

Igor A. Karnovsky

Theory of Arched Structures

Strength, Stability, Vibration

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 Springer

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In memory of Prof. Anatoly B. Morgaevsky

Preface

In modern engineering, as a basis of construction, arches have a diverse range of applications. Today the theory of arches has reached a level that is suitable for most engineering applications. Many methods pertaining to arch analysis can be found in scientific literature. However, most of this material is published in highly specialized journals, obscure manuals, and inaccessible books. This is not surprising, as the intensive development of arch theory, particularly stability and vibration have mostly occurred in the 1940s to the 1960s. Therefore, most engineers lack the opportunity to utilize these developments in their practice.

The author has committed to the goal of presenting a book which encompasses essential and tested methods on fundamental methods of arch analysis and equally important problems.

The objective of the Book is to provide to readers with detailed procedures for analysis of the strength, stability, and vibration of various types of arched structures, using exact analytical methods of classical Structural Analysis.

In 2004, professor L.A. Godoy published the article “*Arches: A Neglected Topic in Structural Analysis Courses.*” This in-depth investigation highlights a deep rift between the modern level of development of arch theory and the level of presentation of this theory in existing material on structural analysis.

In 2009, the author of this book, with co-author O. Lebed published the textbook “*Advanced Methods of Structural Analysis*” (Springer), in which arch theory is presented in a much greater depth and volume than in existing textbooks. However, the issue of producing a single book which covers both general and specialized problems of arches remained unsolved. The book presented here sheds light on issues of strength, stability, and vibrations, as well as special problems of arches and arched structures.

In this book special attention is directed toward the discussion of fundamental properties of structures. An engineer who is armed with fundamental knowledge and means of computation is essentially set to succeed in modern day engineering. Solutions of problems of strength, stability, and vibrations of arches in most cases

are broken down to basic formulas which can be easily applied to engineering practice.

This book is based on the author's experience as a teacher and consultant in structural mechanics. It is intended for senior undergraduate students in structural engineering and for postgraduate students who are concerned with different problems of arches structures. The book will be a useful reference for engineers in the structural industry.

Vancouver, Canada

Igor A. Karnovsky

Distribution of Material in the Book

This book contains an introduction, four parts (nine chapters), and an appendix.

The first part “Strength” contains three chapters. Chapter 1 is devoted to fundamental methods of determining displacement of elastic structures in general accompanied by examples specifically for arches.

Chapter 2 covers the analysis of three-hinged arches, while analysis of redundant arches is considered in Chap. 3; in these chapters a special attention is dedicated to the analysis of arched structures using influence lines.

Second part “Stability” contains two chapters. Chapter 4 provides analytical methods of the stability of arches. These methods are based on the integration of differential equations.

Chapter 5 presents Smirnov’s matrix method and approximate method. Approximate method is based on the approximation of the arch by straight members with subsequent application of the precise displacement method in canonical form.

The third part, “Vibration” contains two chapters. Chapter 6 deals with computation of eigenvalues and eigenfunctions for arches. For analysis of the circular uniform arch, Lamb’s differential equation is used; for analysis of parabolic uniform arch the Rabinovich’s model is applied. The frequency of vibration for arches with different ratio “rise/span” of an arch are presented on the basis of this model.

Chapter 7 presents forced vibrations of arches.

The fourth part of the book, “Special Topics” holds the goal of presenting introductory information regarding problems which until now have only been discussed in specialized literature. Chapter 8 contains the static nonlinear problems. They are plastic analysis of the arches and arched structures with one-sided constraints. Chapter 9 is devoted to dynamical stability of arches, and dynamics of arched structures subjected to moving inertial load.

Finally, the appendix contains the fundamental tabulated data essential for engineering practice involving arches.

Sections 2.1, 2.2, 2.4, and 2.6 were written by Olga Lebed.

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Finally, I would like to thank my relatives, many friends, and colleagues, who have supported me through all stages of research and development of this book.

The author appreciates comments and suggestions to improve the current edition. All constructive criticism will be accepted with gratitude.

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Introduction

Arches and arched structures have a wide range of uses in bridges, arched dams and in industrial, commercial, and recreational buildings. They represent the primary structural components of important and expensive structures, many of which are unique. Current trends in architecture heavily rely on arched building components due to their strengths and architectural appeal.

Complex structural analysis of arches is related to the analysis of the arches strength, stability, and vibration. This type of multidimensional analysis aims at ensuring the proper functionality of an arch as one of the fundamental structural elements.

Terminology

We start our consideration from terminology for a bridge arch (Fig. 1a). The arch is supported by abutments. The heels and crown are the lowest and highest points of the arch, respectively; supports may be rolled, pinned, or fixed. Horizontal distance between two heels is span l , a vertical distance between heels line and crown is rise f . Extrados is the top outer surface of the arch. Intrados is the lower inner surface of the arch. A body of the arch itself may be solid or with webbed members.

As a bridge trusses, the bridge arches are connected using arch bracing. All structural members over the arch are called overarched construction. Deck and arch are connected by vertical members called posts. If the roadway is located below an arch, then vertical members are called hangers. If movement of vehicles is at the intermediate level, then a loaded deck is partially connected with arch by poles and partially by hangers. The posts are compressed, while the hangers are extended.

For structural analysis, a real structure has to be presented in the idealized and simplified form using the axial line of the structural components. For this, a so-

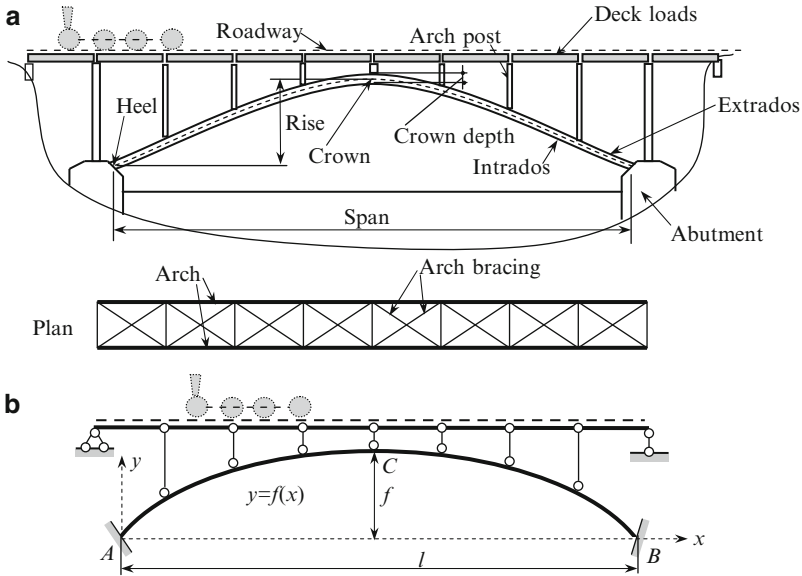


Fig. 1 (a, b) Components of an arch bridge and design diagrams for a deck-arch bridge

called design diagram of the real structure is used. Design diagram is a critically important concept of structural analysis. Design diagram of a real structure reflects the most important and primary features of the structure such as types of members, types of supports, types of joints, while some features of secondary importance (shapes of cross-sections of members, existence of local reinforcements or holes, size of supports and joints, etc.) are ignored.

Few general rules of representing a real structure by its design diagram are:

- A structure is presented as a set of simple structural members
- Real supports are replaced by their idealized supports
- Any connection between members of a structure are replaced by idealized joints
- Cross-section of any member is characterized by its area or/and moment of inertia

It is obvious that a real structure may be represented using different design diagrams.

An arch with overarched members and its design diagram is shown in Fig. 1b. Design diagram also contains information about the shape of the neutral line of the arch. Usually this shape is given by the expression $y = f(x)$.

Note that posts or hangers are connected to the arch itself by means of hinges.

In bridge construction the arches are subdivided into deck-bridge arch (Fig. 1), through-bridge arch, and arch with deck at some intermediate level (Fig. 2). Also, double-deck bridges exist with the lower deck designed for a railway, and the upper deck is utilized for a roadway.

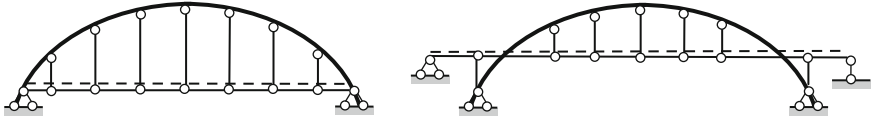


Fig. 2 Design diagrams of the through-bridge arch and arch with deck at intermediate level

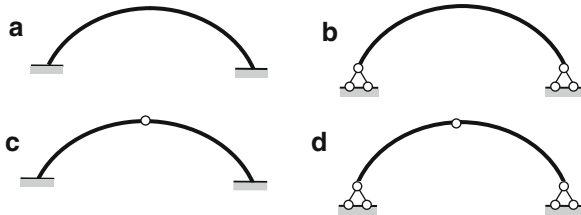


Fig. 3 Design diagrams of arches: (a) hingeless arch; (b) two-hinged arch; (c) one-hinged arch; (d) three-hinged arch

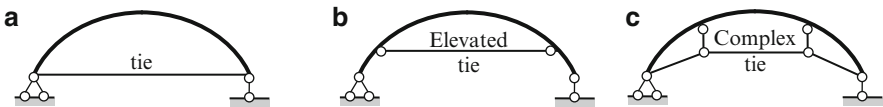


Fig. 4 Arches with tie

Based on their design, arches are divided into hingeless (arch with fixed ends), one-hinged, two-hinged, and three-hinged ones (Fig. 3a–d). All arches presented in Fig. 3, except for the three-hinged arch (d), are statically indeterminate (redundant) ones.

A tie is an additional member which allows us to reinforce an arch. A single tie may be installed on the level of the supports (Fig. 4a), or elevated (b). The tie may also be complex (c). Prestressed tie allows us to control the internal forces in the arch itself.

The arches may be constructed with supports at different elevations. In this case they are called askew arches.

Peculiarities of Arch Behavior

Since posts have hinges at the ends (Fig. 2), then only axial force arises in them. If the posts with fixed ends are thin elements with small flexural stiffness, then they cannot perceive and transmit the bending moments. In both cases, the loads from deck are transferred through posts (hangers) on the arch as concentrated forces.

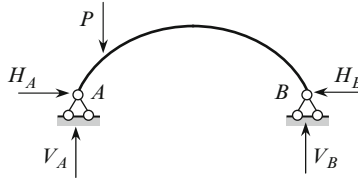


Fig. 5 Reactions of the arch

The fundamental feature of an arched structure is that horizontal reactions appear even if the structure is subjected to vertical load only. These horizontal reactions $H_A = H_B = H$ are called a *thrust* (Fig. 5). If structure has a curvilinear axis but thrust does not exist then this structure cannot be treated as an arch. The presence of thrust leads to a fundamental difference in behavior between arches and beam – the bending moments in arches are smaller than in beams of the same span and loads. Advantages of arches over beams increase as the length of a span increases.

Presence of thrust demands reinforcement of the part of a structure which is subjected to horizontal force.

However, the thrust may be absorbed by a tie; with this, supports of the arch are only subjected to vertical forces.

In addition to the bending moments and shear forces that arise in beams, axial compressive forces are also present in arches. These forces may cause a loss of stability of the arch.

There are advantages and disadvantages of each type of arches. Different design diagrams of the arches may be compared, taking into account different criteria. These include differences in their deformability, internal forces, critical loads, frequencies of vibration, sensitivity of arches to settling of supports, temperature changes, fabrication errors, etc.

Three-hinged arches have less rigidity than two-hinged and hingeless arches. Breaks in elastic curve over a hinge leads to additional forces in the cases where a moving load is present. In the cases when a structure is built on weak soil, three-hinged arches are preferred over hingeless arches since additional stresses caused by the settling of supports do not arise in these structures [Bro99], [Sch80].

Figure 6 shows characteristic distribution of the maximum bending moments in different arches in the presence of a moving load; each arch (diagrams a–d) has a unique bending moment (diagram e) [Kis60]. It is evident that a one-hinged arch (curve c) is the least efficient in regards to bending moment at its supports. In hingeless arches (curve d), the distribution of bending moment is most favorable because of its smoothness.

In the three-hinged arch (a), internal forces arise as a result of external load only. The rest of the arches (b–d) are sensitive to the displacements of supports, changes in temperature, and errors of fabrication. For masonry or concrete arches, material shrinkage should be taken into account, since this property of material leads to additional stresses.

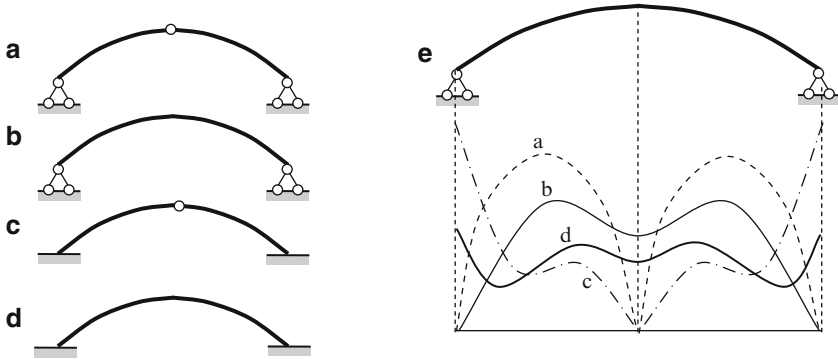


Fig. 6 (a–d) Types of arches; (e) approximate distribution of maximum bending moments across the span of different types of arches. In Fig. 6e design diagram as two-hinged arch is shown arbitrary

Initial Data for Structural Analysis

A comprehensive structural analysis includes the strength, stability, and vibration analysis. Strength analysis (static analysis) deals with the determination of internal forces and deflections of the arch due to action of static loads only. Stability analysis deals with the determination of loads which leads new forms of equilibrium (the loss of stability) of the arch. Vibration analysis considers determination of frequencies of free vibration of arch, as well as determination of internal forces and displacements of the arch subjected to specific external disturbing loads.

For analysis of arches, the following data have to be clearly outlined and specified: type of arch (hingeless, two-hinged, etc.); its shape (circle, parabolic, etc.); its dimensions (span and rise); location of supports (same or different elevation); presence of the tie, its type (single or complex), and its location. In the case of an arched bridge, it is necessary to show location of a loaded deck (Figs. 1–2), location of the hangers (or/and posts), and ways of their connections with arch itself and with loaded deck.

Computation of internal forces for two-hinged and hingeless arches requires knowing the law of change of cross-sectional area $A(x)$ and corresponding moment of inertia $I(x)$, along the axis of the arch. For a tie it is necessary to present the ratio $EI_{\text{arch}}/EA_{\text{tie}}$. For computation of deflections for all types of arches it is necessary to know $A(x)$ and $EI(x)$.

Assumptions

Some of the common assumptions made in this book include the following:

1. Material of the arch obeys Hooke's law (physically linear statement)
2. Deflections of the arches are small compared with the span of the arch (geometrically linear statement). The cases of nonlinear statement are specifically mentioned.
3. All constraints, which are introduced into the arched structure are two-sided, i.e., each constraint prevents displacements in two directions. The case of one-sided constraints is specifically mentioned.
4. In the case of elastic supports the relationship between deflection of constraint and corresponding reaction is linear.
5. The load is applied in the longitudinal plane of symmetry of the arch. The case of out-of-plane loading is specifically mentioned.

Besides the above assumptions, supplementary assumptions are introduced in corresponding parts of the book.

Some remarks related to structural analysis of the arches:

1. Since arches are represented by curvilinear rods, then their analysis, strictly speaking, should be performed using the theory of the curvilinear rods. However, curvature of the arches used in the construction is small ($R/h > 10$), therefore, the curvature of the arch may be neglected and deflections of the arch are assumed to be calculated as for straight rods [Kis60].
2. The superposition principle is valid under assumptions 1–4. In the case of one-sided constraints the superposition principle requires special treatment.

Shape of the Arches

As it is shown below, distribution of internal forces in arches depends on the shape of the central line of an arch. According to their shapes, arches are divided into the circular arch, parabolic arch, etc. Equation of the central line and some necessary formulae for circular and parabolic arches are presented below. For both cases, origin of coordinate axis is located at point *A* as shown in Fig. 7.

Circular arch. Ordinate *y* of any point of the central line of the circular arch is calculated by the formula

$$y = \sqrt{R^2 - \left(\frac{l}{2} - x\right)^2} - R + f; \quad R = \frac{f}{2} + \frac{l^2}{8f}, \quad (1)$$

where *x* is the abscissa of the same point of the central line of the arch; *R* is the radius of curvature of the arch; *f* and *l* are the rise and span of the arch.

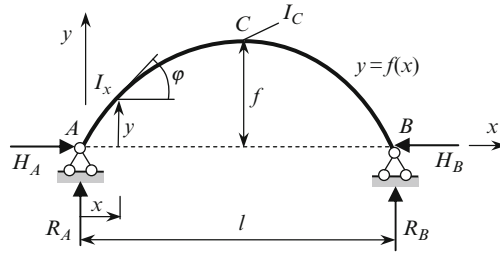


Fig. 7 Design diagram of two-hinged arch

The angle φ between the tangent to the center line of the arch at point (x, y) and horizontal axis is determined as follows:

$$\sin \varphi = (l - 2x) \frac{1}{2R}; \quad \cos \varphi = (y + R - f) \frac{1}{R}. \quad (2)$$

Parabolic arch. Ordinate y of any point of the central line of the parabolic arch is

$$y = 4fx(l - x) \frac{1}{l^2}. \quad (3)$$

Trigonometric functions of the angle between the tangent to the center line of the arch at point (x, y) and a horizontal axis are as follows:

$$\begin{aligned} \tan \varphi &= \frac{dy}{dx} = \frac{4f}{l^2} (l - 2x); & \cos \varphi &= \frac{1}{\sqrt{1 + \tan^2 \varphi}}; \\ \sin \varphi &= \cos \varphi \times \tan \varphi. \end{aligned} \quad (3a)$$

For the left half-arch the functions $\sin \varphi > 0$, $\cos \varphi > 0$, and for the right half-arch the functions $\sin \varphi < 0$ and $\cos \varphi > 0$.

Length S of half-axis of symmetrical arch and length of the axis of the arch S_k from the origin (point A) to an arbitrary point k with coordinates $x_k = \xi_k l$, $y_k = \eta_k l$ are

$$\begin{aligned} S &= \frac{l}{4} \left[\sec \varphi_0 + \frac{1}{4m} \ln(4m + \sec \varphi_0) \right], \\ S_k &= S - \frac{l}{16m} \left(\frac{\tan \varphi_k}{\cos \varphi_k} + \ln \frac{1 + \sin \varphi_k}{\cos \varphi_k} \right). \end{aligned} \quad (4)$$

where φ_0 is a slope at the support A; parameter $m = f/l$.

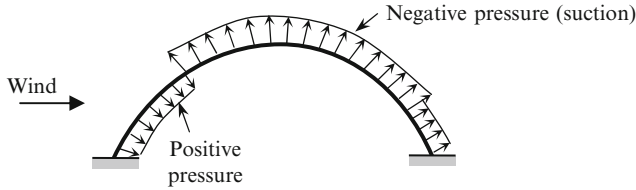


Fig. 8 Pressure of the wind on the surface of the arch

Catenary arch. Ordinate y of any point of the central line of the catenary arch as a function of load may be calculated by the formula which is presented in Sect. 2.3.2.

More expressions $y(x)$ for different arch shapes are presented in Tables A.1–A.5 [Kar01].

Strictly speaking, the concept of arch shape includes not only equation of central line as shown above, but also the law of flexural rigidity along the axis of the arch [Kis60]. The flexural rigidity $EI(x)$ may be constant or variable along the axis of the arch depending on expected distribution of internal forces, requirements of a constructive nature and aesthetic considerations. Usually the variable rigidity of the arch $EI(x)$ expresses in terms of rigidity of the arch at crown, EI_C , where E is a modulus of elasticity, I_C is a moment of inertia of a cross section at the crown C of an arch. This will be considered in more details in Sect. 3.1.

Loads

Arches, as main structural components, are subject to a variety of loads depending on the purpose of the arch and conditions of its operation.

For arches in public and industrial buildings the main loads are deadweight, live-load, and snow. These loads act in the longitudinal plane of symmetry of the arch and lead to in-plane bending. A significant load for arched structures is a wind pressure. The wind leads to the positive and negative loads onto the arch. A simplified scheme of the wind pressure is shown in Fig. 8.

In the case of a tall arch, the in-plane wind loads leads to significant internal forces in the arch. If a tall arch has a small own weight, then the formation of the negative reactions is possible; this dangerous phenomenon leads to the separation of the arch from abutment.

Pressure of the wind, which is directed perpendicular to the plane of the arch, leads to out-of-plane bending of the arch. These loads are absorbed by bracing between arches.

A dangerous phenomenon is observed in the case of an arched cover with open sides. Wind pressure, which is parallel to an open aperture, flows around them and creates a vacuum inside. As a result, the positive pressure onto the arch increases and suction decreases.

For arched bridges the main loads, which lead to the in-plane bending of the arch, are the following: deadweight, vertical loads from vehicles, and horizontal load caused by their longitudinal deceleration. Also, in the case of a bridge with curvature in the horizontal plane, one should take into account horizontal loads, which are caused by moving vehicles in a curvilinear trajectory.

The settlement of supports may induce in-plane and out-of-plane bending. Out-of-plane bending also arises by horizontal out-of-plane wind pressure, and seismic loads. Asymmetric location of the load with respect to the longitudinal plane of symmetry also leads to out-of-plane bending of the arch.

Some types of loads have a distinctly dynamic nature. Among them are seismic loads, wind gusts, moving inertial loads and their deceleration, impacts of wheels on the joints of rails on railway bridges. In the case of road bridges one should take into account the roughness of their surface.

If the shell is reinforced with ribs and is immersed into a liquid, then the pressure on the shell is transmitted on ribs and each rib can be considered as an arch due to a uniformly distributed radial load.

Determination of loads on the arch and the consideration of all possible combinations of loads is an important part of engineering analysis

Part I

Strength Analysis