Progress in Mathematics
296

Ilia Itenberg
Burglind Jöricke
Mikael Passare
Editors

Perspectives in Analysis, Geometry, and Topology

On the Occasion of the 60th Birthday of Oleg Viro

Birkhäuser

Birkhäuser

Progress in Mathematics

Volume 296

For further volumes:
http://www.springer.com/series/4848

Ilia Itenberg • Burglind Jöricke • Mikael Passare
Editors

Perspectives in Analysis, Geometry, and Topology

On the Occasion of the 60th Birthday of Oleg Viro

Birkhäuser

Editors
Ilia Itenberg
Université Pierre et Marie Curie
and Institut Universitaire de France
Institut de Mathématiques de Jussieu
4 place Jussieu
75005 Paris
France
itenberg@math.jussieu.fr

Burglind Jöricke
Institut des Hautes Études Scientifiques
Le Bois-Marie
35, route de Chartres
Bures-sur-Yvette 91440
France
joericke@googlemail.com

Mikael Passare
Department of Mathematics
Stockholm University
Stockholm SE-10691
Sweden
passare@math.su.se

ISBN 978-0-8176-8276-7 e-ISBN 978-0-8176-8277-4
DOI 10.1007/978-0-8176-8277-4
Springer New York Dordrecht Heidelberg London

Library of Congress Control Number: 2011939742

Mathematics Subject Classification (2010): 11H06, 14P25, 14T05, 14H99, 20F36, 31A25, 32U05, 53D05, 53D10, 53D40, 53D42, 57Mxx, 57M25, 57M27

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.birkhauser-science.com)

This volume is dedicated to Oleg Viro on the occasion of his 60th birthday.

Contents

Preface

Encounters between the fields of analysis, geometry, and topology are widespread and often provide major impetus for breakthroughs in these domains. Recent developments in low-dimensional topology, algebraic geometry, symplectic geometry, complex analysis, and tropical geometry are so rich with impressive examples that any attempt to reflect a major portion of them in a single volume would be too ambitious.

For example, recent exciting progress in low-dimensional topology and geometry would have been unthinkable without powerful techniques from analysis; new achievements in topology and symplectic geometry have been influencing complex analysis; methods of tropical geometry are being used in algebraic geometry, statistical physics, and complex analysis; differential equations coming from complex analysis have applications in algebraic geometry.

The variety of topics presented by leading experts in this volume should nonetheless give some sense of recent developments in these fields and their interactions with one another.

A Marcus Wallenberg Symposium on Perspectives in Analysis, Geometry, and Topology was held at Stockholm University in May 2008. The choice of subjects for this symposium and this volume was motivated by the work and the mathematical interests of Oleg Viro, to whom the symposium and this collection are dedicated. As a mathematician with broad education and interests, Viro has a deep feeling for the unity of mathematics. Viro is famous for fundamental results in several areas of geometry and topology. As a professor at Uppsala University, Viro has made invaluable contributions to Swedish research by complementing the country's longstanding tradition in analysis with his own renowned expertise in geometry and topology.

A Glimpse into Viro's Work

Oleg Viro graduated from the Leningrad (now St. Petersburg) school of topology, one of the strongest that ever existed. He defended his PhD thesis in 1974 under the supervision of V. A. Rokhlin, the founder and head of this school.

Viro's first publications date back to his undergraduate years. They are devoted to branched coverings and braids. He proved that any closed orientable 3-manifold of genus two is a two-fold branched covering of the three-dimensional sphere branched over a link with three bridges (this was proved independently by J. Birman and H. Hilden).

Viro's results on branched coverings also include interpretations of the signature invariants of a link of codimension 2 as signature invariants of cyclic branched covering spaces of a ball, and estimates for the slice genus of links and the genus of non-locally-flat surfaces in 4-manifolds.

During the year of Viro's thesis defense, Rokhlin was stricken with a severe heart attack, and Viro, then 26 years old, took over Rokhlin's course in topology for second-year undergraduates. Teaching Rokhlin's course was no easy task. On one hand the brilliance of Rokhlin's lectures was unique, meeting his standard of lecturing seemed to be out of reach. On the other hand, the novelty and level of abstractness provided difficulties for many students, which was reflected in examination scores that displeased the University administration.

As a result of increasing antisemitism and suppression of intellectual freedom in the Soviet Union during the 1970s, Viro was at that time the only graduate of Rokhlin's extremely strong school of topology to be hired by the department. He took on himself the responsibility for topology as a research subject at the department and for the education of students in this field. In the years 1975–1985 he became a central figure in the scientific and pedagogical life at the Mathematical–Mechanical Department of Leningrad University.

A. Vershik has communicated to us that in 1979, Rokhlin remarked to him, "Do you know why it is worthwhile for a student to study in this department? Because Oleg Viro is working there." Anyone who is aware of Rokhlin's high standards and restrained praise can imagine the value of these words of appreciation.

In this period, Oleg Viro introduced the *patchworking technique*, his best-known contribution to *real algebraic geometry*. Viro's investigation followed a breakthrough made in topology of real algebraic varieties in the late 1960s and 1970s. The work of Gudkov, Arnold, Rokhlin, Utkin, and Kharlamov provided answers to questions formulated in the first part of Hilbert's 16th problem (isotopy classification of nonsingular curves of degree 6 in the real projective plane $\mathbb{R}P^2$ and topological classification of nonsingular quartic surfaces in the three-dimensional real projective space) and gave a conceptional understanding of many general phenomena concerning the topology of real algebraic varieties.

For curves of higher degree, an isotopy classification was not accessible by the previous techniques. The patchworking construction allowed Oleg Viro to complete the isotopy classification of nonsingular curves of degree 7 in $\mathbb{R}P^2$ (the largest degree for which such a classification is available even today) and to disprove Ragsdale's conjecture from 1906 concerning curves of even degree.

The impact of the patchworking technique is much wider. Patchworking revolutionized real algebraic geometry by linking together real algebraic geometry, toric geometry, and combinatorics of convex polytopes. It results in a powerful construction of real algebraic varieties with prescribed topology. Almost all recent constructions of real algebraic varieties use Viro's theorem. Patchworking was a fundamental idea in motivating the appearance of tropical geometry. In fact, Viro established a relation between patchworking and Maslov's dequantization of positive real numbers that was one of the starting points of tropical geometry.

A combinatorial version of Viro's theorem is as follows. Let n be a positive integer and Δ a convex lattice polytope in $\mathbb{R}^n$. Consider a lattice triangulation of Δ and a distribution of signs at the vertices of the triangulation. In this situation, Viro constructs a piecewise-linear hypersurface H in the real part $\mathbb{R}T(\Delta)$ of the toric variety $T(\Delta)$ associated with Δ. His theorem then produces, under a certain condition on the triangulation, a real algebraic hypersurface in $\mathbb{R}T(\Delta)$ that is isotopic to H and defined by a polynomial of Newton polytope Δ. This version of Viro's theorem is called *combinatorial patchworking* and provides a rich collection of real algebraic hypersurfaces. The piecewise-linear hypersurfaces appearing in this construction can be seen as real tropical hypersurfaces.

The achievements of Viro in real algebraic geometry also include important restrictions on the topology of real algebraic curves and a notion of complex orientations of real algebraic varieties of dimension ≥ 2 (generalizing the notion of complex orientations of a real algebraic curve dividing its complexification).

Viro was invited to the International Congress of Mathematicians in Warsaw in 1983. His article on real algebraic varieties was published in the proceedings, but he was forbidden by the administration of Leningrad University to attend the congress. The reason behind the denial was the following story. In a wave of antisemitism and personal attacks against Rokhlin, whose independence and nonconformity were taken by the authorities as a provocation, the administrators of the Mathematics Department tried to force Rokhlin, then 60 years old and mathematically active, into early retirement. One of the measures was an attempt to prevent Rokhlin from supervising his graduate student Mikhail Goussarov and to assign Goussarov to

Oleg Viro. Viro declined, and Goussarov remained Rokhlin's student. Viro was thus punished for his display of loyalty and human decency. He never lost the remarkable personality trait that emerged on this occasion: a disposition to protect others, to ease their burdens, and to expose himself to the slings and arrows of outrageous fortune. His empathy and his courage to speak the truth and take a principled stand have always been unquestioned.

Viro's best-known contribution to *low-dimensional topology* is his joint work with Vladimir Turaev that defines "quantum invariants" of 3-manifolds as a certain state sum over an arbitrary triangulation. The state sum is based on so-called quantum 6j-symbols related to the representation theory of the quantum group $U_q\mathfrak{sl}(2)$ for a root of unity q. Notably, the construction leads to a $(2+1)$-dimensional topological quantum field theory – the first rigorous realization of an approach by the physicists Ponzano and Regge to $(2+1)$-dimensional quantum gravity. The paper contributes to the relationship – for mathematicians still largely mysterious – between topology and mathematical physics. Remarkable parallels were revealed concerning the relationship between analysis and theoretical physics: for instance, the interest of Sweden's world-renowned analyst Lennart Carleson in two-dimensional conformal field theory paved the way for Stanislav Smirnov to prove a formula for critical site percolation predicted by the physicist Cardy.

We also mention the following articles among the numerous papers of Viro in *topology*. The volume of Lecture Notes in Mathematics dedicated to the memory of V. A. Rokhlin contains a paper that caused McPherson to refer jokingly to Viro as the guy "who is able to integrate the Euler characteristic." In this paper Viro discussed one of the manifestations of certain "universality" of the Euler characteristic. The same volume contains a construction of the first infinite series of surfaces smoothly embedded in the four-dimensional sphere, which are pairwise ambiently homeomorphic but not diffeomorphic (joint work with S. Finashin and M. Kreck). A joint work of Viro with M. Goussarov and M. Polyak treats diagrammatic formulas for finite-type knot invariants for classical and virtual knots.

Viro has spent a great deal of time and energy with his graduate students. His former students, many of them now renowned professors of mathematics, represent diverse areas of geometry and topology. Viro's principle is to put high demands on his students but to be generous and supportive of them. He believes that education of the next generation of mathematicians cannot be reduced to imparting knowledge and skills. It is also important to mold responsible personalities capable of mastering the challenges of the age, which is often accomplished less through words than by example, through nonverbal messages conveyed by the personality of the teacher, as Viro experienced himself as a student of Rokhlin.

Viro's activities are not limited to research in topology and geometry and the education of graduate students. He is concerned as well about the role of mathematics as an irreplaceable part of human culture and about its survival – in his view a very complex task. This includes an understanding of the development of mathematics as a whole and of its interaction with other sciences, as well as an open mind for promising areas of mathematics, a careful choice of research subjects, and above all, the education of the next generation beginning with the foundations of mathematics. Viro has striven to return geometry to its important place in the educational program. While still in St. Petersburg, he participated in a project to provide students their first approach to mathematics and mathematical rigor through developing geometric intuition rather than through developing the ability to formally manipulate mathematical symbols. In Uppsala, Viro initiated a special program for strong students following similar lines and offered a newly developed course on basic geometry.

Viro taught at Uppsala University until his forced resignation in 2007. Now he is professor at State University of New York, Stony Brook.

Despite all the difficulties he has experienced, Viro has maintained his ability to meet people with a delightfully warm and sincere smile.

Paris, France — Ilia Itenberg
Bures-sur-Yvette, France — Burglind Jöricke
Stockholm, Sweden — Mikael Passare

On the Content of This Volume

The papers of the volume represent various topics in geometry, topology, and analysis. To give an impression of the diversity of these papers, we have divided them provisionally into groups and have included a short description of each. A complete list of the symposium talks appears after the description of the content of this volume.

Algebraic Geometry

K. Kaveh and A. Khovanskii: *Algebraic Equations and Convex Bodies*. This paper is a summary of the authors' recent work on intriguing relationships between algebraic geometry on the one hand, and convex bodies and their combinatorics on the other. Their results can be seen as far-reaching generalizations of the Bernstein–Kushnirenko theorem, which relates algebraic geometry and the theory of mixed volumes.

N. Mok: *Projective Algebraicity of Minimal Compactifications of Complex-Hyperbolic Space Forms of Finite Volume*. This paper considers quotients of the unit ball in complex affine space by lattices (i.e., by discrete subgroups of the holomorphic automorphism group with quotients of finite Bergman volume). The author shows that for (possibly nonarithmetic) quotients, the minimal compactification (obtained by adding a finite number of points to the cusps) is projective algebraic.

E. Shustin: *Tropical and Algebraic Curves with Multiple Points*. This paper is devoted to a new patchworking theorem allowing one to construct algebraic curves with multiple points. This theorem can be seen as a generalization of Viro's patchworking construction and has important applications. It can be used to obtain a new correspondence theorem between the complex algebraic and tropical worlds.

Real Algebraic Geometry

The topics of this group are intimately related to the work of Oleg Viro in topology of real algebraic varieties.

A. Degtyarev: *Toward a Generalized Shapiro and Shapiro Conjecture*. The Shapiro and Shapiro conjecture, proposed in 1993, is the following: *if all flattening points of a rational curve* $\mathbb{C}P^1 \to \mathbb{C}P^n$ *lie on the real line* $\mathbb{R}P^1 \subset \mathbb{C}P^1$*, then the curve is conjugate to a real algebraic curve under an appropriate projective automorphism of* $\mathbb{C}P^n$. The conjecture was proved in 2005 by E. Mukhin, V. Tarasov, and A. Varchenko. The present paper is devoted to a generalization of the conjecture to curves of arbitrary genus (this generalization was proposed by T. Ekedahl, B. Shapiro, and M. Shapiro). A new, asymptotically better, bound on the genus of a curve that may violate the generalized conjecture is obtained.

A. Degtyarev, I. Itenberg, and V. Kharlamov: *On the Number of Components of a Complete Intersection of Real Quadrics*. This paper concerns the topology of complete intersections of three real quadrics. The main result is the following: The maximal possible number $B_2^0(N)$ of connected components of a regular complete intersection of three real quadrics in $\mathbb{P}^N$ differs by at most one from the maximal number of ovals of the submaximal depth $[(N-1)/2]$ of a real plane projective curve of degree $d = N+1$.

S. Orevkov: *Some Examples of Real Algebraic and Real Pseudoholomorphic Curves*. The paper contains several results concerning the embedded topology of algebraic and pseudoholomorphic curves in the real projective plane. The comparison between real algebraic and real pseudoholomorphic curves can be seen as a natural continuation of the study of *flexible curves* which was initiated by Viro.

E. Shustin: *Tropical and Algebraic Curves with Multiple Points* (see under *Algebraic Geometry*).

Differential Geometry and Differential Equations

Y. Eliashberg and N. Mishachev: *Topology of Spaces of S-immersions*. This paper discusses the *h*-principle for maps between equidimensional manifolds whose only singularities are prescribed folds.

Symplectic and Contact Geometry

K. Baker and J. Etnyre: *Rational Linking and Contact Geometry*. The authors extend the self-linking number of transverse knots and the Thurston–Bennequin invariant and rotation number of Legendrian knots to the case of rationally null-homologous knots. The paper contains a generalization of Bennequins inequality for these knots, a study of sharpness of the Bennequin bound for fibered knot types, and a proof of the fact that rational unknots in tight contact structures on lens spaces are weakly transversely simple and Legendrian simple.

T. Ekholm: *Rational SFT, Linearized Legendrian Contact Homology, and Lagrangian Floer Cohomology*. This paper relates the version of rational symplectic field theory for exact Lagrangian cobordisms introduced in a previous article of the author with linearized Legendrian contact homology.

M. Entov, L. Polterovich, and P. Py: *On Continuity of Quasimorphisms for Symplectic Maps. With an appendix by Michael Khanevsky*. This paper is devoted to C^0-continuous homogeneous quasimorphisms on the identity component of the group of compactly supported symplectomorphisms of a symplectic manifold. The authors give a topological characterization of such quasimorphisms in the case of surfaces and show that for standard symplectic balls of any dimension and for compact oriented surfaces other than the sphere, the space of such quasimorphisms is infinite-dimensional.

D. Gay and A. Stipsicz: *On Symplectic Caps*. This paper gives an explicit construction of symplectic caps (concave fillings) of links of minimal rational singularities. As an application, new examples of surface singularities are obtained that do not admit rational homology disk smoothing.

C. Manolescu and Ch. Woodward: *Floer Homology on the Extended Moduli Space*. The goal of this paper is to approach the Atiyah–Floer conjecture by suitably modifying its symplectic side. The Atiyah–Floer conjecture relates Floer's instanton homology and his Lagrangian homology. Given a Heegaard splitting of a 3-manifold, the authors construct, using Lagrangian Floer homology, a relatively $\mathbb{Z}/8\mathbb{Z}$-graded abelian group that is conjecturally a 3-manifold invariant.

S. Orevkov: *Some Examples of Real Algebraic and Real Pseudoholomorphic Curves* (see under *Real Algebraic Geometry*).

Complex Analysis

R. Berman and J.-P. Demailly: *Regularity of Plurisubharmonic Upper Envelopes in Big Cohomology Classes*. The subject of this paper originates from constructions of Hermitian metrics with minimal singularities on a big line bundle over a compact complex manifold. The developed analytic tool, the proof of a certain regularity of an upper envelope of "quasiplurisubharmonic" functions, is applied to study the Dirichlet problem for a degenerate Monge–Ampère operator as well as geodesics in the space of Kähler metrics.

G. Henkin: *Cauchy–Pompeiu-Type Formulas for $\overline{\partial}$ on Affine Algebraic Riemann Surfaces and Some Applications*. The author gives explicit formulas for solving $\bar{\partial}$- and perturbed ∂-problems on complex affine algebraic curves in complex two-space and applies them to the two-dimensional inverse conductivity problem on surfaces with boundary. This is related to electric impedance tomography, geophysics, and other topics.

N. Mok: *Projective Algebraicity of Minimal Compactifications of Complex-Hyperbolic Space Forms of Finite Volume* (see under *Algebraic Geometry*).

Three- and Four-Dimensional Manifolds, Invariants of Links

S. Akbulut: *Exotic Structures on Smooth Four-Manifolds*. This survey treats topological methods of constructions of exotic copies of 4-manifolds (smooth manifolds that are homeomorphic but not diffeomorphic to the original one). One of the methods (gluing "corks") consists of the following. Cut off a smoothly bounded domain that admits the structure of a relatively compact strictly pseudovonvex domain in a Stein manifold. Glue it back by an involution of the boundary that extends to the domain homeomorphically but not diffeomorphically.

L. Kauffman: *Remarks on Khovanov Homology and the Potts Model*. This paper explores relationships between the Potts model in statistical mechanics and Khovanov homology. The author proves in particular that the Euler characteristics of Khovanov homology (precisely, of its subcomplexes for fixed quantum grading) figure in the computation of the Potts model at certain imaginary temperatures.

N. Reshetikhin, C. Stroppel, and B. Webster: *Schur-Weyl-Type Duality for Quantized* $\mathfrak{gl}(1|1)$*, the Burau Representation of Braid Groups, and Invariants of Tangled Graphs*. This paper is a concise survey of the authors' results in the subject. The paper clarifies the relationship between reduced Burau representations of braid groups, nonreduced Burau representations, and the representation of the braid group defined by R-matrices related to $U_q(\mathfrak{gl}(1|1))$.

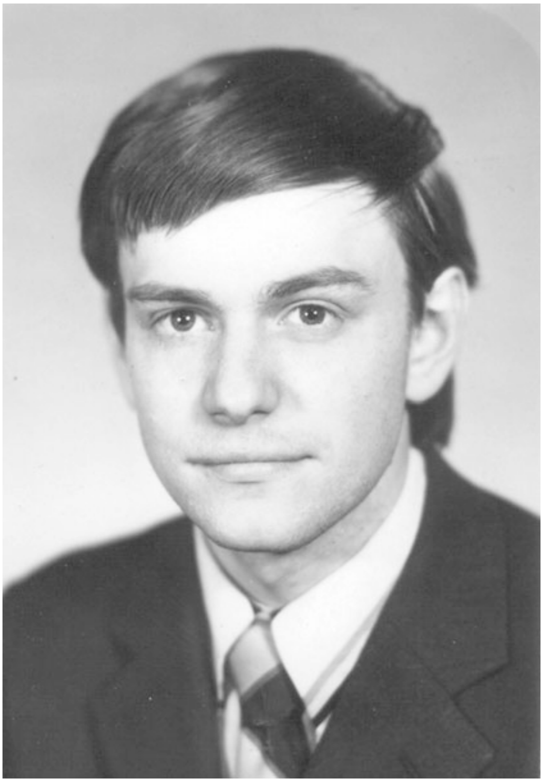
A. Shumakovich: *Khovanov Homology Theories and Their Applications*. This survey presents various versions of Khovanov homology theories. The author discusses relationships between these theories, as well as their properties and applications to other areas of knot theory and low-dimensional topology.

Ilia Itenberg
Burglind Jöricke
Mikael Passare

Acknowledgments The editors want to express deep gratitude to many friends and colleagues of Oleg Viro who generously provided information and assistance. The photographs are courtesy of his family, friends, and colleagues. The computing staff at MSRI and IHES provided help with editing photographs. We would like to thank them all. We are grateful to all speakers at the Marcus Wallenberg Symposium on Perspectives in Analysis, Geometry, and Topology, to all colleagues who contributed to the present volume, and to the anonymous referees of the papers. Lennart Carleson kindly granted permission to include the transcript of his opening remarks. His words have not been edited, and hence some of the charm of a vivid speech has been retained. The symposium was made possible by the generous support of the Marcus Wallenberg Foundation for International Scientific Cooperation, from Stockholm University, and from the Mittag–Leffler Institute.

In memoriam notice On September 15, 2011 our friend, colleague and co-editor Mikael Passare unexpectedly passed away in the result of an accident. He has been widely appreciated as a mathematician, teacher, supervisor and unresting organizer on behalf of mathematics. The symposium would hardly have taken place without his energy and organizational skills. He is missed sadly.

Ilia Itenberg
Burglind Jöricke

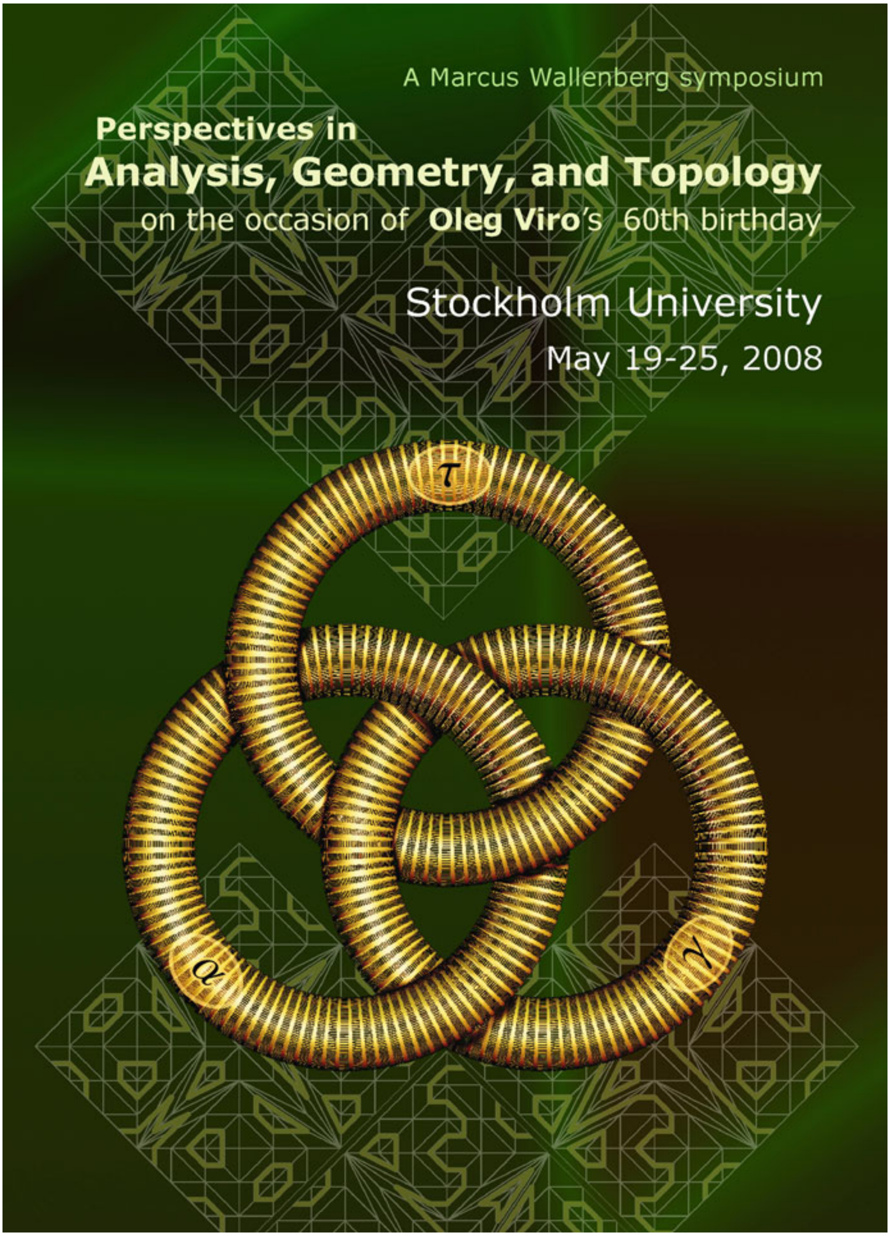


Oleg Viro 1976

Oleg Viro around 1986

Lennart Carleson and Oleg Viro 2008

Oleg Viro 2002



The poster of the Marcus Wallenberg Symposium on Perspectives in Analysis, Geometry, and Toplogy

Program of the Symposium

Monday, May 19:

10.00–10.45	Opening, with a welcoming address by Vice-Chancellor Kåre Bremer and a speech by Lennart Carleson on the "Unity of Mathematics"
11.00–12.00	Louis Kauffman: "An Extended Bracket State Summation for Virtual Knots and Links"
LUNCH	
14.00–15.00	Michael Polyak: "Enumerative Geometry and Finite Type Invariants"
15.30–16.30	Alexander Degtyarev: "Toward a Generalized Shapiro and Shapiro Conjecture"
17.00–18.00	Ngaiming Mok: "Geometric Structures on Fano Manifolds of Picard Number 1"

Tuesday, May 20:

09.30–10.30	Gang Tian: "Ricci Flow and Projective Manifolds"
11.00-12.00	Ludwig Faddeev: "Discrete Series of Representations for the Modular Double of $U_q(\mathfrak{sl}(2,R))$"
LUNCH	
14.00–15.00	Tobias Ekholm: "A Surgery Exact Sequence in Linearized Contact Homology"
15.30–16.30	Paolo Lisca: "Heegaard Floer Invariants of Legendrian and Transverse Knots"
17.00–18.00	John Etnyre: "Fibered Knots and the Bennequin Bound"

Wednesday, May 21:

09.30–10.30	Gennadi Henkin: "Cauchy–Pompeiu-Type Formulas for $\overline{\partial}$ on Affine Algebraic Riemann Surfaces and Some Applications"
11.00–12.00	Charles Epstein: "Solving Maxwell's Equations in Exterior Domains"

LUNCH
14.00–15.00 Elliott Lieb: “Four Decades of ‘Stability of Matter’ and Analytic Inequalities”
15.30–16.30 Eric Bedford: “Dynamics of Rational Surface Automorphisms of Positive Entropy”
17.00–18.00 Mikhail Lyubich: “Yang–Lee Zeros for Diamond Lattices and 2D Rational Dynamics”

Thursday, May 22:
09.00–10.00 Viatcheslav Kharlamov: “On the Number of Connected Components in the Intersection of Three Real Quadrics”
10.30–11.30 Grigory Mikhalkin: “Patchworking of Real Algebraic Knots and Links”
12.00–13.00 Stepan Orevkov: “Classification of Algebraic Links in RP^3 of Degree 5 and 6”
LUNCH
14.00–18.30 Excursion
19.00 Reception at Town Hall

Friday, May 23:
09.30–10.30 Selman Akbulut: “On Exotic Structures on 4-Manifiolds”
11.00–12.00 Nicolai Reshetikhin: “Invariants of Links and Quantum Groups at Roots of Unity”
LUNCH
14.00–15.00 Robert MacPherson: “The Geometry of Grains”
15.30–16.30 Laszlo Lempert: “Two Examples of Complex Manifolds, Motivated by (Theoretical) Physics”
07.00-18.00 Boris Khesin: “A Nonholonomic Moser Theorem and Diffeomorphism Groups”
19.00 Banquet

Saturday, May 24:
09.30–10.30 Tomasz Mrowka: “Knot Invariants from Instantons”
11.00–12.00 Askold Khovanskii: “Algebraic Equations, Convex Bodies, and Bernstein Theorem for Some Spherical Varieties”
LUNCH
14.00–15.00 Ronald Fintushel: “Constructions of 4-Manifolds”
15.30–16.30 Stefan Nemirovskii: “Lagrangian Embeddings and Complex Analysis”
17.00–18.00 Stanislav Smirnov: “Conformal Invariance and Universality in 2D Ising Model”

Sunday, May 25:

09.30–10.30	Eugenii Shustin: "Computing Real Algebraic and Tropical Enumerative Invariants"
11.00–12.00	Alexander Shumakovitch: "Khovanov Homology, Its Properties and Applications"
12.00	Closure

On the Unity of Mathematics

Opening speech by Lennart Carleson
Transcript by the Editors

It is a great pleasure for me to have this opportunity to welcome you all to Stockholm (and on such a beautiful day as we have today—we are not always so lucky!), and to this conference on "Perspectives in Analysis, Geometry, and Topology."

There is, of course, also a very special reason for such a conference at this time and in this place, namely Sweden, and this is that it gives us an opportunity to congratulate Oleg Viro on his 60th birthday, which will happen this year—and may have happened. He had a great influence on the development of mathematics in Uppsala, and on the infusion of geometry and topology in this land of analysis and algebra. Therefore his work is very much in the spirit of this conference, and it is rightly dedicated to him.

Many mathematicians, like myself, are very upset by the way that he was treated by the administration in the University of Uppsala, and by the very unconstructive and exceptional ways in which the administration decided to handle the difficulties there. We were not able to reverse anything, but you should think of this conference as a way of expressing our sentiments to what has happened here.

The subject of the conference defines a subset of the greater issue of the unity of Mathematics. This problem arose really during the 1800s in two different ways. It started with the introduction of the notion of applied mathematics. Gauss might be considered the last universal mathematician in the sense of also including applied mathematics, and what happened during the nineteenth century is the introduction of serious rigor in what we now call pure mathematics, and also the separation of the applied fields, first into different kinds of sciences, and later now also into all kinds of biology and social sciences. And this is a development that still goes on.

This is of course a great issue, which I am not really going to talk about, because the conference here belongs to the second movement, namely the separation of smaller areas of what we now call pure mathematics. This fragmentation I think is a serious business that has been going on for one hundred years now. You can probably say that Hilbert and Poincaré were the last generation of people who could somehow be considered to represent all of mathematics.

Then the fragmentation has continued, and this is seen in the names of professorships and the creation of journals and I don't know what. We have journals in, for example, approximation, in semigroups, or in inequalities. This specialization has really accelerated with the introduction of the electronic way of searching for information, which means that you can now define what you are looking for so well that you are sure that you don't see anything else. It is really difficult to have general colloquia and information outside the specialties.

I remember when I was a student and I was looking for information, I went to the library and found the book which contained the article in question, and I was reading that. Often I took it home and and kept it there, and I looked at the other articles in the journal. Very often you found something that interested you, and very often (mostly, I would say) you found something you couldn't understand. But still you got some information, and you got some impression of the field and of what you had around you.

I think that this possibility of finding unexpected information is what corresponds really to what one has in experimental sciences by making mistakes. There are very famous examples here, for instance the Curies, who found radioactivity by putting a stone on a photographic plate, or Fleming, who found penicillin by similarly having contamination of bacteria.

I think the closest we can get here is that we get information that we didn't really expect about something that we didn't know in advance what it was. I have been trying to find an example of this, but I can't really remember any well-known problem being solved by somebody by mistake reading an article on something else. The most well known story that I can recall (probably untrue, but still) is the story about Heisenberg going to the wrong lecture and learning about matrices. But it could be true!

This conference is, of course, an example of this, but I should like to make a personal illustration of something that happened recently to me. I was in Helsinki to give a description of Lars Ahlfors's work, so I had a reason to look at what he had done, and I looked at his famous 1935 paper on Riemann surfaces. It occurred to me that his approach to Nevanlinna theory, which is purely geometric and doesn't contain any formulas or any integration, but still gives a complete description of the value distribution of a meromorphic function, could be the way to understand Vojta's observations on the formal similarity between valuations and Nevanlinna theory, which nobody understands today. I think the reason one doesn't understand it is that one has the wrong model, and Ahlfors's model is of course completely different from the Nevanlinna model. So if I could suggest a good subject where infusion of ideas from a different field could give something, I would suggest that one look more carefully at this.

So I think it is very important that we take actions in the organization of education which make us all and future students more aware of fields outside their specialty, and that we avoid specialization in the narrow way that has been done here. What you could do is to make sure that the programs of graduate studies are more varied and that specialization comes later.

In terms of journals I think the development accelerates now with the electronic journals, and I should think it is very important that we keep a number of the really best journals, so that they still get printed, and so that people still can go to the library and by mistake read an article on something that they didn't really expect. I think it is a great risk that we make everything computerized, and by doing that, as I said, you make sure that you don't learn anything unexpected.

One more and final point would be that one organizes conferences, not only like this one, but also makes sure that the International Congresses are kept. There is a great risk that people criticize them and say that they don't give anything, it is too wide, there are too many people, and so on. One should resist that. It is very important that we keep the subject together.

And then I have avoided the greatest issue, that is, the relation to applied mathematics, which I think takes a very serious reorganization of mathematics. But this is not the time and place for that.

Thank you!

Exotic Structures on Smooth Four-Manifolds

Selman Akbulut

Dedicated to Oleg Viro on the occasion of his 60th birthday

Abstract A short survey of exotic smooth structures on 4-manifolds is given with a special emphasis on the corresponding cork structures. Along the way we discuss some of the more recent results in this direction, obtained jointly with R. Matveyev, B. Ozbagci, C. Karakurt, and K. Yasui.

Keywords Four-manifold • Exotic structure • Lefschetz fibration • Cork

1 Corks

Let M be a smooth closed simply connected four-manifold, and M' an exotic copy of M (a smooth manifold homeomorphic but not diffeomorphic to M). Then we can find a compact contractible codimension-zero submanifold $W \subset M$

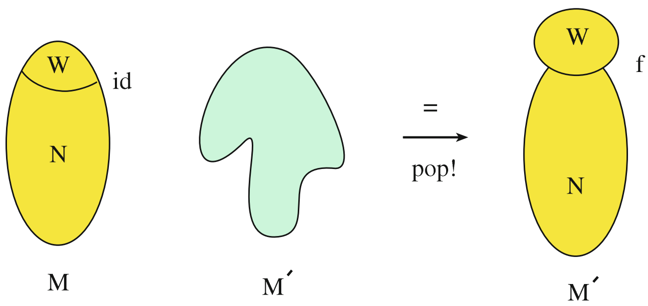


Fig. 1 Popping a cork

S. Akbulut (✉)
Department of Mathematics, Michigan State University, MI 48824, USA
e-mail: akbulut@math.msu.edu

I. Itenberg et al. (eds.), *Perspectives in Analysis, Geometry, and Topology: On the Occasion of the 60th Birthday of Oleg Viro*, Progress in Mathematics 296,
DOI 10.1007/978-0-8176-8277-4_1,