EDITED BY
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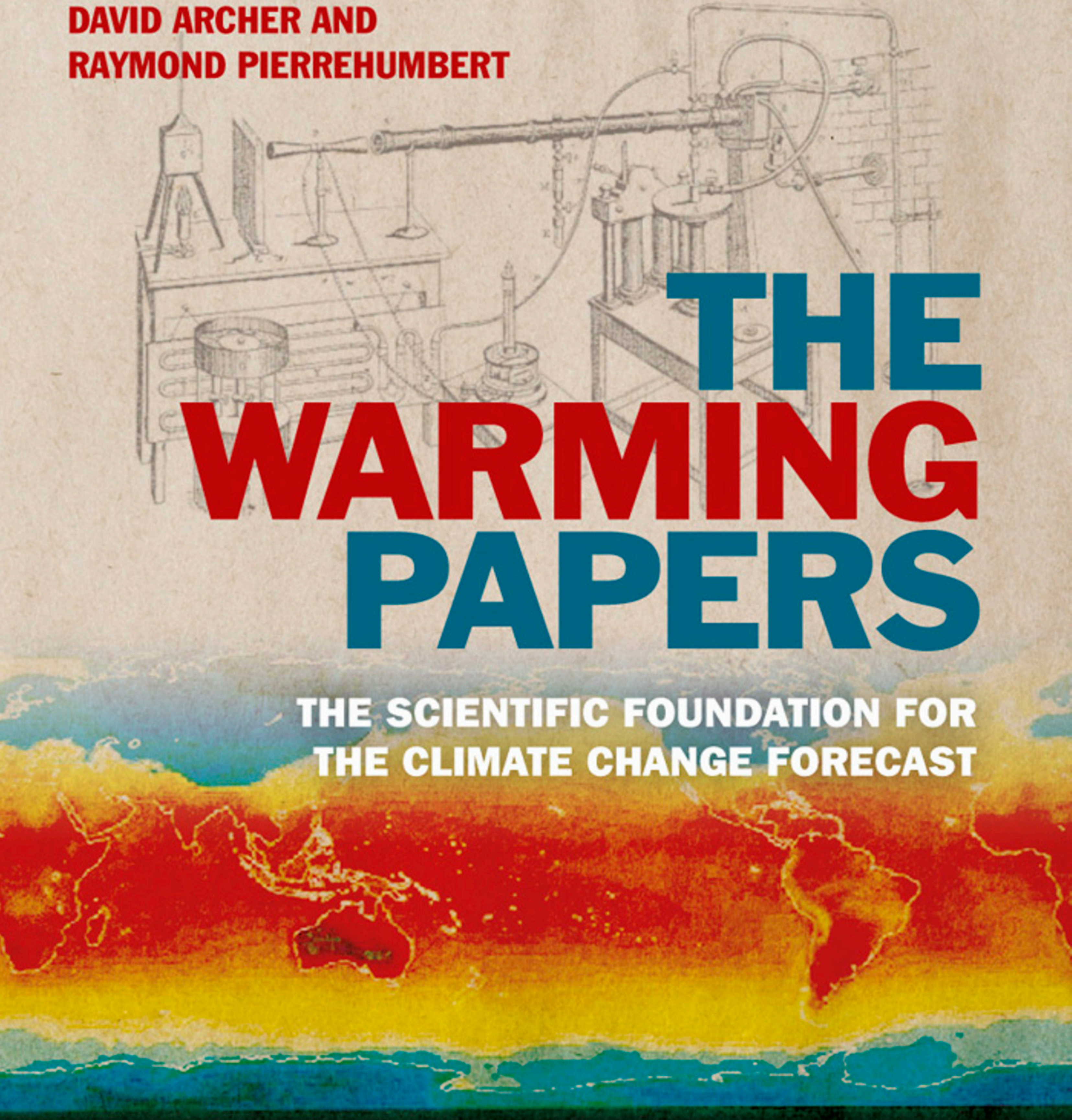

THE WARMING PAPERS

THE SCIENTIFIC FOUNDATION FOR THE CLIMATE CHANGE FORECAST

WILEY-BLACKWELL

The Warming Papers

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The Scientific Foundation for the Climate Change Forecast

Edited by

David Archer

and

Raymond Pierrehumbert

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Preface

Global warming is arguably the defining scientific issue of modern times, but it is not widely appreciated that the foundations of our understanding are almost two centuries old. The sensitivity of climate to changes in atmospheric CO_2 was first estimated about one century ago, and the rise in atmospheric CO_2 concentration was discovered half a century ago. The fundamentals of the science underlying the forecast for human-induced climate change were being published and debated long before it started to appear in the newspapers.

The aim of this book is to gather together the classic scientific papers that are the scientific foundation for the forecast of global warming and its consequences. These are not necessarily the latest in the state of play; there can be subsequent quantitative revision. But these papers are the big ideas. Some of the good old good ones can be heavy going, it must be admitted, so we will try to guide the reader with some verbage of our own, unworthy though it may be. We summarize the results for you, and provide the latest revisions from the ongoing literature, how strong the water vapor feedback turned out to be, for example. We will fill in the context, the personalities, and the aftermath of the ideas in the papers. We'll also presume to provide short comments where they occur to us in boxes throughout the papers, signposts to help guide the casual reader.

Part I

Climate Physics

1

The Greenhouse Effect

Fourier, J. (1827). Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires. *Mémoires de l'Académie Royale des Sciences*, **7**, 569–604. 25 pages.

Joseph Fourier (1768–1830) is generally credited with the discovery of what is now known as the greenhouse effect. In fact, his contribution to the study of planetary temperature is even more profound than that. Fourier introduced the problem of planetary temperature as a proper object of study in physics, and established a largely correct physical framework for attacking the problem. His work set the stage for most of the further developments in this area over the remainder of the nineteenth century. Indeed, it was only toward the end of that century that physics had caught up to the point that the first quantitative estimates of the Earth's temperature based on Fourier's concepts could be attempted.

If much of Fourier's reasoning in this paper seems qualitative, it should be recognized that most of the areas of physics that Fourier needs to call on were in their infancy in Fourier's day. Infrared radiation (called "dark heat" or "dark radiation" at the time) had been discovered in 1800 by the astronomer Sir Frederick William Herschel, and it was the subject of intense inquiry. Infrared was the "dark energy" of its day and it was perhaps no less mysterious to physicists of Fourier's day than is the dark energy talked about by today's physicists. There was some understanding from the work of Fourier's contemporaries, Dulong and Petit, that the rate of heat loss by infrared radiation increases with temperature, and it was known that infrared could carry heat through a near-vacuum. There was, however, only a limited ability to do quantitative calculations involving infrared heat transfer. Thermodynamics was in its infancy. The very nature of heat was still being hotly debated; the landmark energy conservation experiments of Joule that showed the equivalence of mechanical work and heat would not be carried out until 1843. Against this context, the general correctness of Fourier's great leap of intuition seems all the more remarkable.

In his 1827 paper, Fourier introduces five key concepts:

1. The temperature of the Earth, or indeed any planet, is determined by a balance between the rate at which the energy is received and the rate at which the energy is lost. There is therefore a need to determine the sources and sinks of a planet's energy.
2. There are three possible sources of heat: Sunlight, heat diffusing from the hot interior of the planet, and heat communicated from the general "temperature of space." Of these, the amount of heat leaking out of the Earth's interior is too small to play a significant role in the Earth's surface temperature.
3. Emission of infrared radiation is the only means by which a planet loses heat. Since the rate of energy loss by infrared radiation increases with the temperature of the body, the planet can come into equilibrium by heating up until the rate at which it loses energy by infrared emission equals the rate at which it gains energy from its energy sources.
4. Visible light is converted into infrared light when it is absorbed at a solid or liquid surface.
5. The atmosphere has an asymmetric effect on the incoming sunlight and the outgoing infrared, because the atmosphere is largely transparent to sunlight but is relatively opaque

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> to infrared. This retards the rate at which the planet loses energy, for any given temperature. The result is that the atmosphere keeps the planet warmer than it would have been if the atmosphere had been transparent to infrared radiation.

Fourier's inferences concerning the minimal influence of the Earth's interior heat on climate are drawn from observations of the way temperature varies with depth below the Earth's surface. Of all Fourier's claims in the 1827 paper, this is the one that is most backed up by quantitative reasoning, though the actual mathematical analysis appears in Fourier's other papers and is not reproduced in the 1827 essay. Fourier's greatest work as a mathematical physicist was the formulation of the partial differential equation describing the diffusion of heat within a body, and the development of the mathematical techniques required to solve it. The full range of these developments were engaged in Fourier's interpretation of the Earth's subsurface temperature variations. Indeed, Fourier states that the problem of planetary temperatures provided the main impetus for his formulation of the analytical theory of heat. His theory of heat was applied to the problem in two basic ways. First, since the rate of heat flow is proportional to the temperature gradient, the measured increase of time-mean temperature with depth itself shows that the interior of the Earth is hotter than the surface, and gives an estimate of the heat flux, provided that one can estimate the thermal conductivity of the Earth. The flux Fourier arrived at using this procedure was an overestimate compared to modern calculations because he used the thermal conductivity of iron, but his calculation nonetheless showed the diffusion of heat from the interior to be an insignificant factor in surface temperature. The second kind of problem Fourier did was to impose the observed time-periodic daily and seasonal fluctuations of temperature at the surface as a boundary condition, and then calculate what the subsurface temperature fluctuations should look like. It was this kind of calculation that led Fourier to develop what we now call Fourier series, so as to decompose the complex time-periodic boundary condition into a sum of simple sines and cosines for which the problem is analytically tractable. This calculation correctly predicts that the diurnal variation of temperature should decay rapidly with depth and the annual variation more slowly. The calculation also gives an estimate of the amount of heat that flows into and out of the surface from sunlight in the course of the diurnal and seasonal cycle, and thus provides an additional check on the importance of solar energy in determining the Earth's surface temperature.

It takes away nothing from Fourier's brilliance to point out the one stupendous blunder in his paper. Fourier thought that the heat the Earth receives from the general temperature of interplanetary space was a crucial factor in the Earth's climate, on a par with energy received from the Sun. He thought the temperature of space to be somewhat below the minimum temperatures observed in Winter in the Arctic – roughly 200 K in modern terms. He viewed this as one of his principle discoveries, and claimed that without this source of heat, the Earth would become infinitely cold at night and in the winter, and that no life would be possible. In essence, Fourier's view was that 200 K was the natural temperature that all Solar System planets would relax to if there were no absorption of sunlight. Conceptually, he was not entirely wrong, though the correct number for the "temperature of space" in this sense would be more nearly 5 K than 200 K, but Fourier's estimate of the temperature of space was based on highly dubious reasoning that did not justify his level of certainty by any means. The assumption that Arctic night temperatures represent the temperature of space neglects the role played by the long time required for the ocean to cool down ("thermal inertia") and by the ability of air and ocean currents to transport heat from warmer parts of the planet to the poles. Fourier knew about these effects, and even mentions them explicitly elsewhere in the essay. Evidently, he thought they were too ineffective to account for the observed winter and night-time temperature, though his reasons for preferring the more exotic solution of a high temperature of space remain obscure.

In any event, Fourier's misconception about the temperature of space was corrected by Claude Pouillet in 1838. Pouillet's main contribution to science was a largely correct measurement of the

Solar Constant, though his estimate of the corresponding temperature of the Sun was in error because of shortcomings of then-current representations of blackbody radiation. In the course of these measurements, Pouillet found that the temperature of space was far below the value supposed by Fourier, and nothing more was heard thereafter about the role of the temperature in space in climate.

de Saussure's Hot Boxes

In thinking about the effect of the atmosphere on the Earth's energy balance, Fourier drew on the behavior of a simple device invented by the Swiss Alpinist Horace-Bénédict de Saussure (1740–99). This device, called a *heliothermometer*, consisted of a wooden box insulated with cork and wool, with a lid consisting of one or more panes of transparent glass (Fig. 1). The interior walls were painted black so as to absorb nearly all the sunlight entering the box, and a thermometer was placed in the box so that its temperature could be determined. de Saussure devised this instrument as a means of measuring the intensity of sunlight, so that he could test the hypothesis that it is colder atop mountains because the sunlight is weaker there. The idea was to trap the energy of sunlight inside the box, and keep the interior isolated from the surrounding so that the temperature in the box would be responsive to the intensity of the sunlight rather than the temperature of the surroundings. Using the heliothermometer, de Saussure correctly concluded that sunlight becomes, if anything, more intense at higher elevations, so that some other physical process must come into play. "Hot-Boxes" such as de Saussure's were popular toys among scientists throughout the nineteenth century, and many succumbed to the temptation to use them as solar cookers. de Saussure writes that "Fruits … exposed to this heat were cooked and became juicy." Herschel himself took a hot-box with him to South Africa in 1830, and reported: "As these temperatures [up to 240°F] far surpass that of boiling water, some amusing experiments were made by exposing eggs, meat, etc. [to the heat inside the box], all of which, after a moderate length of exposure, were found perfectly cooked. … [On] one occasion a very respectable stew of meat was prepared and eaten with no small relish by the entertained bystanders."

Neither de Saussure nor Fourier hit on the correct explanation of the decline of temperature with altitude, which involves the cooling of air parcels as they are lifted and expand. Nonetheless, the behavior of the heliothermometer provoked a lot of useful thinking about the energy carried by sunlight. Fourier's use of the analogy was to show that if one keeps the rate of energy *input* by sunlight the same, but retards the rate of energy *loss* by putting on a pane of glass, then when the system comes into equilibrium its temperature will be greater than it would have been without the glass in place. Fourier knew that the glass was transparent to sunlight and largely opaque to infrared, but he also knew that in the typical experiment the glass retards heat loss, in part, by simply trapping warm air in the box and keeping it from blowing away. He alludes to the fact that the experiment would still yield an elevation of temperature even if performed in a vacuum, but his use of the subjunctive in the original French suggests that this is a thought experiment, rather than one he actually carried out.

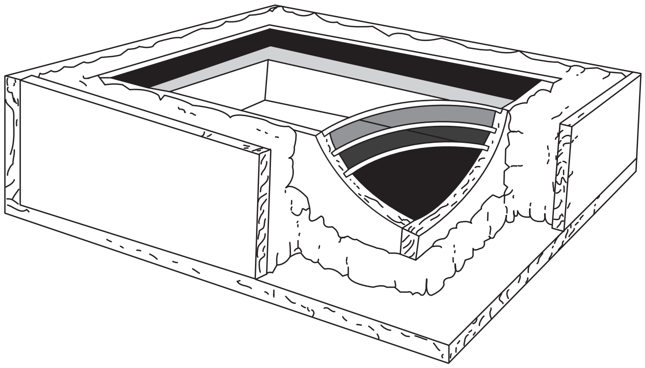

Fig. 1.1 **Artist's conception of the Saussure's improved hot box.**

What Fourier Did Not Do

One thing Fourier did not do was coin the term “greenhouse effect,” though his use of de Saussure’s heliothermometer as an analogue could be considered similar to a greenhouse analogue. de Saussure’s box is indeed a kind of miniature greenhouse. In any event, Fourier showed a clear awareness of the imperfection of the analogy, stating explicitly that the temperature in the hot box was influenced by turbulent heat transfers that have no proper counterpart in the planetary temperature problem.

Further, Fourier did not compute the temperature of the Earth in the absence of an atmosphere and concluded that it was colder than the observed temperature. In fact, he never actually computed the Earth’s temperature based on a balance between incoming sunlight and outgoing infrared, though he could have attempted this using the Dulong–Petit radiation law. It is not clear why Fourier thought the atmosphere had to have a warming role. Rather than this being demanded by too cold temperatures in the absence of an atmosphere, Fourier seems to be inferring that the atmosphere ought to act like a pane of glass in being transparent to sunlight but opaque to infrared; he shows awareness of the downward infrared radiated by the atmosphere, but it is not clear what the basis of Fourier’s leap of intuition about the atmosphere was. In any event, he was right, and his work stimulated a great deal of further research on the effect of the atmosphere on infrared, and ultimately Tyndall’s definitive experiments to be discussed next.

On the Temperatures of the Terrestrial Sphere and Interplanetary Space

JEAN-BAPTISTE JOSEPH FOURIER

Translator's note. This is a translation of Jean-Baptiste Joseph Fourier's "Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires," which originally appeared in *Mémoires dl'Académie Royale des Sciences de l'Institute de France* **VII** 570–604 1827. The original text is most readily accessible in the 1890 edition of Fourier's collected *Oeuvres*, Volume 2, edited by M. Gaston Darboux (Gauthier-Villars et Fils:Paris). This work is available online from the Bibliothèque Nationale de France (search catalogue.bnf.fr for author "Fourier, Jean-Baptiste-Joseph"). In the version reprinted in the *Oeuvres*, it is noted that a very slightly different version of the essay also appeared in the *Annales de Chimie et de Physique*, vol XXVII, pp 136–167; 1824, under the title "Remarques générales sur les températures du globe terrestre et des espaces planétaires."

An English translation of Fourier's article has not been available in print for more than a century. Although the article is widely cited, it is my experience that its actual contents are not well known in the Anglophone community (and they are hardly better known among Francophones). My object in doing a new translation is to help rectify this situation, while using some of my own knowledge of physics of climate to help put Fourier's arguments in the clearest possible light. I have put a premium on readability rather than literal translation, and in some cases I have taken the liberty of rephrasing some sentences so as to make Fourier's reasoning more evident; I do not think that in doing so I have read more into the text than Fourier himself put there, but readers seeking the finer nuances of Fourier's meaning will of course have to read the original. I have not consulted any of the existing translations in carrying out the present one, though I can recommend to the reader's attention the annotated translation by W. M. Connolley, available online only at www.wmc.care4free.net/sci/fourier_1827.

I have provided some commentary in the form of footnotes, which are marked by my initials.

Note that for variety, Fourier often uses *globe terrestre* for "Earth," This also serves to remind the reader of the connection with Fourier's earlier idealized work on heat diffusion in a sphere. In the title, I have preserved this sense, but for the most part the phrase has simply been translated as "Earth" in the text.

R. T. Pierrehumbert
1 September, 2004
Chicago, IL, USA

The question of the Earth's temperature distribution, one of the most important and most difficult of all Natural Philosophy, is made up of rather diverse elements that must be considered from a general point of view. It has occurred to me that it would be useful to unite in a single work the principle consequences of this theory; the analytical details that have been omitted here can for the most part be found in the Works which I have already published. Above all, I wish to present to physicists, in a broader picture, the collection of pertinent phenomena and the mathematical relations amongst them.

It is first necessary to distinguish the three sources from which the Earth derives its heat:

(1) The Earth is heated by solar radiation, the unequal distribution of which produces the diversity of climates;
(2) It participates in the common temperature of interplanetary space, being exposed to irradiation by countless stars which surround all parts of the solar system;
(3) The Earth has conserved in the interior of its mass, a part of the primordial heat which it had when the planets originally formed.

By considering each of these three causes and the phenomena which it produces, we will come to understand as clearly as possible, within the limitations of the current state of science, the principal characteristics of these phenomena. In order

Fourier, J.-B. F. (1827) On the temperatures of the terrestrial sphere and interplanetary space. *Mémoires de l'Académie Royale des Sciences* 7: 569–604.

to provide an overview of this grand question, and to give a first indication of the results of our investigations, we shall present them first in summary form. This summary, in a manner of speaking, serves as an annotated table of contents to my work on the subject.

Our solar system is located in a region of the universe of which all points have a common and constant temperature, determined by the light rays and the heat sent by all the surrounding stars. This cold temperature of the interplanetary sky is slightly below that of the Earth's polar regions. The Earth would have none other than this same temperature of the Sky, were it not for two causes which act together to further heat it. The first is the interior heat which the globe possessed when the planetary bodies were formed, and of which only a part has escaped through the surface. The second cause is the continual action of solar radiation, which has penetrated the whole mass of the Earth and which leads at the surface to the difference in climates from one place to another.

The primordial heat of the globe no longer has any significant effect at the surface, but it can still be immense in the interior of the Earth. The temperature of the surface does not exceed by more than a thirtieth of a degree the value that it will eventually achieve after a long time has passed: At first, it diminished very rapidly; however, at present the diminution continues only exceedingly slowly.

The observations collected so far indicate that the points of a vertical line continued into the solid earth become warmer with increasing depth, and this rate of increase has been estimated at 1 degree for each 30 to 40 meters. Such a result implies a very high temperature for the interior of the Earth; it can not arise from the action of solar radiation: rather, it is naturally explained by the heat the Earth has retained from the time of its origin.

This rate of increase, on the order of 1 degree per 32 m, will not always remain the same: It will diminish progressively; however, a great many centuries (much more than 30,000 years) will pass before it will be reduced to half of its present value.

It is possible that other yet-unknown causes can explain the same facts, and that there are other general or incidental sources of terrestrial heat. If so, one will discover them through comparison of the results of the present theory against observations.

The heat rays which the Sun incessantly sends to the Earth produce two very distinct effects there: The first is periodic and affects the outer envelope of the planet, while the other is constant; one observes it in deep places, for example at 30 m below the surface. The temperature of these locations is subject to hardly any change in the course of the year, it is fixed; however the deep temperature varies substantially from one climatic zone to another: it results from the perpetual action of solar radiation and the inequal exposure of the surface to these rays, from the equator to the poles. One can determine the time which had to pass in order for the solar radiation to produce the diversity of climates observed today. All these results are in accord with dynamical theories which have led us to recognize the stability of the Earth's axis of rotation.

The periodic effect of solar heating consists of both diurnal and annual variations. Observations of this type are reproduced exactly and in all details by the theory. The comparison of results with observations can be used to measure the thermal conductivity of the material of which the crust of the Earth is formed.

The presence of the atmosphere and surface waters has the effect of rendering the distribution of heat more uniform. In the Ocean and in lakes, the most cold molecules – or more precisely, those with the greatest density – direct themselves continually towards lower regions, and the transport of heat due to this cause is much more rapid than that which can be accomplished in solid bodies by means of thermal conductivity. Mathematical examination of the former effect will require numerous and exact observations: they will serve to clarify how these internal fluid motions keep the internal heat of the globe from having a notable effect in the depths of the waters.[1] Liquids conduct heat very poorly; but they have, as do gaseous materials, the the property of being able to transport it rapidly in certain directions through fluid motions. It is this same property which, in combination with centrifugal force, displaces and mixes all parts of the atmosphere and those of the Ocean; it involves organized and immense currents.

The interposition of air greatly modifies the effects of heat at the surface of the globe. The rays of the Sun, in traversing the layers of the atmosphere compressed by their own weight, heats them very inequally: Those which are the most tenuous are also the most cold, because they attenuate and absorb a lesser quantity of these rays.[2] The heat of the Sun, arriving in the form of visible light, has the ability to penetrate transparent solid or liquid substances, but loses this ability almost completely when it is converted, by its interaction with the terrestrial body, into dark radiant heat.

This distinction between luminous heat and dark heat explains the increase of temperature caused by transparent bodies. The body of water which covers a great part of the globe and the polar ice pose less of an obstacle to the incident luminous heat than to the dark heat, which returns in the opposite sense to exterior space.[3] The presence of the atmosphere produces an effect of the same sort, but which, in the present state of theory and owing further to lack of observations with which theory may be compared, cannot yet be exactly defined. However great the effect may be, one would not suppose that the temperature caused by the incidence of the rays of the Sun on an extremely large solid body would greatly exceed that which one would observe on exposing a thermometer to the light of that star.

The radiation from the highest layers of the atmosphere, whose temperature is very cold and nearly constant, influences all meteorological features which we observe: this radiation can be rendered more easily detectible by means of reflection from concave mirrors. The presence of clouds, which intercept these rays, tempers the cold of the nights.[4]

One thus sees that the surface of the Earth is located between one solid mass, whose central heat may surpass that of incandescent matter, and an immense region whose temperature is below the freezing point of mercury.

All the preceding considerations apply equally well to other planetary bodies. One can consider them as being placed in an environment whose common temperature is constant and somewhat below that of the terrestrial polar regions. This temperature – the temperature of the heavens – is the temperature that would be found at the surface of the most distant planets, for the Solar radiation would be too weak, even augmented by the state of the surface, to have a significant effect; From the state of the Earth we know further, that on other planets (whose formation could hardly have been much later than that of the Earth) the interior remanent heat no longer causes any significant elevation of surface temperature.

It is similarly likely that, for most of the planets, the polar temperature is only slightly greater than that of interplanetary space. As for the mean temperature caused by the action of the Sun on each of these bodies, we are in a state of ignorance, because it can depend on the presence of an atmosphere and the state of the surface. One can only assign, in a very imprecise manner, the mean temperature which the Earth would acquire if it were transported to the same position as the planet in question.

After this discussion, we will treat in succession the various parts of the question. First we must set forth a remark the significance of which bears on all these parts, because it is founded on the nature of the differential equations governing the movement of heat. Namely, we make use of the fact that the effects which arise from each of the three causes which we have discussed above can be calculated separately, as if each of these causes existed in isolation. It suffices then to combine the partial effects; they can be freely superposed, just as for the problem of final oscillations of bodies.[5]

We shall describe first the principal results caused by the prolonged action of solar rays on the Earth.

If one places a thermometer at a considerable depth below the surface of the solid Earth, for example at 40 meters, this instrument indicates a fixed temperature.[6]

One observes this fact at all points of the globe. This deep subsurface temperature is constant for any given location; however, it is not the same in all climates. Generally speaking, it decreases as one moves towards the poles.

If one observes the temperature of points much closer to the surface, for example at 1 m or 5 m or 10 m of depth, one notices very different behavior. The temperature varies during the course of a day or a year; however, we will for the moment idealize the problem by supposing that the skin of the Earth wherein such temperature variations occur is eliminated. We then consider the fixed temperatures of the new surface of the globe.

One can imagine that the state of the mass has varied continually in accord with the heat received from the heat source. This variable temperature state gradually alters, and more and more approaches a final state which no longer varies in time. At that time, each point of the solid sphere has acquired – and conserves – a fixed temperature, which depends only on the position of the point in question.

The final state of the mass, of which the heat has penetrated through all parts, is precisely analogous to that of a vessel which receives, through its upper opening, a liquid which furnishes a constant source, and which allows liquid to escape at a precisely equal rate through one or more openings.

Thus, the solar heat accumulates in the interior of the globe and is continually renewed. It penetrates the portions of the surface near the equator, and escapes through the polar regions. The first question of this type which has been subjected to calculation can be found in a dissertation which I read at the Institute of France at the end of 1807, article 115, p. 167.[7] This work has been deposited in the Archives of the Academy of Sciences. At the time, I took up this first question in order to offer a remarkable example of the application of the new theory presented in the article, and to show how analysis reveals the routes followed by solar heat in the interior of the globe.

Let us now restore the upper envelope of the Earth, for which the points are not sufficiently deep for their temperatures to be time-independent. One then notices a more intricate range of phenomena, which can be completely accounted for by our analysis. At a moderate depth, such as 3 m to 4 m, the temperature observed does not vary in the course of the day; however, it changes very noticably in the course of a year; it alternately rises and falls. The amplitude of these variations, that is to say the difference between the *maximum* and the *minimum* of temperature, is not the same at all depths; it becomes less as the distance from the surface becomes greater. The points lying on a vertical line do not all achieve their extremes of temperature at the same time. The amplitude of the variations, and the time of year at which the highest, mean and lowest temperatures are achieved, change with the position of the point on the vertical. The same applies to the quantities of heat which alternately descend and rise: all these quantities have very definite relations amongst each other, which are indicated by experiment and which analysis expresses very distinctly. The observations conform to the results furnished by the theory; there is not any natural effect more completely explained than this. The mean annual temperature of any given point of the vertical, that is, the mean of all values observed at this point in the course of a year, is independent of depth. It is the same for all points of the vertical, and in consequence, the same as that observed immediately below the surface: it is the invariable temperature of deep places.

It is obvious that, in the statement of this proposition, we have idealized away the interior heat of the globe, and with greater reason the accessory effects which could modify this result in any given place. Our principle object is to bring to light the general nature of the phenomena.

We have said above that the diverse effects can be considered separately. It should also be noted that all of the numerical evaluations given in this article are presented only as examples of how the calculation may be performed. The meteorological observations needed to reveal the heat capacity and permeability of the materials which make up the globe are too uncertain and limited to permit the calculation of precise results; nonetheless, we present these numbers in order to show how the formulae are applied. However inexact these evaluations may be, they serve to give a more correct idea of the phenomena than would general mathematical expressions bereft of numerical application.

In the portions of the envelope closest to the surface, a thermometer would rise and fall in the course of each day. These diurnal variations become insignificant at a depth of 2 m to 3 m. Below these depths, one observes only annual variations, which themselves disappear at yet greater depths.

If the speed of rotation of the Earth about its axis were to become incomparably greater, and if the same were to occur for the movement of the planet about the Sun, one would no longer find diurnal and annual temperature variations of the sort described above; the points of the surface would attain and conserve the fixed deep-Earth temperature. In general, the depth to which one must go in order for the variations to be significant has a very simple relation with the length of the period with which the effects repeat at the surface. This depth is exactly proportional to the square root of the period. It is for this reason that the diurnal variations penetrate to a depth nineteen times less than that at which one can still detect annual variations.

The question of the periodic movement of solar heat was treated for the first time and solved in a separate writing which I submitted to the Institute of France in October 1809. I reproduced this solution in a piece sent at the end of 1811, which was printed in our Collected Works.

The same theory provides the means of measuring the total quantity of heat which, in the course of a year, determines the alternation of the seasons. Our goal in choosing this example of the application of the formulae is to show that there exists a necessary relation between the law of periodic variations and the total quantity of heat transfer which accompanies this oscillation; once this law is known from observations of one given climate, one can deduce the quantity of heat which is introduced into the Earth and which later returns to the air.

By considering a law similar to that which holds in the interior of the globe, one finds the following results. One eighth of a year before the temperature of the surface rises to its mean value, the Earth begins to accumulate heat; the rays of the Sun penetrate the Earth for six months. Then, the movement of the Earth's heat reverses direction; it exits and expands through the air and outer space: the quantity of heat exchanged in these oscillations over the course of a year is expressed by the calculation. If the terrestrial envelope were formed of a metallic substance, such as wrought iron (a substance which I chose as an example because its thermal coefficients have been measured), the heat which produces the alternation of the seasons would, for the climate of Paris and for each square meter of surface, be equivalent to that required to melt a cylindrical column of ice with a base of one square meter and a height of about 3 m.[8] Though the value of the thermal coefficients specific to the material of which the globe is formed have not been measured, one sees easily that they would give a result much less that that which I have just indicated. The result is proportional to the square root of the product of the heat capacity per unit volume and the thermal conductivity.

Let us now consider the second cause of the terrestrial heat, which resides, according to us, in interplanetary space. The temperature of this space is that which a thermometer would show if the Sun and all planetary bodies which accompany it were to cease to exist, assuming the instrument to be placed anywhere in the region of the heavens presently occupied by the solar system.

We shall now indicate the principle facts which have led us to recognize the existence of this characteristic temperature of interplanetary space, independent of the presence of the Sun, and independent of the primitive heat that the globe has been able to retain. To obtain knowledge of this remarkable phenomenon, one must consider what the temperature of the Earth would be if it received only the heat of the Sun; further, to render the problem more tractable, one can at first neglect the effect of the atmosphere. Then, if there were no agency maintaining a common and constant temperature in interplanetary space, that is to say if the Earth and all bodies forming the solar system were located in a region deprived of all heat, one would observe effects completely contrary to those which which we are familiar. The polar regions would be subject to intense cold, and the decrease of temperature from equator to pole would be incomparably more rapid and more extreme than is observed.[9]

Under this hypothesis of absolutely cold space, if such a thing is possible to conceive of, all effects of heat, such as we observe at the surface of the globe, would be due solely to the presence of the Sun. The least variation of the distance of the Earth from this star would lead to considerable changes in the temperature, and the eccentricity of the Earth's orbit would give rise to new forms of seasonal variations.

The alternation of day and night would produce effects both sudden and totally different from those we observe. The surface of bodies would be exposed all of a sudden, at the beginning of night, to an infinitely intense cold. The living world, both animal and vegetable, could not survive such a rapid and strong action, which repeats in the opposite sense at sunrise.

The primitive heat conserved in the interior of the terrestrial mass cannot supplant the exterior temperature of space, and would not prevent any of the effects we have just described; for we know with certainty, by theory and observations, that this central heat has long ago become insensible at the surface, notwithstanding that it can be very great at a moderate depth.

From these various remarks we conclude, and principally from the mathematical examination of the question, that there must be a physical cause which is always present, which moderates the temperatures of the surface of the globe, and which gives this planet a fundamental heat independent of the action of the Sun and of the primitive heat retained in the interior of the planet. This fixed temperature which the Earth receives from space differs little from that which one measures at the Earth's poles. Of necessity, the temperature of space is below the temperature characterizing the coldest lands; however, in making this comparison, one must admit only selected observations, and not consider episodes of extremely intense cold caused by accidental effects such as evaporation, violent winds or extraordinary expansion of the air.

Having recognized the existence of this fundamental temperature of space without which the observed pattern of temperature at the Earth's surface would be inexplicable, we note that the origin of this phenomenon is obvious. It is due to the radiation of all the bodies of the universe, whose light and heat can reach us. The stars which we can see with our own eyes, the countless multitude of stars visible by telescope, or the dark bodies which fill the universe, the atmospheres which surround these immense bodies, the tenuous material strewn through various parts of space, act together to form these rays which penetrate all parts of the planetary regions. One cannot conceive of the existence of such an assemblage of luminous or heated bodies, without admitting also that any given point of the space containing it must acquire a definite temperature.[10]

The immense number of bodies compensates for the inequality of their temperatures, and renders the radiation essentially uniform.

This temperature of space is not the same in all parts of the universe; however, it does not vary much over the dimensions of a planetary system, since this size is incomparably smaller than the distance separating the system from the radiant bodies. Thus, the Earth finds the same temperature of the heavens at all parts of its orbit.

The same applies to the other planets of our system; they all participate equally in the communal temperature, which is more or less augmented by the incidence of the rays of the Sun, according to the distance of the planet from this star. As for the problem of assigning the temperature that each planet is expected to attain, the principles which furnish an exact theory are as follows. The intensity and distribution of heat at the surface of these bodies depends on the distance from the Sun, the inclination of the axis of rotation, and the state of the surface. The temperature is very different, even in the mean, from that which an isolated thermometer placed at the location of the planet would measure, for the solidity of the planet, its great size and doubtless also the presence of the atmosphere and the nature of the surface act together to determine the mean temperature.

The original heat conserved in the interior of the mass has long ago stopped having any noticable effect at the surface; the present state of the terrestrial envelope allows us to know with certainty that the primitive heat of the surface has almost entirely dissipated. We regard it as very likely, given the construction of our solar system, that the polar temperatures of each planet, or at least most of them, is little different from that of space. This polar temperature is essentially the same for all bodies, despite the fact that their distances from the Sun differ greatly.

One can determine with reasonable precision the amount of heat which the Earth would acquire if it were substituted for each of the planets; however, the temperature of the planet itself cannot be assigned, because one would need to know the state of its surface and of its atmosphere. This difficulty no longer applies for the bodies situated at the extremities of the solar system, such as the planet discovered by Herschel. The exposure of this planet to sunlight is insignificant. Its surface temperature is therefore little different from that of interplanetary space. We have stated this result in a public discourse delivered recently in the presence of the Academy. One sees that this result applies only to the most distant planets. We do not know any means of assigning the mean temperature of the other planets with any precision.

The movements of the air and the waters, the extent of the oceans, the elevation and form of the surface, the effects of human industry and all the accidental changes of the Earth's surface modify the temperature of each climate. The basic character of phenomena arising from fundamental causes survives, but the thermal effects observed at the surface are different from those which would be seen without the influence of these accessory causes.

The mobility of water and air tends to moderate the effects of heat and cold, and renders the temperature distribution more uniform; however it would be impossible for the action of the atmosphere to supplant the universal cause arising from the communal temperature of interplanetary space; if this cause did not exist, one would observe, despite the action of the atmosphere and the oceans, enormous differences between the polar and equatorial temperature.

It is difficult to know just to what extent the atmosphere affects the mean temperature of the globe, and here the guidance of rigorous mathematical theory ceases. One is indebted to the celebrated explorer M. de Saussure[11] for an experiment which appears to be well suited to clarifying this question. The experiment consists of exposing a vessel covered by one or more sheets of highly transparent glass (placed at some distance from each other) to the rays of the Sun. The interior of the vessel is covered with an thick layer of blackened cork, suited to absorb and retain the heat. The heated air is contained in all parts of the apparatus, either in the interior of the box or in each gap between two plates of glass. Thermometers placed in this vessel and in the spaces between the plates register the degree of heat acquired in these cavities. This instrument was exposed to the Sun at or near noontime, and it has been observed, in various experiments, that the thermometer in the vessel raises to 70°, 80°, 100°, 110° or even higher (octogesimal[12] division). Thermometers placed within the gaps between the sheets of glass indicate a much lower degree of heat acquired, decreasing steadily from the bottom of the box up to the top gap.

The effect of solar heat on air contained within a transparent enclosure has been known for a long time. The apparatus which we have just described is designed for the purpose of maximizing the acquired heat, and above all with the purpose of comparing the solar effect on a very high mountain with that on the plain below. This observation is remarkable by virtue of the accurate and extensive conclusions the inventor of the apparatus has drawn: he has repeated the experiments several times at Paris and at Edinburgh, and found consistent results.

The theory of this instrument is easy to formulate. It suffices to remark that: (1) the heat acquired is concentrated, because it is not dissipated immediately by exchange of air with the surroundings; (2) the heat emanated by the Sun has properties different from those of dark heat. The rays of this star are for the most part transmitted through the glass without attenuation and reach the bottom of the box. They heat the air and the surfaces which contain it: the heat communicated in this way ceases to be luminous, and takes on the properties of dark radiant heat. In this state, the heat cannot freely traverse the layers of glass which cover the vessel; it accumulates more and more in the cavity enclosed by materials which conduct heat poorly, and the temperature rises to the point at which the incident heat is exactly balanced by the dissipated heat. One could verify this explanation, and render the consequences more evident, if one were to vary the conditions of the experiment by employing colored or darkened glass, and by making the cavities containing the thermometers empty of air. When one examines this effect by quantitative calculations, one finds results which conform entirely to those which the observations have yielded[13] It is necessary to consider this range of observations and the results of the calculations very carefully if one is to understand the the influence of the atmosphere and the waters on the thermometric state of the Earth.

In effect, if all the layers of air of which the atmosphere is formed were to retain their density and transparency, but lose only the mobility which they in fact possess, this mass of air would become solid, and being exposed to the rays of the Sun, would produce an effect of the same type as that which we have just described. The heat, arriving in the form of light as far as the solid surface of the Earth, suddenly and almost entirely loses its ability to pass through transparent solids; it will accumulate in the lower layers of the atmosphere, which will therefore acquire elevated temperatures. One will observe at the same time a diminution of the degree of heat acquired as one moves away from the surface of the Earth.[14] The mobility of the air, which is displaced rapidly in all directions and which rises when it is heated, and the irradiation by dark heat in the air diminishes the intensity of the effects which would take place in a transparent and solid atmosphere, but it does not completely eliminate these effects. The reduction of heat in elevated regions of the air does not fail to take place; it is thus that the temperature is augmented by the interposition of the atmosphere, because the heat has less trouble penetrating the air when it is in the form of light, than it has exiting back through the air after it has been converted to dark heat.

We will now consider the heat of the Earth itself, which it possessed at epochs when the planets were formed, and which continues to dissipate at the surface, under the influence of the low temperature of interplanetary space.

The notion of an interior fire, as a perpetual cause of several grand phenomena, has recurred in all the ages of Philosophy. The goal which I have set myself is to know exactly the laws by which a solid sphere, heated by long immersion in a medium, loses its primitive heat once it is transported to a space with constant temperature lower than that of the first medium. This difficult question, not treatable by mathematical techniques formerly known, was resolved by a new method of calculation which is also applicable to a variety of other phenomena.

The form of the terrestrial sphere, the regular disposition of interior layers made manifest by experiments with pendula, their growing density with depth, and various other considerations concur to prove that a very intense heat once penetrated all parts of the globe. This heat dissipates by radiation into the surrounding space, whose temperature is much below the freezing point of water. Now, the mathematical expression of the law of cooling shows that the primitive heat contained in a spherical mass of dimension as big as the Earth diminishes much more rapidly at the surface than at parts situated at great depth. The latter retain almost all of their heat for an immense time; there is no doubt about the truth of the conclusions, because I have calculated this time for metallic substances having much greater thermal conductivity than the materials making up the globe.

However, it is obvious that the theory alone can teach us only about the laws to which the phenomena are subject. It remains to examine if, in the layers of the globe we are able to penetrate, one finds some evidence of this central heat. One must verify, for example, that, below the surface, at distances where diurnal and annual variations cease entirely, temperatures increase with depth along a vertical extended into the interior of the solid earth: Now all the facts which have been gathered and discussed by the most experienced observers have taught us the magnitude of this increase: it has been estimated at 1° for each 30 m to 40 m of depth.

The object of the mathematical question is to discover the definite conclusions which one can deduce from this fact alone, considering it as given by direct observation, and to prove that it determines: (1) the location of the source of heat; (2) the temperature excess remaining at the surface.

It is easy to conclude, as a result of exact analysis, that the increase of temperature with depth cannot be produced by prolonged action of the rays of the Sun. The heat emanating from this star does accumulate in the interior of the globe, but the accumulation has long since ceased; further, if the accumulation were still continuing, one would observe a temperature increase in precisely the opposite sense as that which is observed.

The cause which gives greater temperature to layers located at greater depth is therefore a constant or variable interior source of heat, placed somewhere below the points of the globe which it has been possible to penetrate. This cause raises the temperature of the Earth's surface above the value that it would have under the action of the Sun alone. However, this excess of surface temperature has become almost imperceptible; we can be assured of this because there exists a mathematical relation between the value of temperature increase per meter and the amount by which the surface temperature still exceeds the value it would have if there were no interior heat source. For us, measuring the rate of increase of temperature with depth is the same thing as measuring the temperature excess of the surface.

For a globe made of iron, a rate of increase of a thirtieth of a degree per meter would yield only a quarter of a centessimal degree of excess surface temperature at the present. This elevation is in direct ratio to the conductivity of the material of which the envelope is formed, all other things being equal. Thus, the surface temperature excess of the actual Earth caused by the interior heat source is very small; it is certainly less than a thirtieth of a centessimal degree. It should be noted that this last conclusion applies regardless of the supposition which one may make about the nature of the internal heat source, whether it be regarded as local or universal, constant or variable.

When one carefully examines all the observations relating to the shape of the Earth, according to the principles of dynamical theory, one cannot doubt that the planet received a very high temperature at its origin; further, the present distribution of heat in the terrestrial envelope is that which would be observed if the globe had been formed in a medium of a very high temperature, whereafter the globe cooled continually.

The question of terrestrial temperatures has always appeared to me to be one of the greatest objects of cosmological study, and I have had this subject principally in view in establishing the mathematical theory of heat. I first determined the time-varying state of of a solid globe which, after having been kept for a long time in a heated medium, has been transported to a cold space. I also considered the time-varying state of a sphere which, having been plunged successively in two or more media of varying temperature, is subjected to a final cooling in a space having constant temperature. After having remarked on the general consequences of the solution to this problem, I examined more specifically the case where the primitive temperature acquired in the heated medium became constant throughout the mass; further, supposing the sphere to be extremely large, I investigated the progressive diminution of temperature in layers sufficiently close to the surface. If one applies the results of this analysis to the terrestrial globe in order to know what would be the successive effects of an initial formation similar to that which we have just considered, one sees that the increase of a thirtieth of a degree per meter, considered as the result of interior heat, was once much greater. One sees further that this temperature gradient now varies extremely slowly. As for the temperature excess of the surface, it varies according to the same law; the secular diminution or the quantity by which it reduces in the course of a century is equal to the present value divided by twice the number of centuries that have flown by since the beginning of the cooling. The age of historical monuments provides us with a lower limit to this number, whence one concludes that from the time of the Greek school of Alexandria up to our time, the surface temperature has not diminished (by this cause) by three hundredths of a degree. Here one again encounters the stable character presented by all great phenomena of the universe. This stability is, by the way, a necessary result independent of the initial state of the mass, because the present temperature excess is extremely small and can do nothing else but continue to diminish for an indefinitely prolonged time.

The effect of the primitive heat which the globe has retained has therefore become essentially imperceptible at the Earth's surface; however it is still manifest at accessible depths, because the temperature of lower layers increases with distance from the surface. This increase, measured in fixed units, would not have the same value at much greater depths: it diminishes with depth; however the same theory shows us that the temperature excess, which is nearly zero at the surface, can be enormous at a distance of several tens of kilometers; it follows that the heat of intermediate depth layers could far surpass the that of incandescent matter.

The passage of centuries will bring great changes in these interior temperatures; at the surface, however, these changes are essentially done, and the continual loss of primitive heat cannot result in any cooling of the climate.

It is important to observe that the accessory causes can cause temperature variations at any given place which are incomparably more significant than those arising from the secular cooling of the globe.

The establishment and progress of human societies, and the action of natural forces, can notably change the state of the ground surface over vast regions, as well as the distribution of waters and the great movements of the air. Such effects have the ability to make the mean degree of heat vary over the course of several centuries, for the analytic expressions contain coefficients which depend on the state of the surface, and which greatly influence the temperature.

Though the effect of the interior heat is no longer perceptible at the surface of the Earth, the total quantity of this heat which dissipates in a given amount of time, such as a year or a century, is measurable, and we have determined it; that which traverses one square meter of surface during a century and expands into celestial space could melt a column of ice having this square meter as its base, and a height of about 3 m.[15]

This conclusion derives from a fundamental proposition which belongs to all questions regarding the movement of heat, and which applies above all to those of the terrestrial temperature: I speak of the differential equation which expresses for each instant the state of the surface. This equation, whose truth is palpable and easy to demonstrate, establishes a simple relation between the temperature of an element of the surface and the movement of heat in the direction of the normal to the surface. What renders this theoretical result very important, and more suitable than any other to clarify the questions which are the subject of this Article, is that the relation applies independent of the form and the dimensions of the body, and regardless of the nature of the substances – homogeneous or diverse – of which the interior mass is composed. Hence, the consequences which one deduces from this equation are absolute; they hold equally, whatever might have been the material constitution or original state of the globe.

We have published, in the course of the year 1820, a summary of an Article on the secular cooling of the terrestrial globe (*Bulletin des Sciences, Societé philomathique*, 1820, p. 48 ff). One has reported there the principal formulae, and notably those which express the time-varying state of an extremely large solid body uniformly heated up to a given depth. If the initial temperature, instead of being the same up to a very great distance from the surface, results from a successive immersion in several media with different temperatures, the consequences are neither less simple nor less remarkable. When all is said and done, this case and several others which we have considered are included as special cases of the general expressions which have been indicated.

The reading of this extract gives me the occasion to note that formulae (1) and (2) reported there have not been correctly transcribed. I will make up for this omission afterwards. In any case, the error affects neither the other formulae nor the conclusions contained in the extract.

In order to describe the principal thermometric effects which arise from the presence of the oceans, let us suppose for the moment that the water of the Ocean is drained from the basins which contain it, leaving behind immense cavities in the solid Earth. If this state of the Earth's surface, deprived of the atmosphere and the waters, were to persist for a great many centuries, the solar heat would produce alternations of temperature similar to those which we observe on the continents, and subject to the same laws. The diurnal or annual variations cease at certain depths, and a temporally invariable state would form in the interior layers which continually transports equatorial heat toward the polar regions.

At the same time, as the original heat of the globe dissipates through the exterior surface of the basins, one would observe there, as in all other parts of the surface, an increase of temperature with depth along a line normal to the surface of the bottom.

It is necessary to remark here that the increase of temperature due to the original heat depends principally on the normal distance from the surface. If the exterior surface were horizontal, one would find equal temperatures along horizontal lower layers; however if the surface of the solid Earth is convex, these layers of equal temperature would not be at all horizontal, and they would differ from level surfaces. They follow the sinuous form of the surface: it is for this reason that, in the interior of mountains, the central heat can penetrate up to a great height. This is a complex effect, which one can determine by mathematical analysis keeping in mind the form and the absolute elevation of the masses.

If the surface were concave, one would observe an analogous effect in the opposite sense, and this case applies to the hypothetical water-free oceans which we are considering. The layers of equal temperature would be concave, and this state would be found if the Earth were not covered by waters.

Let us suppose now that, after this same state has lasted a great many centuries, one re-establishes the waters at the bottom of the oceans and lakes, and that they remain exposed to the alternation of the seasons. When the temperature of the upper layers of the liquid becomes less than that of the lower parts, though surpassing the freezing point of water by only a few degrees, the density of these upper layers increases; they will descend more and more, and come to occupy the bottom of the basins which they will then cool by their contact: at the same time, the warmer and lighter waters rise to replace the upper waters, whence infinitely varied movements are established in the liquid masses, whose general effect will be to transport heat toward upper regions.

These phenomena are more complex in the interior of the great oceans, because the inhomogeneity of temperature there occasions currents in the opposite sense and thus displaces the waters of far-removed regions.

The continual action of these causes is modified by another property of water, that which limits the growth of density and causes it to reverse when the temperature falls to near the freezing point. The solid bottom of the oceans is therefore subject to a special action which sustains itself forever, and which has perpetually cooled the bottom since time immemorial, by the contact with a liquid having a temperature exceeding by only a few degrees that of melting ice. One finds in consequence that the temperature of the waters decreases with depth; this deep temperature is on the order of 4° at the bottom of most lakes in our climate. In general, if one observes the temperature of the ocean at ever greater depths, one approaches this limit which corresponds to the greatest density; however one must, in questions of this type, keep in mind the nature of the waters, and above all the communication established by the currents: this last cause can totally change the results.

The increase of temperature, which we observe in Europe when carrying a thermometer into the interior of the solid globe at great depths, therefore cannot survive in the interior of the oceans, and more generally the order of temperature variations must be the reverse.

As for the portions located immediately below the bottom of the oceans, the law of increase of heat is not that which applies in continental lands. These temperatures are determined by a peculiar cooling action, the vessel being exposed, as we have said, to perpetual contact with a liquid which retains the same temperature at all times. It is to clarify this part of the problem of terrestrial temperatures that I determined, in the *Analytic Theory of Heat* (Chapter IX, p 427 ff), the expression for the time-variable state of a solid primitively heated in some manner, and for which the surface is kept during an indefinite time at a constant temperature. The analysis of this problem allows us to know precisely the law by which the exterior influence causes the temperature of the solid to vary. In general, after having established the fundamental equations of movement of heat, and the method of calculation which serves to integrate them, I turned to the solution of the questions pertinent to the study of terrestrial temperatures, and made known the relations of this study to the systematic behavior of the world.

After having explained separately the principles governing terrestrial temperatures, one must bring together all the effects we have just described into a general point of view, and from there form a correct idea of the operation of the full range of phenomena.

The Earth receives the rays of the Sun, which penetrate its mass and are converted there into dark heat; the Earth also possesses heat of its own which it retains from its origin, and which dissipates continually at the surface; finally this planet receives rays of light and heat from the countless stars among which the solar system is located. These are the three general causes which determine terrestrial temperatures. The third, that is to say the influence of the stars, is equivalent to the presence of an immense region closed in all parts, whose constant temperature is little inferior to that which we observe in polar lands.

One could without doubt suppose that radiant heat has properties as yet unknown, which could take the place in some way of this fundamental temperature which we attribute to space; however, in the present state of physical science, and without recourse to properties other than those which derive from observations, all the known facts can be explained naturally. It suffices to posit that the planetary bodies are in a space whose temperature is constant. We have therefore investigated the question of what this temperature must be in order for the thermometric effects to be similar to what we observe; now, the predicted effects differ entirely from observations if one supposes that space is absolutely cold; however, if one progressively increases the common temperature of the environment which encloses this space, the results come to approach the observations. One can affirm that the present phenomena are those which would be produced if the irradiation by the stars gives each point of planetary space a temperature of about 40° below zero (octogesimal division).

The primitive interior heat which is still not at all completely dissipated produces only a very small effect at the surface of the Earth; the primitive heat is more evidently manifest by the augmentation of temperature in deep layers of the Earth. At the greatest distances from the surface, the temperature can surpass the highest temperatures ever measured to date.

The effect of the solar rays is periodic in the upper layers of the terrestrial envelope; it is fixed in all the deeper places. This fixed temperature of the lower portions is not the same for all of them; it depends principally on the latitude of the place.

The solar heat accumulates in the interior of the globe, whose state becomes time-independent. That which penetrates in equatorial regins is exactly compensated by the heat which flows out through the polar regions. Thus the Earth returns to celestial space all the heat which it receives from the Sun, and adds to it a part which derives from its own primitive heat.

All the terrestrial effects of the Sun's heat are modified by the interposition of the atmosphere and by the presence of the waters. The grand movements of these fluids renders the temperature distribution more uniform.

The transparency of the waters and that of the air act together to augment the degree of heat acquired, because incident luminous heat penetrates easily to the interior of the mass, but the dark heat exits with more difficulty when following the contrary route.

The alternations of the seasons are accompanies by an immense quantity of solar heat which oscillates in the terrestrial envelope, passing under the surface for six months, and returning from the Earth to the air during the other half of the year. Nothing can shed better light on this question than the experiments which have as their object the precise measurement of the effect produced by the rays of the Sun on the terrestrial surface.

I have summarized, in this Article, all the principle elements of the analysis of the problem of terrestrial temperatures. It is made up of several results of my research, which have been published long ago. When I first endeavored to treat this type of question, there was no mathematical theory of heat, and one could even doubt such a theory to be possible. The Articles and Works which I have set forth contain the exact solution of fundamental questions; they have been submitted and communicated publicly, or printed and analyzed in scientific collections over the past several years.

In the present writing, I have set myself another goal, that of calling attention to one of the greatest objects of Natural Philosophy, and to set forth an overview of the general conclusions. I have hoped that the geometers will not see these researches only as a question of calculation, but that they will consider also the importance of the subject. One cannot at present resolve all the uncertainties in such a vast subject, which embraces, besides the results of a novel and difficult mathematical analysis, exceedingly varied physical concepts. For the future, it remains to take many more precise observations; one will also study the movement of heat in liquids and air. Possibly, additional properties of radiant heat will be discovered, as well as further processes which can modify the temperature distribution of the globe. However, all the principle laws governing the movement of heat are already known; this theory, which rests on invariable foundations, forms a new branch of mathematical Science: it consists at present of the differential equations for the movement of heat in solids and liquids, solutions of these first equations, and theorems relating to the equilibrium properties of radiant heat.

One of the principle features of the analysis which expresses the distribution of heat in solid bodies is the ability to superpose simple solutions in order to build the solution of more complex problems. This property derives from the nature of the differential equations for the movement of heat, and applies also the the problem of the long-term oscillation of bodies; however, the superposition property belongs more particularly to the theory of heat, since the most complex effects can truly be resolved into simple movements. This proposition does not express a law of nature, and I do not mean to imply anything of this sort; it expresses an enduring property, and not a cause. One one would find the same result in dynamical questions wherein one considers resistive forces which cause a rapid cessation of the effect produced.

The applications of the theory of heat have demanded prolonged analytical research, and it was first necessary to formulate the method of calculation, regarding as constant the specific coefficients which enter into the equations; for, this condition establishes itself spontaneously, and endures for an infinite time once the differences in temperature become sufficiently small, as one observes in the problem of terrestrial temperatures. Moreover, in this question (which is the most important application of the theory of heat), the demonstration of the principle results is independent of the homogeneity and the nature of the interior layers of the Earth.

The analytic theory of heat can be extended as required to treat the most varied applications. The list of principles which serve to generalize the theory is as follows:

- Suppose that the coefficients are subject to very small variations, which have been fixed by observation. One can then determine, by the method of successive substitutions, the corrections which go beyond the results of the the first calculation.
- We have demonstrated several general theorems which are not at all dependent on the form of the body, or on its homogeneity. The general equation relating to area is a proposition of this type. One finds another very remarkable example if one compares the movement of heat in similar bodies, whatever may be the nature of these bodies.
- While the complete solution of these differential equations depends on expressions which are difficult to discover, or on tables which have not yet been created, one can nonetheless determine the limits between which the unknown quantities are necessarily bounded. One arrives thus at definite conclusions regarding the object in question.
- In the research on the temperature distribution of the Earth, the large size of the planet allows one to adopt a simplified form of the equations, and allows for much easier interpretation. Though the nature of the interior masses and their thermal properties are unknown, one can deduce solely from observations made at accessible depths conclusions of the greatest importance regarding the stability of climate, the present excess surface temperature due to the primitive heat, and the secular variation of temperature growth with depth. It is in this fashion that we have been able to demonstrate that this increase, which is on the order of 1° per 32 m in diverse European locations, once had a much larger value. At present its rate of diminution is so slow as to be imperceptible, and it will take more than thirty thousand years before the temperature gradient is reduced to half its present value. This conclusion is not at all uncertain, despite the lack of knowledge of the interior state of the globe, for the interior masses, whatever their state and temperature may be, communicate only an insignificant quantity of heat to the surface over immense stretches of time. For example, I wished to know what would be the effect of an extremely heated mass of the same size of the Earth, placed some leagues below the surface. Here is the result of this inquiry.

> If, below a depth of 12 leagues, one were to replace the terrestrial mass down to the center of the globe by a matter whose temperature is five hundred times that of boiling water, the heat communicated by this mass to the neighborhood of the surface would remain imperceptible for a very long time; certainly more than two hundred thousand years would pass before one could observe a single degree of temperature

> increase at the surface. Heat penetrates solid masses – and especially those of which the terrestrial envelope is formed – so slowly that a separation of only a very few leagues suffices to render it inappreciable during twenty centuries application of the most intense heat.

A careful examination of the conditions to which the planetary system are subject leads to the conclusion that these bodies were made from the mass of the Sun, and it can be said that there is no observed phenomenon which fails to buttress this opinion. We do not know how the interior of the Earth has lost this original heat; one can only affirm that at the surface the excess of heat due to this cause has become essentially undetectable; the thermometric state of the globe no longer varies but with extreme lassitude; and, if one were to imagine that the portion a few leagues below the surface were replaced by either ice or the very substance of the Sun having the same temperature of that star, a great number of centuries would flow by before one observed any appreciable change in the surface temperature. The mathematical theory of heat furnishes several other consequences of this type, whose certainty is independent of all hypotheses regarding the state of the interior of the terrestrial globe.

These theories have an extensive and fertile future ahead of them, and nothing will contribute more to their perfection than a numerous set of precise experiments; for, mathematical analysis (if we may be permitted to reiterate this reflection here)[16] can deduce general phenomena and lend simple form to the expression of the laws of nature; however, the application of these laws to very complex effects demands a long series of exact observations.

NOTES

1 Here, Fourier is evidently referring to the fact that temperature decreases with depth in the ocean whereas it increases with depth in the solid crust. The latter is explained easily by Fourier's diffusion equation, whereas the former requires a quite different explanation. *RTP*

2 Here Fourier is attempting to explain the fact that the atmospheric temperature decreases with height. He seeks to explain this by the effect of density on solar absorption, whereas the correct explanation involves the joint action of convection in lifting air parcels with the cooling resulting from expansion of the parcels. Nonetheless, the rest of the paragraph makes clear that Fourier understands that the atmosphere is mostly transparent to solar radiation. *RTP*

3 Fourier seems to imply that the ocean has a greenhouse effect similar to that of the atmosphere. This is a puzzling, since Fourier knows that the ocean gets colder with depth rather than warmer. It is true that water is more transparent to visible light than it is to infrared, and therefore would seem to have the properties necessary to produce a greenhouse effect. The main reason that the Ocean has no greenhouse effect is that the sunlight is absorbed mostly in the top 100 m, and that a well mixed state of water is isothermal, rather than having a temperature decrease with height as is the case for a compressible substance like air. The ocean in fact causes an *anti-greenhouse* effect, in that the temperature of the bottom of the ocean is lower than what it would be if the water were removed. *RTP*

4 This paragraph refers to the warming of the surface by downwelling infrared radiation coming from the atmosphere. Fourier's many articles on infrared radiation make reference to observations documenting the presence of this radiation. *RTP*

5 Fourier here refers to the linearity of the equations of heat diffusion. He is evidently unaware that other parts of the physics to which he refers (notably the intensity of infrared radiation, as described by the yet-to-be-discovered Stefan Boltzman law) are not linear. *RTP*

6 i.e. independent of time of day or time of year. *RTP*

7 see p 3–28 of the *Oeuvres*, vol. 2. *RTP*

8 Equivalent to a mean flux of about 60 W/m^2 into the surface for one half of the year, followed by the same amount out of the surface for the other half. This is considerably in excess of most estimates of the surface energy imbalance over land, probably because Fourier used the conductivity of iron in his estimate. *RTP*

9 It is strange that Fourier neglects the effect of thermal inertia and atmosphere-ocean heat transports, which easily account for the moderation of polar and night-time cooling. Fourier mentions these effects further along, but dismisses them without having any quantitative reason for doing so. *RTP*

10 This argument is qualitatively right, but quantitatively wrong. The actual "temperature of space," which may be identified with the microwave background radiation, is more like 4 degrees Kelvin than 200 degrees Kelvin, as Fourier supposed. *RTP*

11 Horace Bénédict de Saussure, 1740–1799, a scientist and mountaineer who was primarily interested in the factors governing weather and climate on mountains. He is widely regarded as the first mountain meteorologist, and is known also as the grandfather of the celebrated linguist Ferdinand de Saussure.

12 The octogesimal temperature scale, also known as the Reaumur scale, divides the temperature range between the freezing and boiling points of water into 80 equally spaced degrees. A comparison with de Saussure's data suggests that Fourier may have actually converted the values to centigrade here, but erroneously continued to refer to them as octogesimal. *RTP*

13 Fourier refers to the existence such calculations, but I have not located them anywhere in his published works. In his discussion of variations on de Saussure's experiment, Fourier is probably describing his

expectation of what the results of such experiments would be rather than referring to experiments which have actually been carried out and reported. This is underscored by his use of the conditional tense in the original. In any event de Saussure could not have performed experiments with an evacuated box, given the technology available to him. On the other hand, many other investigators did reproduce de Saussure's results, so it is not out of the question that Fourier had actual knowledge of some results from experiments such as he describes. *RTP*

14 This reasoning is partially correct for an atmosphere which does not move, but fails to capture the true reason that atmospheric temperature decreases with height. In fact, buoyancy driven motion greatly enhances the vertical decay of temperature, through the cooling of lifted air parcels as they expand. It is clear that Fourier understood that air cools as it expands (see his remark about episodic bouts of intense cold), but he doesn't seem to have connected this effect with the general decrease of atmospheric temperature with height. He also fails to identify the important role this temperature decrease plays in limiting radiation of infrared to space, via reducing the temperature of the "radiating surface." *RTP*

15 Equivalent to 318 mW/m^2, which is 3–4 times modern estimates of geothermal heat flux. As Fourier implies, the overestimate arises from using the conductivity of iron.

16 Discours Préliminaire of *Théorie Analytique de la Chaleur*