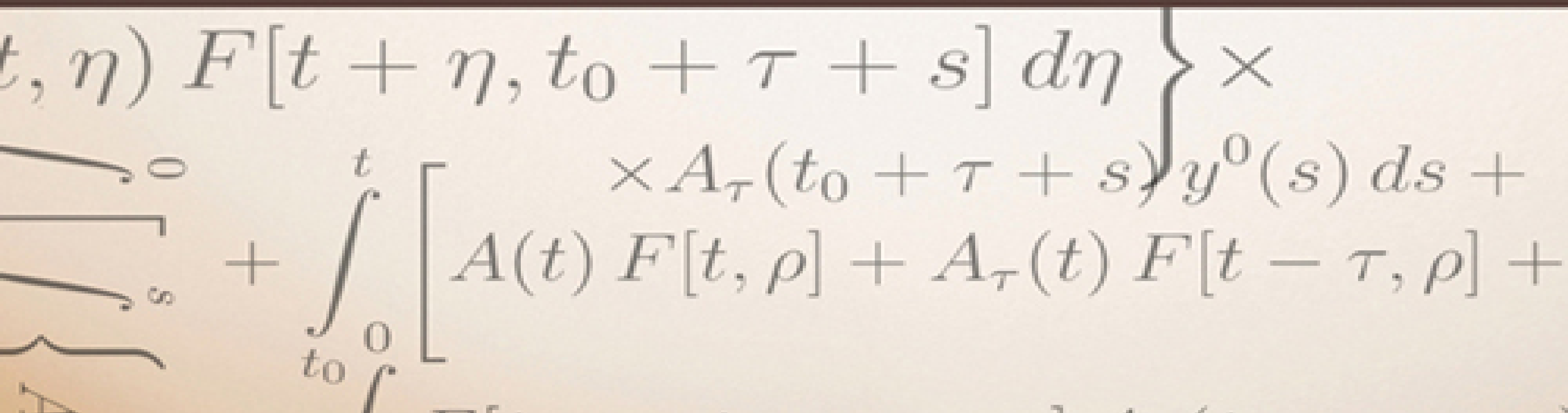
SYSTEMS WITH DELAYS

ANALYSIS, CONTROL, AND COMPUTATIONS



A.V. Kim
A. V. Ivanov

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Scrivener Publishing
100 Cummings Center, Suite 541J
Beverly, MA 01915-6106

Publishers at Scrivener
Martin Scrivener(martin@scrivenerpublishing.com)
Phillip Carmical (pcarmical@scrivenerpublishing.com)

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Preface

At present there are elaborated effective control and numerical methods and corresponding software for analysis and simulating different classes of *ordinary differential equations* (ODE) and *partial differential equations* (PDE). The progress in this direction results in wide application of these types of equations in practice. Another class of differential equations is represented by *delay differential equations* (DDE), also called systems with delays, time-delay systems, hereditary systems, functional differential equations.

Delay differential equations are widely used for describing and mathematical modeling of various processes and systems in different applied problems [3, 5, 1, 27, 32, 33, 34, 40, 50, 62, 63, 183, 91, 107, 108, 111, 127, 183].

Delay in dynamical systems can have several causes, for example: technological lag, signal transmission and information delay, incubational period (infection diseases), time of mixing reactants (chemical kinetics), time of spreading drugs in a body (pharmaceutical kinetics), latent period (population dynamics), etc.

Though at present different *theoretical aspects* of time-delay theory (see, for example, [3, 1, 27, 32, 34, 50, 62, 63, 67, 72, 73, 183, 91, 107, 111, 127] and references therein) are developed with almost the same completeness as the corresponding parts of ODE theory, *practical* implementation of many methods is very difficult because of infinite dimensional nature of systems with delays.

Also it is necessary to note that, unlike ODE, even for linear DDE there are no methods of finding solutions in explicit forms, and the absence of generally available general-purpose software packages for simulating such systems cause a big obstacle for analysis and application of time-delay systems.

In this book we try to fill up this gap.

The main aim of the book is to present new constructive methods of DDE theory and to give readers *practical tools for analysis, control design and simulating of linear systems with delays*[1].

The main outstanding features of this book are the following:

1. on the basis of *i-smooth analysis* we give a complete description of the structure and properties of quadratic Lyapunov-Krasovskii functionals[2];

2. we describe a new control design technique for systems with delays, based on an explicit form of solutions of linear quadratic control problems;

3. we present new numerical algorithms for simulating DDE.

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[1] The present volume is devoted to linear time-delay system theory. We plan to prepare a special volume devoted to analysis of nonlinear systems with delays.

[2] Including properties of positiveness, and constructive presentation of the total derivative of functionals with respect to time-delay systems.

Chapter 1

Linear time-delay systems

1.1 Introduction

1.1.1 Linear systems with delays

In this book we consider methods of analysis, control and computer simulation of linear systems with delays

(1.1)

$$\dot{x}(t) = A(t)\, x(t) + A_\tau(t)\, x(t - \tau(t)) + \int\limits_{-\tau(t)}^{0} G(t,s)\, x(t+s)\, ds + u\,,$$

where $A(t)$, $A_\tau(t)$ are $n \times n$ matrices with piece-wise continuous elements, $G(t, s)$ is $n \times n$ matrix with piece-wise continuous elements on $\mathbf{R} \times [-\tau, 0]$, u is a given n-dimensional vector-function, $\tau(t) : \mathbf{R} \to [-\tau, 0]$ is a continuous function, τ is a positive constant.

Much attention will be paid to the special class of linear **time-invariant** systems

$$\dot{x}(t) = A\, x(t) + A_\tau\, x(t-\tau) + \int\limits_{-\tau}^{0} G(s)\, x(t+s)\, ds + u\,,$$

(1.2)

where A, A_τ are $n \times n$ constant matrices, $G(s)$ is $n \times n$ matrix with piece-wise continuous elements on $[-\tau, 0]$, τ is a positive constant[1].

Usually we will consider u as the vector of control parameters. There are two possible variants:

1) $u = u(t)$ is the function of time t;

2) u depend on the current and previous state of the system, for example,

$$u = C\,x(t) + \int_{-\tau}^{0} D(s)\,x(t+s)\,ds\,. \tag{1.3}$$

Consider some models of control systems with delays.

1.1.2 Wind tunnel model

A linearized model of the high-speed closed-air unit wind tunnel is [134, 135]

$$\begin{aligned} \dot{x}_1(t) &= -a\,x_1(t) + a\,k\,x_2(t-\tau)\,, \\ \dot{x}_2(t) &= x_3(t)\,, \\ \dot{x}_3(t) &= -\omega^2\,x_2(t) - 2\,\xi\,\omega\,x_3(t) + \omega^2 u_3(t)\,, \end{aligned} \tag{1.4}$$

with $a = \dfrac{1}{1.964}$, $k = -0.117$, $\omega = 6$, $\xi = 0.8$, $\tau = 0.33$ s.

The state variable x_1, x_2, x_3 represent deviations from a chosen operating point (equilibrium point) of the following quantities: x_1 = Mach number, x_2 = actuator position guide vane angle in a driving fan, x_3 = actuator rate. The delay represents the time of the transport between the fan and the test section.

The system can be written in matrix form

$$\dot{x}(t) = A_0 x(t) + A_\tau x(t-\tau) + B\,u(t)\,, \tag{1.5}$$

where

$$A_0 = \begin{bmatrix} -a & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -\omega^2 & -2\,\xi\,\omega \end{bmatrix},$$

$$A_\tau = \begin{bmatrix} 0 & a\,k & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \omega^2 \end{bmatrix}.$$

1.1.3 Combustion stability in liquid propellant rocket motors

A linearized version of the feed system and combustion chamber equations, assuming nonsteady flow, is given by[2]

$$\begin{aligned} \dot{\phi}(t) &= (\gamma - 1)\,\phi(t) - \gamma\,\phi(t-\delta) + \mu(t-\delta) \\ \dot{\mu}_1(t) &= \frac{1}{\xi J}\left[-\psi(t) + \frac{p_0 - p_1}{2\Delta p}\right] \\ \dot{\mu}(t) &= \frac{1}{(1-\xi)J}\left[-\mu(t) + \psi(t) - P\,\phi(t)\right] \\ \dot{\psi}(t) &= \frac{1}{E}\left[\mu_1(t) - \mu(t)\right]. \end{aligned} \tag{1.6}$$

Here

$\phi(t)$ = fractional variation of pressure in the combustion chamber,

t is the unit of time normalized with gas residence time, θ_g, in steady operation,

$\tilde{\tau}$ = value of time lag in steady operation,
$\tilde{p}$ = pressure in combustion chamber in steady operation,
$\overline{\tau p^{\gamma}}$ = *const* for some number γ,

$$\delta = \frac{\tilde{\tau}}{\theta_g}$$

μ(*t*) = fractional variation of injection and burning rate,
ψ(*t*) = relative variation of p_1,
p_1 = instantaneous pressure at that place in the feeding line where the capacitance representing the elasticity is located,
ξ = fractional length for the constant pressure supply,
J = inertial parameter of the line,
P = pressure drop parameter,
$\mu_1(t)$ = fractional variation of instantaneous mass flow upstream of the capacitance,
Δ*p* = injector pressure drop in steady operation,
p_0 = regulated gas pressure for constant pressure supply,
E = elasticity parameter of the line.

For our purpose we have taken

$$u = \frac{p_0 - p_1}{2\Delta p}$$

to be a control variable and guided by [36] have adopted the following representative numerical values:
γ = 0.8, ξ = 0.5, δ = 1, *J* = 2, *P* = 1, *E* = 1.

This gives, for $x(t) = (\varphi(t), \mu_1(t), \mu(t), \psi(t))'$,

$$\dot{x}(t) = A_0 x(t) + A_\tau x(t-1) + Bu(t)\,, \quad (1.7)$$

where

$$A_0 = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix},$$

$$A_\tau = \begin{bmatrix} -0.8 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

The system (1.7) has two roots with positive real part: $\lambda_{1,2} = 0.11255 \pm 1.52015\, i$.

1.2 Conditional representation of differential equations

1.2.1 Conditional representation of ODE and PDE

Let us remember that for ODE

(1.8) $\dot{x}(t) = g(t, x(t))\,,$

the conditional representation is

(1.9) $$\dot{x} = g(t, x) \,,$$

i.e. the argument *t* is not pointed out in state variable *x*(*t*).

The conditional representation of the partial differential equation

$$\frac{\partial y(t,x)}{\partial t} = a\frac{\partial^2 y(t,x)}{\partial x^2} \,,$$

is

(1.10) $$\frac{\partial y}{\partial t} = a\frac{\partial^2 y}{\partial x^2} \,,$$

i.e. the arguments *t* and *x* are not pointed out in the function *y*(*t, x*).

Thus in order to obtain the conditional representation of an ODE it is necessary to make in this equation the following substitutions

(1.11) $$\begin{cases} x(t) & \text{to replace by} \quad x, \\ x'(t) & \text{to replace by} \quad x'. \end{cases}$$

Example 1.1. The linear control ODE

$$x'(t) = a(t)x(t) + u(t) \,,$$

can be written in the conditional form as

$$x' = a(t)x + u(t) \,,$$

note, we omit variable *t* only in the state variable *x*(*t*) but not in the coefficients *a*(*t*) and *u*(*t*). One can omit *t* also in the control variable *u*(*t*), in this case the conditional representation will be

$$x' = a(t)x + u \,.$$

Remark 1.1 It is necessary to emphasize, conditional representation is very useful for describing local properties of differential equations, for application of geometrical language and methods.

1.2.2 Conditional representation of DDE

Let us introduce the conditional representation of systems with delays (1.1). First of all it necessary to note, differential equations with time lags differ from ODE by presence (involving) point $x(t-\tau)$ and/or segment $x(t+s)$, $-\tau \le s < 0$, which characterize previous history (*pre-history*) of the solution $x(t)$.

The conditional representation of time-delay systems (1.1) can be introduced in the following way. In H an element of trajectory of the system is written as a pair $x_t \equiv \{x(t); x(t+s), -\tau \le s < 0\} \in H$. Then, using the notation

$$x_t \equiv \{x(t);\, x(t+s), -\tau \le s < 0\} \equiv$$

$$\equiv \{x(t);\, x(t+\cdot)\} \equiv \{x, y(\cdot)\}_t \qquad (1.12)$$

we obtain the conditional representation

(1.13)

$$\dot{x} = A(t)\,x + A_\tau(t)\,y(-\tau(t)) + \int\limits_{-\tau(t)}^{0} G(t,s)\,y(s)\,ds + u\,,$$

for system (1.1) in the space H.

Correspondingly, the conditional representation of time-invariant system (1.2) is

$$\dot{x} = A\,x + A_\tau\,y(-\tau) + \int\limits_{-\tau}^{0} G(s)\,y(s)\,ds + u\,. \qquad (1.14)$$

Conditional representations (1.9), (1.13) and (1.14) have no “physical sense”, and formulas (1.13) and (1.14) are understood as systems (1.1) and (1.2) considered in the phase space H. It is convenient to use representations (1.9), (1.13) and (1.14) for investigating local properties of differential equations.