx@cmu.edu

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for a coordination of interactions between the smart balancing authorities (SBAs) responsible for frequency regulation in the changing industry. Given the rapid deployment of synchrophasors, the proposed E-AGC can be easily implemented.

1 Introduction

A high quality of electricity service assumes near-ideal nominal frequency, which is the result of almost instantaneous power supply and demand balancing. In actual systems, deviations from this perfect balance always exist and are mainly caused by hard-to-predict demand and by the lack of direct power control exchange between neighboring utilities. Therefore, the resulting frequency must be regulated back to nominal by means of output feedback control responding to the frequency errors caused by the power imbalances. Much is known about Automatic Generation Control (AGC), and this control scheme is considered one of the most ingenious elegant feedback schemes in large-scale man-made network systems. Its effectiveness comes from viewing the problem as quasi-stationary and accounting for the fact that, at equilibrium, the system operates at a single frequency. This, together with the assumption that utilities are weakly connected, forms the basis for using Area Control Error (ACE) as the single output to which several fast-responding power plants respond in each utility (control area) and drive the error to zero within 10 min or so. It can be shown that, at theoretical equilibrium, the entire interconnected system will balance and frequency should return to nominal, provided that each control area (CA) responds to its own ACE without communicating with other CAs.

However, the question of "correct" AGC standards and a utility's ability to meet these standards has been a long-standing subject of many industry committees led by the North American Electric Reliability Organization (NERO). Many of the industry's great thinkers, starting with the late Nathan Cohn, the father of AGC, have brought the scheme to near perfection over the years. Understanding the fundamentals of AGC, including the elusive notions of Inadvertent Energy Exchange (IEE) and Time Error Correction (TEC), requires an in-depth treatment, which is outside the objectives of this chapter; the interested reader should explore the extensive literature on this subject. Ensuring that the standards are met has been difficult up until very recently because of the lack of synchronized measurements. As a result, there have been growing concerns about the worsening of frequency quality. These concerns have become magnified with the prospects of deploying highly variable energy resources, such as wind and solar power farms. This overall situation requires a rethinking of today's AGC industry practice from the viewpoint of the key assumptions underlying its design. In particular, given the recent deployments of synchrophasor technologies capable of providing fast and synchronized measurements across wide areas, the question of an enhanced AGC (E-AGC) design and its purpose presents itself quite forcefully.

In this chapter, we pose the problem of E-AGC as a control design problem based on a carefully designed dynamic model directly relevant for frequency regulation in complex power systems. We derive a model which can capture the effects of disturbances likely to be seen in future electric power systems with many intermittent resources. We observe that today's AGC is used for the fine tuning of frequency deviations caused by small hard-to-predict power imbalances around the generation dispatched to supply forecast demand. As such, it is viewed as regulating steady-state frequency offsets over the time interval of 10–15 min; primary governor control at the same time compensates for local fluctuations very quickly, without considering dynamic interactions with the rest of the system. Simplicity is achieved by having fast local primary control, which is tuned without modeling fast dynamic interactions with the rest of the system. AGC, on the other hand, compensates for steady-state net power imbalances in each utility, assuming that the electrical distances between the generators in the CA are negligible; the effects of other CAs are compensated for by responding to the ACE, which is a measure of the power imbalance created by the internal demand deviations from the forecast and the deviations in net power exchange from the scheduled exchanges at the time of dispatch in each CA.

In emerging electric energy systems, the implied spatial simplifications when designing fast-stabilizing primary control, as well as the implied temporal simplifications assuming near steady-state conditions when performing AGC and accounting for the effects of net imbalances within a large multi-CA interconnected power system, will become hard to justify. Depending on the electrical properties of the system, the nature of the disturbances causing the frequency changes, and the dynamics of the power plants and loads, it is plausible that frequency response may become very different than the historic response has been. Models for analyzing and predicting the likely frequency response and, consequently, designing controls for stabilizing and regulating frequency have become a very difficult problem. A power system driven by continuously varying persistent disturbances does not lend itself well to separating stabilization and regulation objectives.

In order to begin to answer these difficult questions, we pose in Sect. 2 the problem of frequency regulation by first reviewing a general dynamic model of an interconnected power system capable of representing typical system response to these new disturbances. To start with, this general model is very complex and without obvious structure. In Sect. 3, we propose a systematic model reduction which lends itself well to the enhanced frequency regulation design. Of particular interest is the derivation of models capable of capturing dynamic interactions between (groups of) system users given the ultimate objective of designing the minimum coordination architecture across these groups of system users necessary to control power imbalance interactions causing potentially unacceptable frequency response. Notably, the concepts introduced early on by Joe Chow and his collaborators for model reduction in electric power systems are shown to be key to arriving at the models for enhanced frequency regulation of interest here. Model simplification using standard singular perturbation is used to introduce acceptable temporal simplifications of the generator models used. Similarly, the lesser-known nonstandard singular perturbation method is used to prove the existence of the interactions variable, which can capture the dynamics of power imbalances across large power grid interconnections. The question of aggregation within an interconnected system and the implications of the grouping selected on the achievable frequency quality and the complexity of the required sensing, communications and control architectures is discussed in considerable detail. We recognize that aggregation principles could be based on: (1) the pre-defined organizational boundaries of the consumers and producers responsible for balancing supply and demand; the most typical representatives are utilities and/or CAs; (2) the bottom-up created portfolio of users and producers; representatives of such aggregated system users can likely be entities comprising users with their own distributed energy resources; and (3) best technical decomposition of a given dynamical system from the point of view of having coherent dynamic response and being controllable and observable without relying on help from the others. In Sect. 4, the model relevant for frequency regulation of each SBA is simulated to show the qualitatively different interactions variables resulting from the different relative electrical distances internal and external to the SBAs. In Sect. 5, we illustrate the use of coherency-based method introduced by Joe Chow for the four qualitatively different physical systems of seemingly identical design. We show the system aggregation which results from this approach. In Sect. 6, we compare the aggregation obtained using our proposed interactions variable-based modeling of SBAs with coherency-based aggregation. Interestingly, when inquiring whether interconnection-level coordination may be needed, the two methods arrive at the same conclusion via different paths. For SBAs with strong internal interactions, the coherency-based method leads to the conclusion that a meaningful aggregation would be to have one single system. The interactions variable-based approach concludes that, because the interactions are strong, it is essential to coordinate them. Notably, the model proposed here does not neglect the effects of electrical distances; this is in sharp contrast with today's AGC. We show how the interactions variables are affected by the relative strength of electrical interconnections, both internal to the SBA and also in-between the SBAs.

We next show how these qualitatively different cases lend themselves to different measurement, communications, and control architectures. In Sect. 7, a required sensing, communications and control architecture based on the interactions variable-based model is drawn up, and the results of using such a control architecture are illustrated for the four cases of a small system studied. In Sect. 8, a required sensing, communications, and control architecture based on coherency-based aggregation is sketched out. The complexity of the two, and a comparison of the two, are summarized. In Sect. 9, we discuss the relationship between the proposed cyber architecture for ensuring acceptable frequency response and the AGC architecture of today. We highlight and confirm how the complexity of a relevant dynamic model for frequency regulation depends on the locations and the type of disturbances, and on the electrical characteristics of the interconnected system comprising the CAs with the same boundaries. This complexity will determine the most adequate cyber architecture. We stress that as the industry undergoes both technological and organizational changes, it is going to become difficult to break down a complex interconnected system into weakly connected subsystems, which would lend themselves to uncoordinated frequency regulation. For example, a CA with significant wind power capacity and very few fast-responding power plants may need to rely on power sent by other areas which have this type of plants. Provided this is done, the cost of regulating frequency at the interconnection level as a whole will be lower than without coordination. Examples of such sharing of resources across CAs already exist and are known as dynamic scheduling. A hydro power plant in CA_2 may provide regulation to the CA_1, for example. This all leads to the observation that a dynamic model relevant for the effective frequency regulation of a given interconnected system, independent of how it is partitioned, must represent the relative importance of dynamic power imbalance interactions. It is with this in mind that we formally define the notion of a dynamic interactions variable for each SBA. We show that this variable is driven by the disturbances internal to the SBA, and by the disturbances from the rest of the system as seen by the SBA model.

In the closing Sect. 10, we point out that the case of E-AGC described in this chapter is only an illustration of the enormous need to rethink what the role of cyber is and what it might become in future electric energy systems. While much effort has been made toward major breakthroughs in the fundamental science of energy processing, it is important to recognize the huge opportunities for the enhanced utilization of energy resources presented to us by the soft technologies of sensing, communications, computing, and control. These opportunities are clear but not very tangible at present in part because of the major lack of viewing the cyber design of energy systems with a full understanding of the physical characteristics of these systems, and of their temporal, spatial, and contextual structures. We hope that this chapter will illustrate the enormous importance of relating the physical understanding of the systems problem at hand, the models used, and the implications of these on the type of cyber required. Conversely, the models are also cyber-dependent and the recent progress in cyber technologies for power systems has opened the doors wide to progress in this area. We have attempted with great pleasure and honor to discuss the technical problem of interest, keeping in mind the early work of Joe Chow, which paved our way.

2 A Dynamic Model of Electric Energy Systems with Persistent Disturbances

In this section, the general frequency dynamic model for interconnected power systems is reviewed. We assume that only generators contribute to the frequency dynamics, and we use a module-based approach to derive a dynamic model by subjecting the dynamics of generator modules and loads to the transmission network constraints. Persistent disturbances in the system are caused by both variable loads and renewable power sources.

2.1 Modeling Dynamics of Generator Modules

We model the frequency dynamics of generator modules by assuming that the effects of reactive power and voltage change can be ignored and focus only on the governor-turbine model, which captures the real power-frequency dependence. In addition, in the governor-turbine model, we assume that the primary control loop has already been designed and its parameters are known. Recall that our purpose is to propose a secondary level regulation and therefore the control variable is viewed as being predefined.

The continuous-time frequency dynamics of each individual generator module with closed-loop primary control is modeled as follows [1]:

$$\begin{bmatrix} \Delta\dot{\delta}_G \\ \Delta\dot{f}_G \\ \Delta\dot{P}_T \\ \Delta\dot{a} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ 0 & -\frac{D}{M} & \frac{1}{M} & \frac{e_t}{M} \\ 0 & 0 & -\frac{1}{T_t} & \frac{Kt}{T_t} \\ 0 & -\frac{1}{T_g} & 0 & -\frac{r}{T_g} \end{bmatrix} \begin{bmatrix} \Delta\delta_G \\ \Delta f_G \\ \Delta P_T \\ \Delta a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix} \Delta f_G^{ref} + \begin{bmatrix} 0 \\ -\frac{1}{M} \\ 0 \\ 0 \end{bmatrix} \Delta P_G, \quad (1)$$

where the state variables $\Delta\delta_G$, Δf_G, ΔP_T, and Δa correspond to the deviations of generator voltage phase angle, frequency, turbine mechanical power output and the incremental change of the steam valve position, respectively, around the system equilibrium. Δf_G^{ref} is the governor set-point adjustment, which will be used as the control on the secondary level. ΔP_G refers to the deviation of electrical power output of the generator around the equilibrium value. ω_0 which is the rated angular velocity. M, D, T_g, and T_t are used to denote the inertia constant of the generator, its damping coefficient and the time constants of the governor and turbine, respectively. K_t and e_t are the constant parameters of the governor-turbine primary control loop. r is defined so that $\frac{1}{r}$ is the generator speed droop.

For the ith generator module, we denote the state variables and the secondary control input as

$$\begin{aligned} \boldsymbol{x}_{G,i} &= [\Delta\delta_{G,i} \quad \Delta f_{G,i} \quad \Delta P_{T,i} \quad \Delta a_i], \\ u_{G,i} &= \Delta f_{G,i}^{ref}, \end{aligned}$$

and the system, input and coupling matrices of the module as

$$\boldsymbol{A}_{G,i} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ 0 & -\frac{D_i}{M_i} & \frac{1}{M_i} & \frac{e_{t,i}}{M_i} \\ 0 & 0 & -\frac{1}{T_{t,i}} & \frac{K_{t,i}}{T_{t,i}} \\ 0 & -\frac{1}{T_{g,i}} & 0 & -\frac{r_i}{T_{g,i}} \end{bmatrix}, \quad \boldsymbol{B}_{G,i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{g,i}} \end{bmatrix}, \quad \boldsymbol{F}_{G,i} = \begin{bmatrix} 0 \\ -\frac{1}{M_i} \\ 0 \\ 0 \end{bmatrix}.$$

The generator module can then be represented in the state-space form

$$\dot{\boldsymbol{x}}_{G,i} = \boldsymbol{A}_{G,i}\boldsymbol{x}_{G,i} + \boldsymbol{B}_{G,i}u_{G,i} + \boldsymbol{F}_{G,i}\Delta P_{G,i}. \quad (2)$$

2.2 Modeling of the Variable Load

For the purposes of deriving a dynamic model for frequency regulation the load is characterized as a constant forecasted real power $\boldsymbol{P}_L(0)$. Deviations of load power around its forecast $\Delta \boldsymbol{P}_L(t)$ create a disturbance in the dynamics of the interconnected power system. We consider $\Delta \boldsymbol{P}_L(t)$ to be either a hard-to-predict deviation in demand and/or a hard-to-predict deviations in the renewable source located at the load bus. The dynamics of renewable power sources are not modeled here in order to explicitly define them as negative variable loads, which inject randomly disturbed power into the grid. Therefore, the actual load $\boldsymbol{P}_L(t)$ can be represented as

$$\boldsymbol{P}_L(t) = \boldsymbol{P}_L(0) + \Delta \boldsymbol{P}_L(t). \tag{3}$$

2.3 Modeling of the Transmission Network Constraints

Both the dynamics of generators and load deviations are subject to transmission network constraints. The network constraints are typically expressed in terms of nodal algebra equations. When modules get interconnected through a transmission network the basic Kirchhoff's laws have to be satisfied. Let $\boldsymbol{S} = \boldsymbol{P} + \boldsymbol{jQ}$ be the vector of the net complex power injections to all the buses; the algebraic complex power flow equation can be written as [1]

$$\boldsymbol{S} = \mathrm{diag}(\boldsymbol{V})(\boldsymbol{Y}_{bus}\boldsymbol{V})^*, \tag{4}$$

where diag$(\cdot)$ stands for the diagonal matrix with each element of the vector as a diagonal element. $\boldsymbol{V}$ is the vector of all the bus voltage phasors. The kth element of $\boldsymbol{V}$ is given by $V_k e^{j\delta_{G,k}}$. $\boldsymbol{Y}_{bus}$ is the admittance matrix of the power grid.

The net real part of complex power $\boldsymbol{S}$, in general, is comprised of active power injection of the generator $\boldsymbol{P}_G$ and consumption of the load $\boldsymbol{P}_L$, which is $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_G & -\boldsymbol{P}_L \end{bmatrix}^T$. Linearizing the real part of the complex power flow equation (4) around the system equilibrium yields

$$\Delta \boldsymbol{P}_G = \boldsymbol{J}_{GG}\Delta\boldsymbol{\delta}_G + \boldsymbol{J}_{GL}\Delta\boldsymbol{\delta}_L, \tag{5a}$$

$$-\Delta \boldsymbol{P}_L = \boldsymbol{J}_{GL}\Delta\boldsymbol{\delta}_G + \boldsymbol{J}_{LL}\Delta\boldsymbol{\delta}_L, \tag{5b}$$

where

$$\boldsymbol{J}_{ij} = \left.\frac{\partial \boldsymbol{P}_i}{\partial \boldsymbol{\delta}_j}\right|_{\delta_j = \delta_j^*}, \quad \boldsymbol{i,j} \in \{G, L\}$$

is the Jacobian matrix evaluated at the system equilibrium. $\Delta\boldsymbol{\delta}_L$ stands for the phase angle deviations on the load buses. Assuming that $\boldsymbol{J}_{LL}$ is invertible in normal operating conditions, we can substitute $\Delta\boldsymbol{\delta}_L$ from (5b) to (5a) and obtain the system-level algebraic network coupling equation:

$$\Delta\boldsymbol{P}_G = \boldsymbol{K}_p\Delta\boldsymbol{\delta}_G + \boldsymbol{D}_p\Delta\boldsymbol{P}_L, \tag{6}$$

where

$$\begin{aligned}\boldsymbol{K}_p &= \boldsymbol{J}_{GG} - \boldsymbol{J}_{GL}\boldsymbol{J}_{LL}^{-1}\boldsymbol{J}_{LG},\\ \boldsymbol{D}_p &= -\boldsymbol{J}_{GL}\boldsymbol{J}_{LL}^{-1}.\end{aligned}$$

2.4 Dynamic Model of the Interconnected System

The dynamic model of the interconnected system with n generators is derived by combing the dynamics of the individual generator modules given in equation 2 whose local states are $\boldsymbol{x}_{G,i}$, $i = 1, 2, \ldots, n$ and the network constraints (6). The system-level state variables, system disturbances and the secondary control inputs are defined as

$$\begin{aligned}\boldsymbol{x} &= [\boldsymbol{x}_{G,1}, \boldsymbol{x}_{G,2}, \ldots \boldsymbol{x}_{G,n}]^T,\\ \boldsymbol{W}(t) &= \Delta\boldsymbol{P}_L(t),\\ \boldsymbol{u} &= [u_{G,1}, u_{G,2}, \ldots u_{G,n}]^T,\end{aligned}$$

and the system matrices of all generator modules $\boldsymbol{A}_{uc}$, $\boldsymbol{B}_{uc}$ and $\boldsymbol{F}_{uc}$ are defined by combining corresponding matrices of all generator modules the $\boldsymbol{A}_{G,i}$, $\boldsymbol{F}_{G,i}$, and $\boldsymbol{B}_{G,i}$ to obtain

$$\begin{aligned}\boldsymbol{A}_{uc} &= \text{blockdiag}\left(\boldsymbol{A}_{G,1}, \boldsymbol{A}_{G,2}, \ldots \boldsymbol{A}_{G,n}\right),\\ \boldsymbol{B}_{uc} &= \text{blockdiag}\left(\boldsymbol{B}_{G,1}, \boldsymbol{B}_{G,2}, \ldots \boldsymbol{B}_{G,n}\right),\\ \boldsymbol{F}_{uc} &= \text{blockdiag}\left(\boldsymbol{F}_{G,1}, \boldsymbol{F}_{G,2}, \ldots \boldsymbol{F}_{G,n}\right).\end{aligned}$$

The interconnected system-level dynamic model then becomes

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_{uc}\boldsymbol{x} + \boldsymbol{B}_{uc}\boldsymbol{u} + \boldsymbol{F}_{uc}\Delta\boldsymbol{P}_G. \tag{7}$$

We define selection matrix $\boldsymbol{S}$ such that

$$\Delta\delta_G = \boldsymbol{S}\boldsymbol{x}, \tag{8}$$

and it can be substituted into the network constraint equation (6), which yields

$$\Delta\boldsymbol{P}_G = \boldsymbol{K}_p\boldsymbol{S}\boldsymbol{x} + \boldsymbol{D}_p\Delta\boldsymbol{W}. \tag{9}$$

The full state-space model is then obtained by combining (7) into (9):

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{F}\boldsymbol{W}, \tag{10a}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}, \tag{10b}$$

where

$$\begin{aligned}
\boldsymbol{A} &= \boldsymbol{A}_{uc} + \boldsymbol{F}_{uc}\boldsymbol{K}_p\boldsymbol{S},\\
\boldsymbol{B} &= \boldsymbol{B}_{uc},\\
\boldsymbol{F} &= \boldsymbol{F}_{uc}\boldsymbol{D}_p,\\
\boldsymbol{w} &= \Delta\boldsymbol{P}_L.
\end{aligned}$$

Note that since no slack generator bus is specified nor removed from the state space model (10), the system matrix $\boldsymbol{A}$ is structurally singular with $rank(\boldsymbol{A}) = 4n - 1$.

3 A New Interactions Variable-based Dynamic Model for Frequency Regulation in Large Interconnected Electric Energy Systems

The interconnected power system model (10) introduced in Sect. 2 may be overly complex and not necessary when designing a sensing, communication, and control architecture for frequency regulation. Both temporal and spatial simplifications are possible. However, as the system changes and the nature of generation and load changes, one must proceed very carefully with such simplifications. A temporal simplification takes into consideration that, at the generator level, the internal states of the generator module consist of the fast states ΔP_T and Δa and the slow states $\Delta\delta_G$ and Δf_G. The spatial simplification mainly considers that, at the system level, the structurally singular matrix $\boldsymbol{A}$ leads to the slow dynamics, which can represent the entire system's response to persistent disturbances. We address both the component and system-level time-scale separation in this section by, respectively, using the standard and nonstandard singular perturbation form model, which had its origin in the early work of Joe Chow and his collaborators [2, 3].

The existence of an interactions variable, which is crucial to the cyber architecture proposed in this chapter, is a direct consequence of the system-level dynamic model having a structurally non-standard singularly perturbed form. This form reflects the fact that system matrix $\boldsymbol{A}$ is not a full rank matrix. We show that the structural singularity also holds at the CA level. We introduce the definition of SBA, which refers to the CA in the new power system environment.

3.1 Use of Standard Singularly Perturbed Form for Temporal Simplifications of the System Model

The standard singularly perturbed form-based method is applied at each generator module level to reduce the full state $\boldsymbol{x}$ to the reduced state $\hat{\boldsymbol{x}}$ by identifying the time-scale separation between the slow states and fast states of the turbine-generator-governor set. The standard state separable form is described by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, z, \boldsymbol{u}, \boldsymbol{\varepsilon}, t), \quad \boldsymbol{x}(t_0) = \mathbf{x}_0, \quad \boldsymbol{x} \in \boldsymbol{R}^n, \tag{11a}$$

$$\boldsymbol{\varepsilon}\dot{\boldsymbol{v}} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{v}, \boldsymbol{u}, \boldsymbol{\varepsilon}, t), \quad \boldsymbol{v}(t_0) = \boldsymbol{v}_0, \quad \boldsymbol{v} \in \boldsymbol{R}^m, \tag{11b}$$

where $\boldsymbol{x}$ refers to the slow system states and $\boldsymbol{v}$ to the fast system states. ε is a small positive scalar, which accounts for the small time constant. If the dynamics of the two states are widely separated, ε will become very small and can be approximated as $\varepsilon = 0$. This approximation is equivalent to setting the speed of $\boldsymbol{v}$ as infinitely large and the transient of $\boldsymbol{v}$ as instantaneous. Hence, (11b) reduces to a set of algebra equations:

$$0 = \boldsymbol{g}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{v}}, \hat{\boldsymbol{u}}, 0, t), \tag{12}$$

and the substitution of a root of (12)

$$\hat{\boldsymbol{v}} = \boldsymbol{\phi}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t), \tag{13}$$

into (11a) yields a reduced model:

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}}{\mathrm{d}t} = \boldsymbol{f}(\hat{\boldsymbol{x}}, \boldsymbol{\phi}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{u}}, t), \hat{\boldsymbol{u}}, 0, t), \quad \hat{\boldsymbol{x}}(t_0) = \boldsymbol{x}_0, \quad \hat{\boldsymbol{x}} \in \boldsymbol{R}^n, \tag{14}$$

where the upper hat is used to indicate that the variables belong to a system with $\varepsilon = 0$. The singularly perturbed model of (2) can be obtained as

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}_{G,i}}{\mathrm{d}t} = \hat{\boldsymbol{A}}_{G,i}\hat{\boldsymbol{x}}_{G,i} + \hat{\boldsymbol{B}}_{G,i}\hat{u}_{G,i} + \hat{\boldsymbol{F}}_{G,i}\Delta\hat{P}_{G,i}, \tag{15}$$

where

$$\begin{aligned} \hat{\boldsymbol{x}}_{G,i} &= [\Delta\delta_{G,i} \quad \Delta f_{G,i}], \\ \hat{u}_{G,i} &= \Delta f_{G,i}^{ref}, \end{aligned}$$

and

$$\hat{\boldsymbol{A}}_{G,i} = \begin{bmatrix} 0 & \omega_0 \\ 0 & -\frac{r_i D_i + K_{t,i} + e_{t,i}}{r_i M_i} \end{bmatrix}, \quad \hat{\boldsymbol{B}}_{G,i} = \begin{bmatrix} 0 \\ \frac{K_{t,i} + e_{t,i}}{r_i M_i} \end{bmatrix}, \quad \hat{\boldsymbol{F}}_{G,i} = \begin{bmatrix} 0 \\ -\frac{1}{M_i} \end{bmatrix}.$$

This temporal simplification at the generator module level leads to the reduced order model of the interconnected system as follows:

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}}{\mathrm{d}t} = \hat{\boldsymbol{A}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{B}}\hat{\boldsymbol{u}} + \hat{\boldsymbol{F}}\hat{\boldsymbol{W}}, \tag{16a}$$

$$\hat{\boldsymbol{y}} = \hat{\boldsymbol{C}}\hat{\boldsymbol{x}}. \tag{16b}$$

3.2 Use of Nonstandard Singularly Perturbed Form for Spatial Simplifications of the System Model

The reduced system dynamic model (16) still has a structural singularity because $rank(\hat{\boldsymbol{A}}) = 2n - 1$. Therefore, the dimension of the null-space $\mathscr{N}$ of $\hat{\boldsymbol{A}}$ is 1. Therefore, a further time-scale separation based on the nonstandard singularly perturbed form is possible. According to [3], the slow variable z can be obtained by deriving a $(1 \times 2n)$ vector $\hat{\boldsymbol{T}}$ transformation, which spans the left null-space of $\hat{\boldsymbol{A}}$, that is, $\hat{\boldsymbol{T}}\hat{\boldsymbol{A}} = 0$. Then in the reduced system model (16a), we multiply by $\hat{\boldsymbol{T}}$ on both sides and obtain

$$\frac{\mathrm{d}\hat{z}}{\mathrm{d}t} = \hat{\boldsymbol{T}}\frac{\mathrm{d}\hat{\boldsymbol{x}}}{\mathrm{d}t} = \hat{\boldsymbol{T}}\hat{\boldsymbol{B}}\hat{\boldsymbol{u}} + \hat{\boldsymbol{T}}\hat{\boldsymbol{F}}\hat{\boldsymbol{W}}. \tag{17}$$

This implies that the time response of the slow variable $\hat{z}$ only depends on the control input and the disturbances. When no control is implemented, $\hat{z}$ represents the response of the system to disturbances. The dynamics of $\hat{z}$ are a consequence of the existence of the zero eigenvalue in system matrix $\hat{\boldsymbol{A}}$. They are defined by equation (17) from which it can be seen that $\hat{z}$ can only be controlled by the secondary level control $\hat{\boldsymbol{u}}$ in response to disturbances $\hat{\boldsymbol{w}}$. This points out the necessity for secondary level frequency control: with only the locally designed primary controllers of the generator-turbine sets, the frequency of the power system exposed to persistent disturbances will never return to the nominal frequency. This is because the interactions variable $\hat{z}$ will not settle to zero, which can be shown to be the necessary condition for system frequency to settle to zero.

3.3 The Dynamics of CA-level Interactions Variable

Motivated by today's AGC approach, we consider the temporally and spatially simplified model at the subsystem level. The subsystem in today's AGC approach is denoted as the CA. Later in this subsection, we will introduce the dynamic model and a new notion to the subsystem.

The subsystem is equivalent to a stand-alone power system with external interconnections represented as disturbances. Therefore, each subsystem can be modeled similar to (16) with the subscript a to differentiate it from (16),

$$\begin{aligned}\frac{\mathrm{d}\hat{\boldsymbol{x}}_a}{\mathrm{d}t} &= \hat{\boldsymbol{A}}_a\hat{\boldsymbol{x}}_a + \hat{\boldsymbol{B}}_a\hat{\boldsymbol{u}}_a + \hat{\boldsymbol{F}}_{a_{uc}}\hat{\boldsymbol{D}}_{p_a}\Delta\hat{P}_{L_a} + \hat{\boldsymbol{F}}_{a_{uc}}\hat{\boldsymbol{D}}_{p_a}\Delta\hat{F}_{L_a} - \hat{\boldsymbol{F}}_{a_{uc}}\Delta\hat{F}_{G_a}\\ &= \hat{\boldsymbol{A}}_a\hat{\boldsymbol{x}}_a + \hat{\boldsymbol{B}}_a\hat{\boldsymbol{u}}_a + \hat{\boldsymbol{F}}_a\hat{\boldsymbol{W}}_a,\end{aligned} \tag{18}$$

where

$$\begin{aligned}\hat{\boldsymbol{F}}_a &= \begin{bmatrix}\hat{\boldsymbol{F}}_{a_{uc}}\hat{\boldsymbol{D}}_{p_a} & \hat{\boldsymbol{F}}_{a_{uc}}\hat{\boldsymbol{D}}_{p_a} & -\hat{\boldsymbol{F}}_{a_{uc}}\end{bmatrix},\\ \hat{\boldsymbol{W}}_a &= \begin{bmatrix}\Delta\hat{P}_{L_a}\\ \Delta\hat{F}_{L_a}\\ \Delta\hat{F}_{G_a}\end{bmatrix}.\end{aligned} \tag{19}$$

The term $\Delta\hat{F}_{G_a}$ stands for the power flow from the neighboring CAs to the generator buses, and $\Delta\hat{F}_{L_a}$ is the real power flow from the neighboring CAs to the load buses. Hence, at the CA level, disturbances to a CA could be caused by its own load fluctuations and/or the tie-line fluctuations connecting to other areas.

Structural singularity also exists in (18), so we can apply the nonstandard form singular perturbation method to the subsystem model as in Sect. 3.2, which leads to

$$\frac{\mathrm{d}\hat{z}_a}{\mathrm{d}t} = \hat{\boldsymbol{T}}_a\frac{\mathrm{d}\hat{\boldsymbol{x}}_a}{\mathrm{d}t} = \hat{\boldsymbol{T}}_a\hat{\boldsymbol{B}}_a\hat{\boldsymbol{u}}_a + \hat{\boldsymbol{T}}_a\hat{\boldsymbol{F}}_a\hat{\boldsymbol{W}}_a. \tag{20}$$

The dynamics of the slow variable $\hat{z}_a$ depend solely on the internal control input and the disturbances from both internal and external energy sources. When no internal control is applied, $\hat{z}_a$ is the interactions variable of the subsystem and it represents power imbalances caused by the internal and external disturbances.

We define the CA interactions variable $\hat{z}_a$ has the unique property given as follows [1]:

Definition 1. The interactions variable $\hat{z}_a$ of a CA is the variable which satisfies

$$\frac{\mathrm{d}\hat{z}_a}{\mathrm{d}t} \equiv 0,$$

when no secondary control input is applied, and when there are no internal disturbances and all interconnections with other CAs are removed.

As in Sect. 3.2, the interactions variable $\hat{z}_a$ is an aggregation of the state variables of a CA, namely,

$$\hat{z}_a = \hat{\boldsymbol{T}}_a\hat{\boldsymbol{x}}_a, \tag{21}$$

and $\hat{\boldsymbol{T}}_a$ is the basis of the eigenspace $\mathcal{N}(\hat{\boldsymbol{A}}_a)$ and can be determined by solving

$$\hat{\boldsymbol{T}}_a\hat{\boldsymbol{A}}_a = 0. \tag{22}$$

We further define $\hat{z}_a$ as the output variable of the subsystem and write the subsystem model as

$$\frac{\mathrm{d}\hat{\boldsymbol{x}}_a}{\mathrm{d}t} = \hat{\boldsymbol{A}}_a\hat{\boldsymbol{x}}_a + \hat{\boldsymbol{B}}_a\hat{\boldsymbol{u}}_a + \hat{\boldsymbol{F}}_a\hat{\boldsymbol{W}}_a, \tag{23a}$$

$$\hat{z}_a = \hat{\boldsymbol{C}}_a\hat{\boldsymbol{x}}_a, \tag{23b}$$

where

$$\hat{\boldsymbol{C}}_a = \hat{\boldsymbol{T}}_a.$$

At the end of this section, we introduce a new notion of SBA as the CA (subsystem), which communicates its interactions variable with the rest of the system in order to balance supply and demand in real time by utilizing on-line adjustments of both the resources internal to the subsystem and the neighboring subsystems. Both the concepts of interactions variable of CA and SBA will be used in the follow up sections for designing the cyber architecture of the frequency regulation system.

4 Interactions Variable-based Dynamical Models for Frequency Regulation: Comparison of Model Complexity for Four Qualitatively Different Cases

In this section, we illustrate the role of the interaction variables in four qualitatively different systems. The five-bus power system shown in Fig. 1 is used to demonstrate the four different scenarios without loss of generality. The pre-defined organizational boundaries divide the system into two interconnected subsystems (SBAs). However, as the internal and external electrical distances and disturbances change qualitatively, the dynamic interaction between these two SBAs can differ dramatically. Tables 1 and 2 list the system parameters and $\hat{\boldsymbol{A}}$ matrices, respectively. In this section, we assume that the magnitude and rate of change of the disturbances remain within a pre-specified range and differentiate among the cases with different electrical distances.

4.1 Cases 1 and 2

In the first case, the system has two weakly interconnected SBAs whose internal states are strongly connected. The system is therefore modeled using the concept of interactions variable proposed in this chapter, for which we have

$$\hat{z}_a^I \triangleq \hat{\boldsymbol{C}}_a^I\hat{\boldsymbol{x}}_a^I, \tag{24a}$$

$$\hat{z}_a^{II} \triangleq \hat{\boldsymbol{C}}_a^{II}\hat{\boldsymbol{x}}_a^{II}, \tag{24b}$$

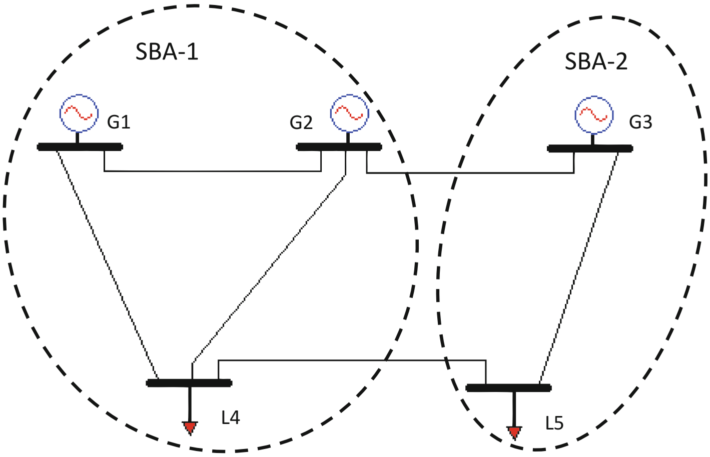

Fig. 1 Five-bus power system

Table 1 Parameters of the five-bus test system ($S_{\text{base}} = 100\,\text{MVA}$)

Transmission line reactance data (p.u.)							
	X_{12}	X_{14}	X_{23}	X_{24}	X_{35}	X_{45}	
Case 1	0.01	0.01	20	0.01	0.01	20	
Case 2	1	1	20	1	1	20	
Case 3	0.01	0.01	0.01	0.01	0.01	0.01	
Case 4	1	1	0.01	1	1	0.01	
Generator data (p.u.)							
	M	D	T_t	T_g	e_t	K_t	r
Gen 1	8	2	0.2	0.25	39.4	250	19
Gen 2	10	2	0.18	0.23	39.4	250	19
Gen 3	9	1.6	0.3	0.3	39	280	21

where $\hat{\boldsymbol{C}}_a^I$ and $\hat{\boldsymbol{C}}_a^{II}$ can be solved from (22). It is shown in Fig. 3a that when the system is driven by fast oscillating disturbances (depicted by Fig. 2), the dynamics of the interactions variables of SBA-2 are significantly affected. By contrast, the interactions variables of SBA-1 have much slower dynamics. Note that the positive direction of the interactions variables can be arbitrarily assigned.

Case 2 represents an overall weakly connected system. The interactions variables for this case are presented in Fig. 3b. The results resemble those of case 1 due to the fact that the dynamic behaviors of $\hat{z}_a^I$ and $\hat{z}_a^{II}$ differ significantly and SBA-1 and SBA-2 interact weakly. These two cases are different with regard to internal dynamics in the CAs.

We conclude that in cases 1 and 2 the dynamics of the interactions variables depend mainly on internal disturbances; and we propose that, because of the insignificant interactions between the SBAs in these cases, there is no need to

Table 2 System $\hat{\boldsymbol{A}}$ matrices in the four cases

Case 1	Case 2
$\hat{\boldsymbol{A}} = \begin{bmatrix} 0 & 377 & 0 & 0 & 0 & 0 \\ -18.75 & -2.15 & 18.75 & 0 & 0.0031 & 0 \\ 0 & 0 & 0 & 377 & 0 & 0 \\ 15 & 0 & -15 & -1.72 & 0.0075 & \\ 0 & 0 & 0 & 0 & 0 & 377 \\ 0.0028 & 0 & 0.0083 & 0 & -0.0111 & -1.87 \end{bmatrix}$	$\hat{\boldsymbol{A}} = \begin{bmatrix} 0 & 377 & 0 & 0 & 0 & 0 \\ -0.19 & -2.15 & 0.19 & 0 & 0.0029 & 0 \\ 0 & 0 & 0 & 377 & 0 & 0 \\ 0.15 & 0 & -0.16 & -1.72 & 0.0073 & \\ 0 & 0 & 0 & 0 & 0 & 377 \\ 0.0026 & 0 & 0.0081 & 0 & -0.0107 & -1.87 \end{bmatrix}$
Case 3	**Case 4**
$\hat{\boldsymbol{A}} = \begin{bmatrix} 0 & 377 & 0 & 0 & 0 & 0 \\ -20 & -2.15 & 17.5 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 377 & 0 & 0 \\ 14 & 0 & -26 & -1.72 & 12 & \\ 0 & 0 & 0 & 0 & 0 & 377 \\ 2.22 & 0 & 13.33 & 0 & -15.55 & -1.87 \end{bmatrix}$	$\hat{\boldsymbol{A}} = \begin{bmatrix} 0 & 377 & 0 & 0 & 0 & 0 \\ -0.17 & -2.15 & 0.14 & 0 & 0.0345 & 0 \\ 0 & 0 & 0 & 377 & 0 & 0 \\ 0.11 & 0 & -10.13 & -1.72 & 10.02 & \\ 0 & 0 & 0 & 0 & 0 & 377 \\ 0.0307 & 0 & 11.14 & 0 & -11.17 & -1.87 \end{bmatrix}$