EDITED BY
DAVID ARCHER AND
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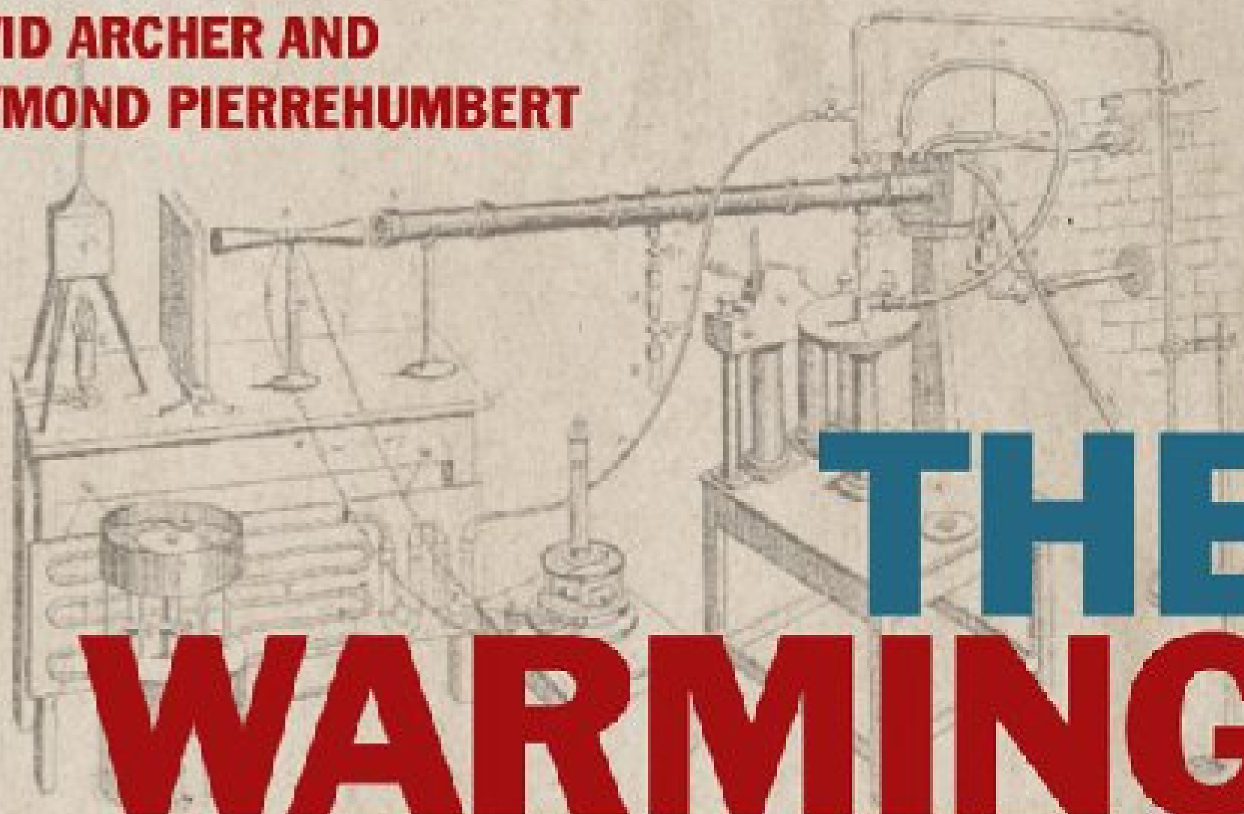
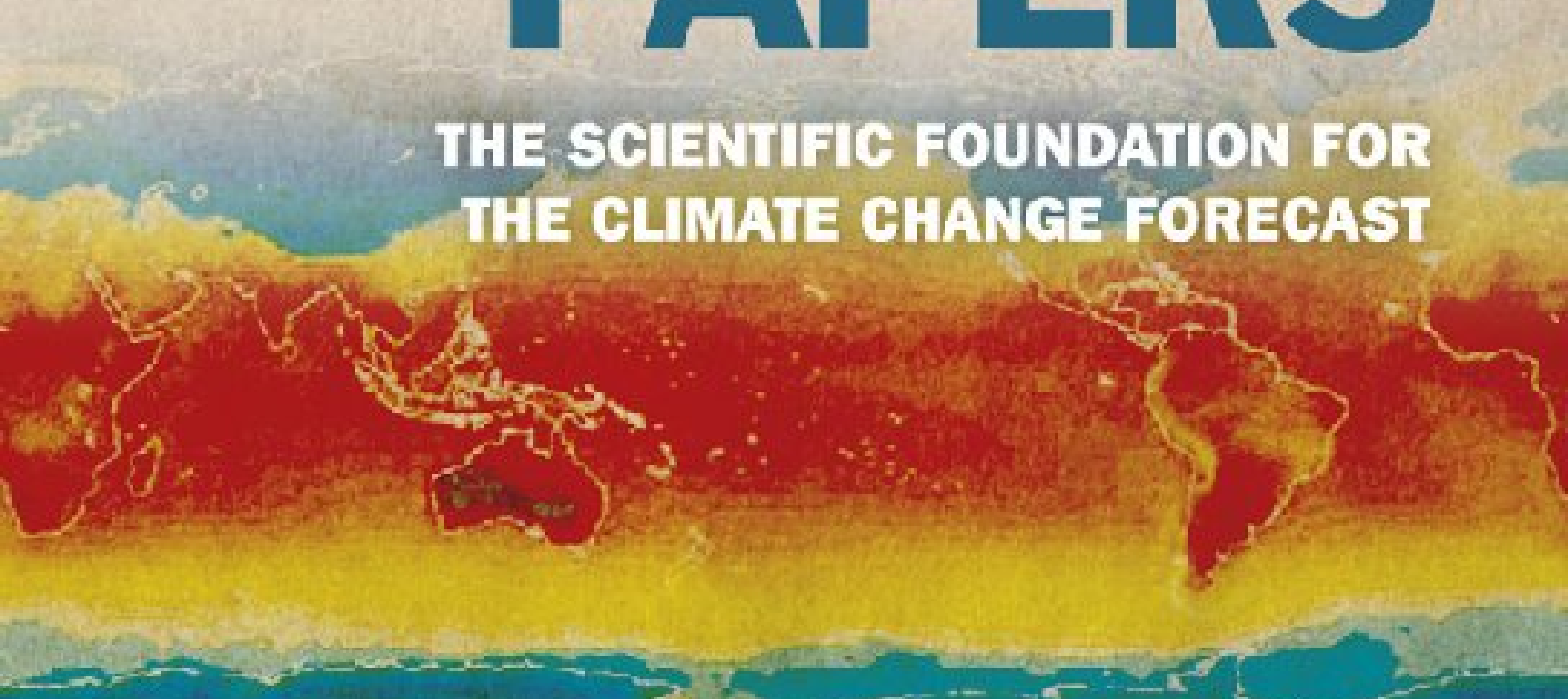


THE WARMING PAPERS

THE SCIENTIFIC FOUNDATION FOR THE CLIMATE CHANGE FORECAST



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The Warming Papers

The Scientific Foundation
for the Climate Change Forecast

Edited by
David Archer
and
Raymond Pierrehumbert

WILEY-BLACKWELL
A John Wiley & Sons, Ltd., Publication

This edition first published 2011 © 2011 by Blackwell Publishing Ltd

Blackwell Publishing was acquired by John Wiley & Sons in February 2007. Blackwell's publishing program has been merged with Wiley's global Scientific, Technical and Medical business to form Wiley-Blackwell.

Registered Office

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, UK

Editorial Office

9600 Garsington Road, Oxford, OX4 2DQ, UK

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Library of Congress Cataloging-in-Publication Data

The warming papers : the scientific foundation for the climate change forecast / edited by David Archer and Raymond Pierrehumbert.

p. cm.

Includes bibliographical references and index.

ISBN 978-1-4051-9616-1 (pbk.) – ISBN 978-1-4051-9617-8 (hardcover)

1. Greenhouse effect, Atmospheric. 2. Greenhouse gases. 3. Global temperature changes. I. Archer, David, 1960–

II. Pierrehumbert, Raymond T.

QC912.3.W37 2011

551.5–dc22

2010040516

A catalogue record for this book is available from the British Library.

1 2011

Preface

Global warming is arguably the defining scientific issue of modern times, but it is not widely appreciated that the foundations of our understanding are almost two centuries old. The sensitivity of climate to changes in atmospheric CO_2 was first estimated about one century ago, and the rise in atmospheric CO_2 concentration was discovered half a century ago. The fundamentals of the science underlying the forecast for human-induced climate change were being published and debated long before it started to appear in the newspapers.

The aim of this book is to gather together the classic scientific papers that are the scientific foundation for the forecast of global warming and its consequences. These are not necessarily the latest in the state of play; there can be subsequent quantitative revision. But these papers are the big ideas. Some of the good old good ones can be heavy going, it must be admitted, so we will try to guide the reader with some verbage of our own, unworthy though it may be. We summarize the results for you, and provide the latest revisions from the ongoing literature, how strong the water vapor feedback turned out to be, for example. We will fill in the context, the personalities, and the aftermath of the ideas in the papers. We'll also presume to provide short comments where they occur to us in boxes throughout the papers, signposts to help guide the casual reader.

Part I

Climate Physics

1

The Greenhouse Effect

Fourier, J. (1827). Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires. *Mémoires de l'Académie Royale des Sciences,* **7,** 569–604. 25 pages.

Joseph Fourier (1768–1830) is generally credited with the discovery of what is now known as the greenhouse effect. In fact, his contribution to the study of planetary temperature is even more profound than that. Fourier introduced the problem of planetary temperature as a proper object of study in physics, and established a largely correct physical framework for attacking the problem. His work set the stage for most of the further developments in this area over the remainder of the nineteenth century. Indeed, it was only toward the end of that century that physics had caught up to the point that the first quantitative estimates of the Earth's temperature based on Fourier's concepts could be attempted.

If much of Fourier's reasoning in this paper seems qualitative, it should be recognized that most of the areas of physics that Fourier needs to call on were in their infancy in Fourier's day. Infrared radiation (called "dark heat" or "dark radiation" at the time) had been discovered in 1800 by the astronomer Sir Frederick William Herschel, and it was the subject of intense inquiry. Infrared was the "dark energy" of its day and it was perhaps no less mysterious to physicists of Fourier's day than is the dark energy talked about by today's physicists. There was some understanding from the work of Fourier's contemporaries, Dulong and Petit, that the rate of heat loss by infrared radiation increases with temperature, and it was known that infrared could carry heat through a near-vacuum. There was, however, only a limited ability to do quantitative calculations involving infrared heat transfer. Thermodynamics was in its infancy. The very nature of heat was still being hotly debated; the landmark energy conservation experiments of Joule that showed the equivalence of mechanical work and heat would not be carried out until 1843. Against this context, the general correctness of Fourier's great leap of intuition seems all the more remarkable.

In his 1827 paper, Fourier introduces five key concepts:

1. The temperature of the Earth, or indeed any planet, is determined by a balance between the rate at which the energy is received and the rate at which the energy is lost. There is therefore a need to determine the sources and sinks of a planet's energy.

2. There are three possible sources of heat: Sunlight, heat diffusing from the hot interior of the planet, and heat communicated from the general "temperature of space." Of these, the amount of heat leaking out of the Earth's interior is too small to play a significant role in the Earth's surface temperature.

3. Emission of infrared radiation is the only means by which a planet loses heat. Since the rate of energy loss by infrared radiation increases with the temperature of the body, the planet can come into equilibrium by heating up until the rate at which it loses energy by infrared emission equals the rate at which it gains energy from its energy sources.

4. Visible light is converted into infrared light when it is absorbed at a solid or liquid surface.

5. The atmosphere has an asymmetric effect on the incoming sunlight and the outgoing infrared, because the atmosphere is largely transparent to sunlight but is relatively opaque to infrared. This retards the rate at

which the planet loses energy, for any given temperature. The result is that the atmosphere keeps the planet warmer than it would have been if the atmosphere had been transparent to infrared radiation.

Fourier’s inferences concerning the minimal influence of the Earth’s interior heat on climate are drawn from observations of the way temperature varies with depth below the Earth’s surface. Of all Fourier’s claims in the 1827 paper, this is the one that is most backed up by quantitative reasoning, though the actual mathematical analysis appears in Fourier’s other papers and is not reproduced in the 1827 essay. Fourier’s greatest work as a mathematical physicist was the formulation of the partial differential equation describing the diffusion of heat within a body, and the development of the mathematical techniques required to solve it. The full range of these developments were engaged in Fourier’s interpretation of the Earth’s subsurface temperature variations. Indeed, Fourier states that the problem of planetary temperatures provided the main impetus for his formulation of the analytical theory of heat. His theory of heat was applied to the problem in two basic ways. First, since the rate of heat flow is proportional to the temperature gradient, the measured increase of time-mean temperature with depth itself shows that the interior of the Earth is hotter than the surface, and gives an estimate of the heat flux, provided that one can estimate the thermal conductivity of the Earth. The flux Fourier arrived at using this procedure was an overestimate compared to modern calculations because he used the thermal conductivity of iron, but his calculation nonetheless showed the diffusion of heat from the interior to be an insignificant factor in surface temperature. The second kind of problem Fourier did was to impose the observed time-periodic daily and seasonal fluctuations of temperature at the surface as a boundary condition, and then calculate what the subsurface temperature fluctuations should look like. It was this kind of calculation that led Fourier to develop what we now call Fourier series, so as to decompose the complex time-periodic boundary condition into a sum of simple sines and cosines for which the problem is analytically tractable. This calculation correctly predicts that the diurnal variation of temperature should decay rapidly with depth and the annual variation more slowly. The calculation also gives an estimate of the amount of heat that flows into and out of the surface from sunlight in the course of the diurnal and seasonal cycle, and thus provides an additional check on the importance of solar energy in determining the Earth's surface temperature.

It takes away nothing from Fourier's brilliance to point out the one stupendous blunder in his paper. Fourier thought that the heat the Earth receives from the general temperature of interplanetary space was a crucial factor in the Earth's climate, on a par with energy received from the Sun. He thought the temperature of space to be somewhat below the minimum temperatures observed in Winter in the Arctic – roughly 200 K in modern terms. He viewed this as one of his principle discoveries, and claimed that without this source of heat, the Earth would become infinitely cold at night and in the winter, and that no life would be possible. In essence, Fourier's view was that 200 K was the natural temperature that all Solar System planets would relax to if there were no absorption of sunlight. Conceptually, he was not entirely wrong, though the correct number for the "temperature of space" in this sense would be more nearly 5 K than 200 K, but Fourier's estimate of the temperature of space was based on highly dubious reasoning that did not justify his level of certainty by any means. The assumption that Arctic night temperatures represent the temperature of space neglects the role played by the long time required for the ocean to cool down ("thermal inertia") and by the ability of air and ocean currents to transport heat from warmer parts of the planet to the poles. Fourier knew about these effects, and even mentions them explicitly elsewhere in the essay. Evidently, he thought they were too ineffective to account for the observed winter and night-time temperature, though his reasons for preferring the more exotic solution of a high temperature of space remain obscure.

In any event, Fourier's misconception about the temperature of space was corrected by Claude Pouillet in 1838. Pouillet's main contribution to science was a largely correct measurement of the Solar Constant, though his estimate of the corresponding temperature of the Sun was in error because of shortcomings of then-current representations of blackbody radiation. In the course of these measurements, Pouillet found that the temperature of space was far below the value supposed by Fourier, and nothing more was heard thereafter about the role of the temperature in space in climate.

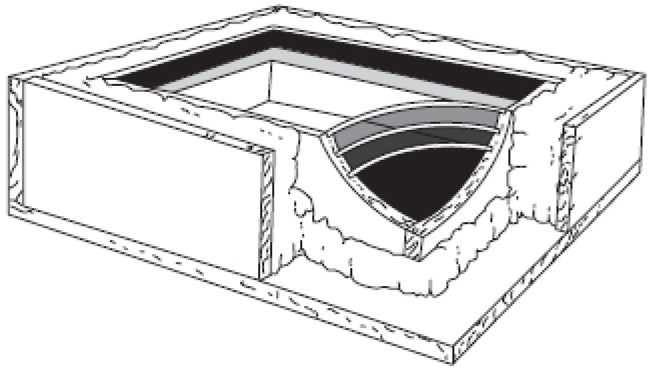
de Saussure's Hot Boxes

In thinking about the effect of the atmosphere on the Earth's energy balance, Fourier drew on the behavior of a simple device invented by the Swiss Alpinist Horace-Bénédict de Saussure (1740–99). This device, called a *heliothermometer*, consisted of a wooden box insulated with cork and wool, with a lid consisting of one or more panes of transparent glass (Fig. 1). The interior walls were painted black so as to absorb nearly all the sunlight entering the box, and a thermometer was placed in the box so that its temperature could be determined. de Saussure devised this instrument as a means of measuring the intensity of sunlight, so that he could test the hypothesis that it is colder atop mountains because the sunlight is weaker there. The idea was to trap the energy of sunlight inside the box, and keep the interior isolated from the surrounding so that the temperature in the box would be responsive to the intensity of the sunlight rather than the temperature of the surroundings. Using the heliothermometer, de Saussure correctly concluded that sunlight becomes, if anything, more intense at higher elevations, so that some other physical process must come into play. "Hot-Boxes" such as de Saussure's were popular toys among scientists throughout the nineteenth century, and many succumbed to the temptation to use them as solar cookers. de Saussure writes that "Fruits … exposed to this heat were cooked and became juicy." Herschel himself took a hot-box with him to South Africa in 1830, and reported: "As these temperatures [up to 240°F] far surpass that of boiling water, some amusing experiments were made by exposing eggs, meat, etc. [to the heat inside the box], all of which, after a moderate length of exposure, were found perfectly cooked. … [On] one occasion a very respectable stew of meat was prepared and eaten with no small relish by the entertained bystanders."

Neither de Saussure nor Fourier hit on the correct explanation of the decline of temperature with altitude, which involves the cooling of air parcels as they are lifted and expand. Nonetheless, the behavior of the heliothermometer provoked a lot of useful thinking about the energy carried by sunlight. Fourier's use of the analogy was to show that if one keeps the rate of energy *input* by sunlight the same, but retards the rate of energy *loss* by putting on a pane of glass, then when the system comes into equilibrium its temperature will be greater than it would have been without the glass in place. Fourier knew that the glass was transparent to sunlight and largely opaque to infrared, but he also knew that in the typical experiment the glass retards heat loss, in part, by simply trapping warm air in the box and keeping it from blowing away. He alludes to the fact that the experiment would still yield an elevation of temperature even if performed in a vacuum, but his use of the subjunctive in the original

French suggests that this is a thought experiment, rather than one he actually carried out.

Fig. 1.1 Artist's conception of the Saussure's improved hot box.



What Fourier Did Not Do

One thing Fourier did not do was coin the term "greenhouse effect," though his use of de Saussure's heliothermometer as an analogue could be considered similar to a greenhouse analogue. de Saussure's box is indeed a kind of miniature greenhouse. In any event, Fourier showed a clear awareness of the imperfection of the analogy, stating explicitly that the temperature in the hot box was influenced by turbulent heat transfers that have no proper counterpart in the planetary temperature problem.

Further, Fourier did not compute the temperature of the Earth in the absence of an atmosphere and concluded that it was colder than the observed temperature. In fact, he never actually computed the Earth's temperature based on a balance between incoming sunlight and outgoing infrared, though he could have attempted this using the Dulong–Petit radiation law. It is not clear why Fourier thought the atmosphere had to have a warming role. Rather than this being demanded by too cold temperatures in the absence of an atmosphere, Fourier seems to be inferring that the atmosphere ought to act like a pane of glass in being transparent to sunlight but opaque to infrared; he shows awareness of the downward infrared radiated by the atmosphere, but it is not clear what the basis of Fourier's leap of intuition about the atmosphere was. In any event, he was right, and his work stimulated a great deal of further research on the effect of the atmosphere on infrared, and ultimately Tyndall's definitive experiments to be discussed next.

On the Temperatures of the Terrestrial Sphere and Interplanetary Space

JEAN-BAPTISTE JOSEPH FOURIER

Translator's note. This is a translation of Jean-Baptiste Joseph Fourier's "Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires," which originally appeared in *Mémoires dl'Académie Royale des Sciences de l'Institute de France* **VII** 570–604 1827. The original text is most readily accessible in the 1890 edition of Fourier's collected *Oeuvres,* Volume 2, edited by M. Gaston Darboux (Gauthier-Villars et Fils:Paris). This work is available online from the Bibliothèque Nationale de France (search catalogue.bnf.fr for author "Fourier, Jean-Baptiste-Joseph"). In the version reprinted in the *Oeuvres,* it is noted that a very slightly different version of the essay also appeared in the *Annales de Chimie et de Physique,* vol XXVII, pp 136–167; 1824, under the title "Remarques générales sur les températures du globe terrestre et des espaces planétaires."

An English translation of Fourier's article has not been available in print for more than a century. Although the article is widely cited, it is my experience that its actual contents are not well known in the Anglophone community (and they are hardly better known among Francophones). My object in doing a new translation is to help rectify this situation, while using some of my own knowledge of physics of climate to help put Fourier's arguments in the clearest possible light. I have put a premium on readability rather than literal translation, and in some cases I have taken the liberty of rephrasing some sentences so as to make Fourier's reasoning more evident; I do not think that in doing so I have read more into the text than Fourier himself put there, but readers seeking the finer nuances of Fourier's meaning will of course have to read the original. I have not consulted any of the existing translations in carrying out the present one, though I can recommend to the reader's attention the annotated translation by W. M. Connolley, available online only at www.wmc.care4free.net/sci/fourier_1827.

I have provided some commentary in the form of footnotes, which are marked by my initials.

Note that for variety, Fourier often uses *globe terrestre* for "Earth," This also serves to remind the reader of the connection with Fourier's earlier idealized work on heat diffusion in a sphere. In the title, I have preserved this sense, but for the most part the phrase has simply been translated as "Earth" in the text.

R. T. Pierrehumbert
1 September, 2004
Chicago, IL, USA

The question of the Earth's temperature distribution, one of the most important and most difficult of all Natural Philosophy, is made up of rather diverse elements that must be considered from a general point of view. It has occurred to me that it would be useful to unite in a single work the principle consequences of this theory; the analytical details that have been omitted here can for the most part be found in the Works which I have already published. Above all, I wish to present to physicists, in a broader picture, the collection of pertinent phenomena and the mathematical relations amongst them.

It is first necessary to distinguish the three sources from which the Earth derives its heat:

(1) The Earth is heated by solar radiation, the unequal distribution of which produces the diversity of climates;

(2) It participates in the common temperature of interplanetary space, being exposed to irradiation by countless stars which surround all parts of the solar system;

(3) The Earth has conserved in the interior of its mass, a part of the primordial heat which it had when the planets originally formed.

By considering each of these three causes and the phenomena which it produces, we will come to understand as clearly as possible, within the limitations of the current state of science, the principal characteristics of these phenomena. In order to provide an overview of this grand question, and to give a first indication of the results of our investigations, we shall present them first in summary form. This summary, in a manner of speaking, serves as an annotated table of contents to my work on the subject.

Our solar system is located in a region of the universe of which all points have a common and constant temperature, determined by the light rays and the heat sent by all the surrounding stars. This cold temperature of the interplanetary sky is slightly below that of the Earth’s polar regions. The Earth would have none other than this same temperature of the Sky, were it not for two causes which act together to further heat it. The first is the interior heat which the globe possessed when the planetary bodies were formed, and of which only a part has escaped through the surface. The second cause is the continual action of solar radiation, which has penetrated the whole mass of the Earth and which leads at the surface to the difference in climates from one place to another.

The primordial heat of the globe no longer has any significant effect at the surface, but it can still be immense in the interior of the Earth. The temperature of the surface does not exceed by more than a thirtieth of a degree the value that it will eventually achieve after a long time has passed: At first, it diminished very rapidly; however, at present the diminution continues only exceedingly slowly.

The observations collected so far indicate that the points of a vertical line continued into the solid earth become warmer with increasing depth, and this rate of increase has been estimated at 1 degree for each 30 to 40 meters. Such a result implies a very high temperature for the interior of the Earth; it can not arise from the action of solar radiation: rather, it is naturally explained by the heat the Earth has retained from the time of its origin.

This rate of increase, on the order of 1 degree per 32 m, will not always remain the same: It will diminish progressively; however, a great many centuries (much more than 30,000 years) will pass before it will be reduced to half of its present value.

It is possible that other yet-unknown causes can explain the same facts, and that there are other general or incidental sources of terrestrial heat. If so, one will discover them through comparison of the results of the present theory against observations.

The heat rays which the Sun incessantly sends to the Earth produce two very distinct effects there: The first is periodic and affects the outer envelope of the planet, while the other is constant; one observes it in deep places, for example at 30 m below the surface. The temperature of these locations is subject to hardly any change in the course of the year, it is fixed; however the deep temperature varies substantially from one climatic zone to another: it results from the perpetual action of solar radiation and the inequal exposure of the surface to these rays, from the equator to the poles. One can determine the time which had to pass in order for the solar radiation to produce the diversity of climates observed today. All these results are in accord with dynamical theories which have led us to recognize the stability of the Earth's axis of rotation.

The periodic effect of solar heating consists of both diurnal and annual variations. Observations of this type are reproduced exactly and in all details by the theory. The comparison of results with observations can be used to measure the thermal conductivity of the material of which the crust of the Earth is formed.

The presence of the atmosphere and surface waters has the effect of rendering the distribution of heat more uniform. In the Ocean and in lakes, the most cold molecules – or more precisely, those with the greatest density – direct themselves continually towards lower regions, and the transport of heat due to this cause is much more rapid than that which can be accomplished in solid bodies by means of thermal conductivity. Mathematical examination of the former effect will require numerous and exact observations: they will serve to clarify how these internal fluid motions keep the internal heat of the globe from having a notable effect in the depths of the waters.[1] Liquids conduct heat very poorly; but they have, as do gaseous materials, the the property of being able to transport it rapidly in certain directions through fluid motions. It is this same property which, in combination with centrifugal force, displaces and mixes all parts of the atmosphere and those of the Ocean; it involves organized and immense currents.

The interposition of air greatly modifies the effects of heat at the surface of the globe. The rays of the Sun, in traversing the layers of the atmosphere compressed by their own weight, heats them very inequally: Those which are the most tenuous are also the most cold, because they attenuate and absorb a lesser quantity of these rays.[2] The heat of the Sun, arriving in the form of visible light, has the ability to penetrate transparent solid or liquid substances, but loses this ability almost completely when it is converted, by its interaction with the terrestrial body, into dark radiant heat.

This distinction between luminous heat and dark heat explains the increase of temperature caused by transparent bodies. The body of water which covers a great part of the globe and the polar ice pose less of an obstacle to the incident luminous heat than to the dark heat, which returns in the opposite sense to exterior space.[3] The presence of the atmosphere produces an effect of the same sort, but which, in the present state of theory and owing further to lack of observations with which theory may be compared, cannot yet be exactly defined. However great the effect may be, one would not suppose that the temperature caused by the incidence of the rays of the Sun on an extremely large solid body would greatly exceed that which one would observe on exposing a thermometer to the light of that star.

The radiation from the highest layers of the atmosphere, whose temperature is very cold and nearly constant, influences all meteorological features which we observe: this radiation can be rendered more easily detectible by means of reflection from concave mirrors. The presence of clouds, which intercept these rays, tempers the cold of the nights.[4]

One thus sees that the surface of the Earth is located between one solid mass, whose central heat may surpass that of incandescent matter, and an immense region whose temperature is below the freezing point of mercury.

All the preceding considerations apply equally well to other planetary bodies. One can consider them as being placed in an environment whose common temperature is constant and somewhat below that of the terrestrial polar regions. This temperature – the temperature of the heavens – is the temperature that would be found at the surface of the most distant planets, for the Solar radiation would be too weak, even augmented by the state of the surface, to have a significant effect; From the state of the Earth we know further, that on other planets (whose formation could hardly have been much later than that of the Earth) the interior remanent heat no longer causes any significant elevation of surface temperature.

It is similarly likely that, for most of the planets, the polar temperature is only slightly greater than that of interplanetary space. As for the mean temperature caused by the action of the Sun on each of these bodies, we are in a state of ignorance, because it can depend on the presence of an atmosphere and the state of the surface. One can only assign, in a very imprecise manner, the mean temperature which the Earth would acquire if it were transported to the same position as the planet in question.

After this discussion, we will treat in succession the various parts of the question. First we must set forth a remark the significance of which bears on all these parts, because it is founded on the nature of the differential equations governing the movement of heat. Namely, we make use of the fact that the effects which arise from each of the three causes which we have discussed above can be calculated separately, as if each of these causes existed in isolation. It suffices then to combine the partial effects; they can be freely superposed, just as for the problem of final oscillations of bodies.[5]

We shall describe first the principal results caused by the prolonged action of solar rays on the Earth.

If one places a thermometer at a considerable depth below the surface of the solid Earth, for example at 40 meters, this instrument indicates a fixed temperature.[6]

One observes this fact at all points of the globe. This deep subsurface temperature is constant for any given location; however, it is not the same in all climates. Generally speaking, it decreases as one moves towards the poles.

If one observes the temperature of points much closer to the surface, for example at 1 m or 5 m or 10 m of depth, one notices very different behavior. The temperature varies during the course of a day or a year; however, we will for the moment idealize the problem by supposing that the skin of the Earth wherein such temperature variations occur is eliminated. We then consider the fixed temperatures of the new surface of the globe.

One can imagine that the state of the mass has varied continually in accord with the heat received from the heat source. This variable temperature state gradually alters, and more and more approaches a final state which no longer varies in time. At that time, each point of the solid sphere has acquired – and conserves – a fixed temperature, which depends only on the position of the point in question.

The final state of the mass, of which the heat has penetrated through all parts, is precisely analogous to that of a vessel which receives, through its upper opening, a liquid which furnishes a constant source, and which allows liquid to escape at a precisely equal rate through one or more openings.

Thus, the solar heat accumulates in the interior of the globe and is continually renewed. It penetrates the portions of the surface near the equator, and escapes through the polar regions. The first question of this type which has been subjected to calculation can be found in a dissertation which I read at the Institute of France at the end of 1807, article 115, p. 167.[7] This work has been deposited in the Archives of the Academy of Sciences. At the time, I took up this first question in order to offer a remarkable example of the application of the new theory presented in the article, and to show how analysis reveals the routes followed by solar heat in the interior of the globe.

Let us now restore the upper envelope of the Earth, for which the points are not sufficiently deep for their temperatures to be time-independent. One then notices a more intricate range of phenomena, which can be completely accounted for by our analysis. At a moderate depth, such as 3 m to 4 m, the temperature observed does not vary in the course of the day; however, it changes very noticably in the course of a year; it alternately rises and falls. The amplitude of these variations, that is to say the difference between the *maximum* and the *minimum* of temperature, is not the same at all depths; it becomes less as the distance from the surface becomes greater. The points lying on a vertical line do not all achieve their extremes of temperature at the same time. The amplitude of the variations, and the time of year at which the highest, mean and lowest temperatures are achieved, change with the position of the point on the vertical. The same applies to the quantities of heat which alternately descend and rise: all these quantities have very definite relations amongst each other, which are indicated by experiment and which analysis expresses very distinctly. The observations conform to the results furnished by the theory; there is not any natural effect more completely explained than this. The mean annual temperature of any given point of the vertical, that is, the mean of all values observed at this point in the course of a year, is independent of depth. It is the same for all points of the vertical, and in consequence, the same as that observed immediately below the surface: it is the invariable temperature of deep places.

It is obvious that, in the statement of this proposition, we have idealized away the interior heat of the globe, and with greater reason the accessory effects which could modify this result in any given place. Our principle object is to bring to light the general nature of the phenomena.

We have said above that the diverse effects can be considered separately. It should also be noted that all of the numerical evaluations given in this article are presented only as examples of how the calculation may be performed. The meteorological observations needed to reveal the heat capacity and permeability of the materials which make up the globe are too uncertain and limited to permit the calculation of precise results; nonetheless, we present these numbers in order to show how the formulae are applied. However inexact these evaluations may be, they serve to give a more correct idea of the phenomena than would general mathematical expressions bereft of numerical application.

In the portions of the envelope closest to the surface, a thermometer would rise and fall in the course of each day. These diurnal variations become insignificant at a depth of 2 m to 3 m. Below these depths, one observes only annual variations, which themselves disappear at yet greater depths.

If the speed of rotation of the Earth about its axis were to become incomparably greater, and if the same were to occur for the movement of the planet about the Sun, one would no longer find diurnal and annual temperature variations of the sort described above; the points of the surface would attain and conserve the fixed deep-Earth temperature. In general, the depth to which one must go in order for the variations to be significant has a very simple relation with the length of the period with which the effects repeat at the surface. This depth is exactly proportional to the square root of the period. It is for this reason that the diurnal variations penetrate to a depth nineteen times less than that at which one can still detect annual variations.

The question of the periodic movement of solar heat was treated for the first time and solved in a separate writing which I submitted to the Institute of France in October 1809. I reproduced this solution in a piece sent at the end of 1811, which was printed in our Collected Works.

The same theory provides the means of measuring the total quantity of heat which, in the course of a year, determines the alternation of the seasons. Our goal in choosing this example of the application of the formulae is to show that there exists a necessary relation between the law of periodic variations and the total quantity of heat transfer which accompanies this oscillation; once this law is known from observations of one given climate, one can deduce the quantity of heat which is introduced into the Earth and which later returns to the air.

By considering a law similar to that which holds in the interior of the globe, one finds the following results. One eighth of a year before the temperature of the surface rises to its mean value, the Earth begins to accumulate heat; the rays of the Sun penetrate the Earth for six months. Then, the movement of the Earth's heat reverses direction; it exits and expands through the air and outer space: the quantity of heat exchanged in these oscillations over the course of a year is expressed by the calculation. If the terrestrial envelope were formed of a metallic substance, such as wrought iron (a substance which I chose as an example because its thermal coefficients have been measured), the heat which produces the alternation of the seasons would, for the climate of Paris and for each square meter of surface, be equivalent to that required to melt a cylindrical column of ice with a base of one square meter and a height of about 3 m.[8] Though the value of the thermal coefficients specific to the material of which the globe is formed have not been measured, one sees easily that they would give a result much less that that which I have just indicated. The result is proportional to the square root of the product of the heat capacity per unit volume and the thermal conductivity.

Let us now consider the second cause of the terrestrial heat, which resides, according to us, in interplanetary space. The temperature of this space is that which a thermometer would show if the Sun and all planetary bodies which accompany it were to cease to exist, assuming the instrument to be placed anywhere in the region of the heavens presently occupied by the solar system.

We shall now indicate the principle facts which have led us to recognize the existence of this characteristic temperature of interplanetary space, independent of the presence of the Sun, and independent of the primitive heat that the globe has been able to retain. To obtain knowledge of this remarkable phenomenon, one must consider what the temperature of the Earth would be if it received only the heat of the Sun; further, to render the problem more tractable, one can at first neglect the effect of the atmosphere. Then, if there were no agency maintaining a common and constant temperature in interplanetary space, that is to say if the Earth and all bodies forming the solar system were located in a region deprived of all heat, one would observe effects completely contrary to those which which we are familiar. The polar regions would be subject to intense cold, and the decrease of temperature from equator to pole would be incomparably more rapid and more extreme than is observed.[9]

Under this hypothesis of absolutely cold space, if such a thing is possible to conceive of, all effects of heat, such as we observe at the surface of the globe, would be due solely to the presence of the Sun. The least variation of the distance of the Earth from this star would lead to considerable changes in the temperature, and the eccentricity of the Earth’s orbit would give rise to new forms of seasonal variations.

The alternation of day and night would produce effects both sudden and totally different from those we observe. The surface of bodies would be exposed all of a sudden, at the beginning of night, to an infinitely intense cold. The living world, both animal and vegetable, could not survive