



VINTAGE

THE ROAD TO REALITY:
A COMPLETE GUIDE
TO THE LAWS
OF THE UNIVERSE
ROGER PENROSE

Contents

Cover

About the Book

About the Author

Also By Roger Penrose

Title Page

Dedication

Preface

Acknowledgements

Notation

Prologue

1 The roots of science

1.1 The quest for the forces that shape the world

1.2 Mathematical truth

1.3 Is Plato's mathematical world 'real'?

1.4 Three worlds and three deep mysteries

1.5 The Good, the True, and the Beautiful

2 An ancient theorem and a modern question

2.1 The Pythagorean theorem

2.2 Euclid's postulates

2.3 Similar-areas proof of the Pythagorean theorem

2.4 Hyperbolic geometry: conformal picture

- 2.5 Other representations of hyperbolic geometry
- 2.6 Historical aspects of hyperbolic geometry
- 2.7 Relation to physical space
- 3 Kinds of number in the physical world
 - 3.1 A Pythagorean catastrophe?
 - 3.2 The real-number system
 - 3.3 Real numbers in the physical world
 - 3.4 Do natural numbers need the physical world?
 - 3.5 Discrete numbers in the physical world
- 4 Magical complex numbers
 - 4.1 The magic number 'i'
 - 4.2 Solving equations with complex numbers
 - 4.3 Convergence of power series
 - 4.4 Caspar Wessel's complex plane
 - 4.5 How to construct the Mandelbrot set
- 5 Geometry of logarithms, powers, and roots
 - 5.1 Geometry of complex algebra
 - 5.2 The idea of the complex logarithm
 - 5.3 Multiple valuedness, natural logarithms
 - 5.4 Complex powers
 - 5.5 Some relations to modern particle physics
- 6 Real-number calculus
 - 6.1 What makes an honest function?
 - 6.2 Slopes of functions
 - 6.3 Higher derivatives; C^∞ -smooth functions
 - 6.4 The 'Eulerian' notion of a function?

6.5 The rules of differentiation

6.6 Integration

7 Complex-number calculus

7.1 Complex smoothness; holomorphic functions

7.2 Contour integration

7.3 Power series from complex smoothness

7.4 Analytic continuation

8 Riemann surfaces and complex mappings

8.1 The idea of a Riemann surface

8.2 Conformal mappings

8.3 The Riemann sphere

8.4 The genus of a compact Riemann surface

8.5 The Riemann mapping theorem

9 Fourier decomposition and hyperfunctions

9.1 Fourier series

9.2 Functions on a circle

9.3 Frequency splitting on the Riemann sphere

9.4 The Fourier transform

9.5 Frequency splitting from the Fourier transform

9.6 What kind of function is appropriate?

9.7 Hyperfunctions

10 Surfaces

10.1 Complex dimensions and real dimensions

10.2 Smoothness, partial derivatives

10.3 Vector fields and 1-forms

10.4 Components, scalar products

10.5 The Cauchy–Riemann equations

11 Hypercomplex numbers

11.1 The algebra of quaternions

11.2 The physical role of quaternions?

11.3 Geometry of quaternions

11.4 How to compose rotations

11.5 Clifford algebras

11.6 Grassmann algebras

12 Manifolds of n dimensions

12.1 Why study higher-dimensional manifolds?

12.2 Manifolds and coordinate patches

12.3 Scalars, vectors, and covectors

12.4 Grassmann products

12.5 Integrals of forms

12.6 Exterior derivative

12.7 Volume element; summation convention

12.8 Tensors; abstract-index and diagrammatic notation

12.9 Complex manifolds

13 Symmetry groups

13.1 Groups of transformations

13.2 Subgroups and simple groups

13.3 Linear transformations and matrices

13.4 Determinants and traces

13.5 Eigenvalues and eigenvectors

13.6 Representation theory and Lie algebras

13.7 Tensor representation spaces; reducibility

13.8 Orthogonal groups

- 13.9 Unitary groups
- 13.10 Symplectic groups

14 Calculus on manifolds

- 14.1 Differentiation on a manifold?
- 14.2 Parallel transport
- 14.3 Covariant derivative
- 14.4 Curvature and torsion
- 14.5 Geodesics, parallelograms, and curvature
- 14.6 Lie derivative
- 14.7 What a metric can do for you
- 14.8 Symplectic manifolds

15 Fibre bundles and gauge connections

- 15.1 Some physical motivations for fibre bundles
- 15.2 The mathematical idea of a bundle
- 15.3 Cross-sections of bundles
- 15.4 The Clifford-Hopf bundle
- 15.5 Complex vector bundles, (co)tangent bundles
- 15.6 Projective spaces
- 15.7 Non-triviality in a bundle connection
- 15.8 Bundle curvature

16 The ladder of infinity

- 16.1 Finite fields
- 16.2 A finite or infinite geometry for physics?
- 16.3 Different sizes of infinity
- 16.4 Cantor's diagonal slash
- 16.5 Puzzles in the foundations of mathematics
- 16.6 Turing machines and Gödel's theorem

16.7 Sizes of infinity in physics

17 Spacetime

17.1 The spacetime of Aristotelian physics

17.2 Spacetime for Galilean relativity

17.3 Newtonian dynamics in spacetime terms

17.4 The principle of equivalence

17.5 Cartan's 'Newtonian spacetime'

17.6 The fixed finite speed of light

17.7 Light cones

17.8 The abandonment of absolute time

17.9 The spacetime for Einstein's general relativity

18 Minkowskian geometry

18.1 Euclidean and Minkowskian 4-space

18.2 The symmetry groups of Minkowski space

18.3 Lorentzian orthogonality; the 'clock paradox'

18.4 Hyperbolic geometry in Minkowski space

18.5 The celestial sphere as a Riemann sphere

18.6 Newtonian energy and (angular) momentum

18.7 Relativistic energy and (angular) momentum

19 The classical fields of Maxwell and Einstein

19.1 Evolution away from Newtonian dynamics

19.2 Maxwell's electromagnetic theory

19.3 Conservation and flux laws in Maxwell theory

19.4 The Maxwell field as gauge curvature

19.5 The energy-momentum tensor

19.6 Einstein's field equation

19.7 Further issues: cosmological constant; Weyl tensor

19.8 Gravitational field energy

20 Lagrangians and Hamiltonians

20.1 The magical Lagrangian formalism

20.2 The more symmetrical Hamiltonian picture

20.3 Small oscillations

20.4 Hamiltonian dynamics as symplectic geometry

20.5 Lagrangian treatment of fields

20.6 How Lagrangians drive modern theory

21 The quantum particle

21.1 Non-commuting variables

21.2 Quantum Hamiltonians

21.3 Schrödinger's equation

21.4 Quantum theory's experimental background

21.5 Understanding wave-particle duality

21.6 What is quantum 'reality'?

21.7 The 'holistic' nature of a wavefunction

21.8 The mysterious 'quantum jumps'

21.9 Probability distribution in a wavefunction

21.10 Position states

21.11 Momentum-space description

22 Quantum algebra, geometry, and spin

22.1 The quantum procedures **U** and **R**

22.2 The linearity of **U** and its problems for **R**

22.3 Unitary structure, Hilbert space, Dirac notation

22.4 Unitary evolution: Schrödinger and Heisenberg

22.5 Quantum 'observables'

22.6 YES/NO measurements; projectors

- 22.7 Null measurements; helicity
- 22.8 Spin and spinors
- 22.9 The Riemann sphere of two-state systems
- 22.10 Higher spin: Majorana picture
- 22.11 Spherical harmonics
- 22.12 Relativistic quantum angular momentum
- 22.13 The general isolated quantum object

23 The entangled quantum world

- 23.1 Quantum mechanics of many-particle systems
- 23.2 Hugeness of many-particle state space
- 23.3 Quantum entanglement; Bell inequalities
- 23.4 Bohm-type EPR experiments
- 23.5 Hardy's EPR example: almost probability-free
- 23.6 Two mysteries of quantum entanglement
- 23.7 Bosons and fermions
- 23.8 The quantum states of bosons and fermions
- 23.9 Quantum teleportation
- 23.10 Quanglement

24 Dirac's electron and antiparticles

- 24.1 Tension between quantum theory and relativity
- 24.2 Why do antiparticles imply quantum fields?
- 24.3 Energy positivity in quantum mechanics
- 24.4 Diffculties with the relativistic energy formula
- 24.5 The non-invariance of $\partial/\partial t$
- 24.6 Clifford-Dirac square root of wave operator
- 24.7 The Dirac equation
- 24.8 Dirac's route to the positron

25 The standard model of particle physics

- 25.1 The origins of modern particle physics
- 25.2 The zigzag picture of the electron
- 25.3 Electroweak interactions; reflection asymmetry
- 25.4 Charge conjugation, parity, and time reversal
- 25.5 The electroweak symmetry group
- 25.6 Strongly interacting particles
- 25.7 'Coloured quarks'
- 25.8 Beyond the standard model?

26 Quantum field theory

- 26.1 Fundamental status of QFT in modern theory
- 26.2 Creation and annihilation operators
- 26.3 Infinite-dimensional algebras
- 26.4 Antiparticles in QFT
- 26.5 Alternative vacua
- 26.6 Interactions: Lagrangians and path integrals
- 26.7 Divergent path integrals: Feynman's response
- 26.8 Constructing Feynman graphs; the S-matrix
- 26.9 Renormalization
- 26.10 Feynman graphs from Lagrangians
- 26.11 Feynman graphs and the choice of vacuum

27 The Big Bang and its thermodynamic legacy

- 27.1 Time symmetry in dynamical evolution
- 27.2 Submicroscopic ingredients
- 27.3 Entropy
- 27.4 The robustness of the entropy concept
- 27.5 Derivation of the second law—or not?
- 27.6 Is the whole universe an 'isolated system'?

- 27.7 The role of the Big Bang
- 27.8 Black holes
- 27.9 Event horizons and spacetime singularities
- 27.10 Black-hole entropy
- 27.11 Cosmology
- 27.12 Conformal diagrams
- 27.13 Our extraordinarily special Big Bang

28 Speculative theories of the early universe

- 28.1 Early-universe spontaneous symmetry breaking
- 28.2 Cosmic topological defects
- 28.3 Problems for early-universe symmetry breaking
- 28.4 Inflationary cosmology
- 28.5 Are the motivations for inflation valid?
- 28.6 The anthropic principle
- 28.7 The Big Bang's special nature: an anthropic key?
- 28.8 The Weyl curvature hypothesis
- 28.9 The Hartle-Hawking 'no-boundary' proposal
- 28.10 Cosmological parameters: observational status?

29 The measurement paradox

- 29.1 The conventional ontologies of quantum theory
- 29.2 Unconventional ontologies for quantum theory
- 29.3 The density matrix
- 29.4 Density matrices for spin $\frac{1}{2}$: the Bloch sphere
- 29.5 The density matrix in EPR situations
- 29.6 FAPP philosophy of environmental decoherence
- 29.7 Schrödinger's cat with 'Copenhagen' ontology
- 29.8 Can other conventional ontologies resolve the 'cat'?
- 29.9 Which unconventional ontologies may help?

30 Gravity's role in quantum state reduction

- 30.1 Is today's quantum theory here to stay?
- 30.2 Clues from cosmological time asymmetry
- 30.3 Time-asymmetry in quantum state reduction
- 30.4 Hawking's black-hole temperature
- 30.5 Black-hole temperature from complex periodicity
- 30.6 Killing vectors, energy flow—and time travel!
- 30.7 Energy outflow from negative-energy orbits
- 30.8 Hawking explosions
- 30.9 A more radical perspective
- 30.10 Schrödinger's lump
- 30.11 Fundamental conflict with Einstein's principles
- 30.12 Preferred Schrödinger-Newton states?
- 30.13 FELIX and related proposals
- 30.14 Origin of fluctuations in the early universe

31 Supersymmetry, supra-dimensionality, and strings

- 31.1 Unexplained parameters
- 31.2 Supersymmetry
- 31.3 The algebra and geometry of supersymmetry
- 31.4 Higher-dimensional spacetime
- 31.5 The original hadronic string theory
- 31.6 Towards a string theory of the world
- 31.7 String motivation for extra spacetime dimensions
- 31.8 String theory as quantum gravity?
- 31.9 String dynamics
- 31.10 Why don't we see the extra space dimensions?
- 31.11 Should we accept the quantum-stability argument?
- 31.12 Classical instability of extra dimensions

- 31.13 Is string QFT finite?
- 31.14 The magical Calabi-Yau spaces; M-theory
- 31.15 Strings and black-hole entropy
- 31.16 The 'holographic principle'
- 31.17 The D-brane perspective
- 31.18 The physical status of string theory?

32 Einstein's narrower path; loop variables

- 32.1 Canonical quantum gravity
- 32.2 The chiral input to Ashtekar's variables
- 32.3 The form of Ashtekar's variables
- 32.4 Loop variables
- 32.5 The mathematics of knots and links
- 32.6 Spin networks
- 32.7 Status of loop quantum gravity?

33 More radical perspectives; twistor theory

- 33.1 Theories where geometry has discrete elements
- 33.2 Twistors as light rays
- 33.3 Conformal group; compactified Minkowski space
- 33.4 Twistors as higher-dimensional spinors
- 33.5 Basic twistor geometry and coordinates
- 33.6 Geometry of twistors as spinning massless particles
- 33.7 Twistor quantum theory
- 33.8 Twistor description of massless fields
- 33.9 Twistor sheaf cohomology
- 33.10 Twistors and positive/negative frequency splitting
- 33.11 The non-linear graviton
- 33.12 Twistors and general relativity
- 33.13 Towards a twistor theory of particle physics

33.14 The future of twistor theory?

34 Where lies the road to reality?

34.1 Great theories of 20th century physics—and beyond?

34.2 Mathematically driven fundamental physics

34.3 The role of fashion in physical theory

34.4 Can a wrong theory be experimentally refuted?

34.5 Whence may we expect our next physical
revolution?

34.6 What is reality?

34.7 The roles of mentality in physical theory

34.8 Our long mathematical road to reality

34.9 Beauty and miracles

34.10 Deep questions answered, deeper questions posed

Epilogue

Bibliography

Copyright

About the Book

The Road to Reality is the most important and ambitious work of science for a generation. It provides nothing less than a comprehensive account of the physical universe and the essentials of its underlying mathematical theory. It assumes no particular specialist knowledge on the part of the reader, so that, for example, the early chapters give us the vital mathematical background to the physical theories explored later in the book.

Roger Penrose's purpose is to describe as clearly as possible our present understanding of the universe and to convey a feeling for its deep beauty and philosophical implications, as well as its intricate logical interconnections.

The Road to Reality is rarely less than challenging, but the book is leavened by vivid descriptive passages, as well as hundreds of hand-drawn diagrams. In a single work of colossal scope one of the world's greatest scientists has given us a complete and unrivalled guide to the glories of the universe that we all inhabit.

About the Author

Roger Penrose is Emeritus Rouse Ball Professor of Mathematics at the University of Oxford. He has received a number of prizes and awards, including the 1988 Wolf Prize for physics which he shared with Stephen Hawking for their joint contribution to our understanding of the universe. His books include *The Emperor's New Mind* and *Shadows of the Mind*.

ALSO BY ROGER PENROSE

*The Emperor's New Mind: Concerning Computers,
Minds, and the Laws of Physics*
*Shadows of the Mind: A Search for the
Missing Science of Consciousness*

Roger Penrose

THE ROAD TO REALITY

A Complete Guide to the
Laws of the Universe

VINTAGE BOOKS
London

I dedicate this book to the memory of
DENNIS SCIAMA
who showed me the excitement of physics

Preface

THE purpose of this book is to convey to the reader some feeling for what is surely one of the most important and exciting voyages of discovery that humanity has embarked upon. This is the search for the underlying principles that govern the behaviour of our universe. It is a voyage that has lasted for more than two-and-a-half millennia, so it should not surprise us that substantial progress has at last been made. But this journey has proved to be a profoundly difficult one, and real understanding has, for the most part, come but slowly. This inherent difficulty has led us in many false directions; hence we should learn caution. Yet the 20th century has delivered us extraordinary new insights—some so impressive that many scientists of today have voiced the opinion that we may be close to a basic understanding of *all* the underlying principles of physics. In my descriptions of the current fundamental theories, the 20th century having now drawn to its close, I shall try to take a more sober view. Not all my opinions may be welcomed by these ‘optimists’, but I expect further changes of direction greater even than those of the last century.

The reader will find that in this book I have not shied away from presenting mathematical formulae, despite dire warnings of the severe reduction in readership that this will entail. I have thought seriously about this question, and have come to the conclusion that what I have to say cannot reasonably be conveyed without a certain amount of mathematical notation and the exploration of genuine mathematical concepts. The understanding that we have of the principles that actually underlie the behaviour of our physical world indeed depends upon some appreciation of

its mathematics. Some people might take this as a cause for despair, as they will have formed the belief that they have no capacity for mathematics, no matter at how elementary a level. How could it be possible, they might well argue, for them to comprehend the research going on at the cutting edge of physical theory if they cannot even master the manipulation of *fractions*? Well, I certainly see the difficulty.

Yet I am an optimist in matters of conveying understanding. Perhaps I am an incurable optimist. I wonder whether those readers who cannot manipulate fractions—or those who claim that they cannot manipulate fractions—are not deluding themselves at least a little, and that a good proportion of them actually have a potential in this direction that they are not aware of. No doubt there are some who, when confronted with a line of mathematical symbols, however simply presented, can see only the stern face of a parent or teacher who tried to force into them a non-comprehending parrot-like apparent competence—a duty, and a duty alone—and no hint of the magic or beauty of the subject might be allowed to come through. Perhaps for some it is too late; but, as I say, I am an optimist and I believe that there are many out there, even among those who could never master the manipulation of fractions, who have the capacity to catch some glimpse of a wonderful world that I believe must be, to a significant degree, genuinely accessible to them.

One of my mother's closest friends, when she was a young girl, was among those who could not grasp fractions. This lady once told me so herself after she had retired from a successful career as a ballet dancer. I was still young, not yet fully launched in my activities as a mathematician, but was recognized as someone who enjoyed working in that subject. 'It's all that cancelling', she said to me, 'I could just never get the hang of cancelling.' She was an elegant and highly intelligent woman, and there is no doubt in my mind that the mental qualities that are required in

comprehending the sophisticated choreography that is central to ballet are in no way inferior to those which must be brought to bear on a mathematical problem. So, grossly overestimating my expositional abilities, I attempted, as others had done before, to explain to her the simplicity and logical nature of the procedure of ‘cancelling’.

I believe that my efforts were as unsuccessful as were those of others. (Incidentally, her father had been a prominent scientist, and a Fellow of the Royal Society, so she must have had a background adequate for the comprehension of scientific matters. Perhaps the ‘stern face’ could have been a factor here, I do not know.) But on reflection, I now wonder whether she, and many others like her, did not have a more rational hang-up—one that with all my mathematical glibness I had not noticed. There is, indeed, a profound issue that one comes up against again and again in mathematics and in mathematical physics, which one first encounters in the seemingly innocent operation of cancelling a common factor from the numerator and denominator of an ordinary numerical fraction.

Those for whom the action of cancelling has become second nature, because of repeated familiarity with such operations, may find themselves insensitive to a difficulty that actually lurks behind this seemingly simple procedure. Perhaps many of those who find cancelling mysterious are seeing a certain profound issue more deeply than those of us who press onwards in a cavalier way, seeming to ignore it. What issue is this? It concerns the very way in which mathematicians can provide an existence to their mathematical entities and how such entities may relate to physical reality.

I recall that when at school, at the age of about 11, I was somewhat taken aback when the teacher asked the class what a fraction (such as $\frac{3}{4}$) actually is! Various suggestions came forth concerning the dividing up of pieces of pie and the like, but these were rejected by the teacher on the

(valid) grounds that they merely referred to imprecise physical situations to which the precise mathematical notion of a fraction was to be *applied*; they did not tell us what that clear-cut mathematical notion actually *is*. Other suggestions came forward, such as $\frac{3}{8}$ is ‘something with a 3 at the top and an 8 at the bottom with a horizontal line in between’ and I was distinctly surprised to find that the teacher seemed to be taking these suggestions seriously! I do not clearly recall how the matter was finally resolved, but with the hindsight gained from my much later experiences as a mathematics undergraduate, I guess my schoolteacher was making a brave attempt at telling us the definition of a fraction in terms of the ubiquitous mathematical notion of an *equivalence class*.

What is this notion? How can it be applied in the case of a fraction and tell us what a fraction actually is? Let us start with my classmate’s ‘something with a 3 at the top and an 8 on the bottom’. Basically, this is suggesting to us that a fraction is specified by an ordered pair of whole numbers, in this case the numbers 3 and 8. But we clearly cannot regard the fraction as *being* such an ordered pair because, for example, the fraction $\frac{6}{16}$ is the same number as the fraction $\frac{3}{8}$, whereas the pair (6, 16) is certainly not the same as the pair (3, 8). This is only an issue of cancelling; for we can write $\frac{6}{16}$ as $\frac{3 \times 2}{8 \times 2}$ and then cancel the 2 from the top and the bottom to get $\frac{3}{8}$. Why are we allowed to do this and thereby, in some sense, ‘equate’ the pair (6, 16) with the pair (3, 8)? The mathematician’s answer—which may well sound like a cop-out—has the cancelling rule just built in to the definition of a fraction: a pair of whole numbers ($a \times n, b \times n$) is deemed to represent the same fraction as the pair (a, b) whenever n is any non-zero whole number (and where we should not allow b to be zero either).

But even this does not tell us what a fraction is; it merely tells us something about the way in which we represent fractions. What *is* a fraction, then? According to the

mathematician's "equivalence class" notion, the fraction $\frac{3}{8}$, for example, simply is the infinite collection of all pairs

$(3, 8), (-3, -8), (6, 16), (-6, -16), (9, 24), (-9, -24), (12, 32), \dots,$

where each pair can be obtained from each of the other pairs in the list by repeated application of the above cancellation rule. ¹ We also need definitions telling us how to add, subtract, and multiply such infinite collections of pairs of whole numbers, where the normal rules of algebra hold, and how to identify the whole numbers themselves as particular types of fraction.

This definition covers all that we mathematically need of fractions (such as $\frac{1}{2}$ being a number that, when added to itself, gives the number 1, etc.), and the operation of cancelling is, as we have seen, built into the definition. Yet it seems all very formal and we may indeed wonder whether it really captures the intuitive notion of what a fraction is. Although this ubiquitous equivalence class procedure, of which the above illustration is just a particular instance, is very powerful as a pure-mathematical tool for establishing consistency and mathematical existence, it can provide us with very top-heavy-looking entities. It hardly conveys to us the intuitive notion of what $\frac{3}{8}$ is, for example! No wonder my mother's friend was confused.

In my descriptions of mathematical notions, I shall try to avoid, as far as I can, the kind of mathematical pedantry that leads us to define a fraction in terms of an 'infinite class of pairs' even though it certainly has its value in mathematical rigour and precision. In my descriptions here I shall be more concerned with conveying the idea—and the beauty and the magic—inherent in many important mathematical notions. The idea of a fraction such as $\frac{3}{8}$ is simply that it is some kind of an entity which has the property that, when added to itself 8 times in all, gives 3.

The magic is that the idea of a fraction actually works despite the fact that we do not really directly experience things in the physical world that are exactly quantified by fractions—pieces of pie leading only to approximations. (This is quite unlike the case of natural numbers, such as 1, 2, 3, which do precisely quantify numerous entities of our direct experience.) One way to see that fractions do make consistent sense is, indeed, to use the ‘definition’ in terms of infinite collections of pairs of integers (whole numbers), as indicated above. But that does not mean that $\frac{a}{b}$ actually *is* such a collection. It is better to think of $\frac{a}{b}$ as being an entity with some kind of (Platonic) existence of its own, and that the infinite collection of pairs is merely one way of our coming to terms with the consistency of this type of entity. With familiarity, we begin to believe that we can easily grasp a notion like $\frac{a}{b}$ as something that has its own kind of existence, and the idea of an ‘infinite collection of pairs’ is merely a pedantic device—a device that quickly recedes from our imaginations once we have grasped it. Much of mathematics is like that.

To mathematicians (at least to most of them, as far as I can make out), mathematics is not just a cultural activity that we have ourselves created, but it has a life of its own, and much of it finds an amazing harmony with the physical universe. We cannot get any deep understanding of the laws that govern the physical world without entering the world of mathematics. In particular, the above notion of an equivalence class is relevant not only to a great deal of important (but confusing) mathematics, but a great deal of important (and confusing) physics as well, such as Einstein’s general theory of relativity and the ‘gauge theory’ principles that describe the forces of Nature according to modern particle physics. In modern physics, one cannot avoid facing up to the subtleties of much sophisticated mathematics. It is for this reason that I have spent the first 16 chapters of this work directly on the description of mathematical ideas.

What words of advice can I give to the reader for coping with this? There are four different levels at which this book can be read. Perhaps you are a reader, at one end of the scale, who simply turns off whenever a mathematical formula presents itself (and some such readers may have difficulty with coming to terms with fractions). If so, I believe that there is still a good deal that you can gain from this book by simply skipping all the formulae and just reading the words. I guess this would be much like the way I sometimes used to browse through the chess magazines lying scattered in our home when I was growing up. Chess was a big part of the lives of my brothers and parents, but I took very little interest, except that I enjoyed reading about the exploits of those exceptional and often strange characters who devoted themselves to this game. I gained something from reading about the brilliance of moves that they frequently made, even though I did not understand them, and I made no attempt to follow through the notations for the various positions. Yet I found this to be an enjoyable and illuminating activity that could hold my attention. Likewise, I hope that the mathematical accounts I give here may convey something of interest even to some profoundly non-mathematical readers if they, through bravery or curiosity, choose to join me in my journey of investigation of the mathematical and physical ideas that appear to underlie our physical universe. Do not be afraid to skip equations (I do this frequently myself) and, if you wish, whole chapters or parts of chapters, when they begin to get a mite too turgid! There is a great variety in the difficulty and technicality of the material, and something elsewhere may be more to your liking. You may choose merely to dip in and browse. My hope is that the extensive cross-referencing may sufficiently illuminate unfamiliar notions, so it should be possible to track down needed concepts and notation by turning back to earlier unread sections for clarification.

At a second level, you may be a reader who is prepared to peruse mathematical formulae, whenever such is presented, but you may not have the inclination (or the time) to verify for yourself the assertions that I shall be making. The confirmations of many of these assertions constitute the solutions of the exercises that I have scattered about the mathematical portions of the book. I have indicated three levels of difficulty by the icons -



very straight forward



needs a bit of thought



not to be undertaken lightly.

It is perfectly reasonable to take these on trust, if you wish, and there is no loss of continuity if you choose to take this position.

If, on the other hand, you are a reader who does wish to gain a facility with these various (important) mathematical notions, but for whom the ideas that I am describing are not all familiar, I hope that working through these exercises will provide a significant aid towards accumulating such skills. It is always the case, with mathematics, that a little direct experience of thinking over things on your own can provide a much deeper understanding than merely reading about them. (If you need the solutions, see the website www.roadsolutions.ox.ac.uk.)

Finally, perhaps you are already an expert, in which case you should have no difficulty with the mathematics (most of which will be very familiar to you) and you may have no wish to waste time with the exercises. Yet you may find that there is something to be gained from my own perspective on a number of topics, which are likely to be somewhat different (sometimes very different) from the usual ones. You may have some curiosity as to my opinions relating to a number of modern theories (e.g. supersymmetry, inflationary cosmology, the nature of the Big Bang, black

holes, string theory or M-theory, loop variables in quantum gravity, twistor theory, and even the very foundations of quantum theory). No doubt you will find much to disagree with me on many of these topics. But controversy is an important part of the development of science, so I have no regrets about presenting views that may be taken to be partly at odds with some of the mainstream activities of modern theoretical physics.

It may be said that this book is really about the relation between mathematics and physics, and how the interplay between the two strongly influences those drives that underlie our searches for a better theory of the universe. In many modern developments, an essential ingredient of these drives comes from the judgement of mathematical beauty, depth, and sophistication. It is clear that such mathematical influences can be vitally important, as with some of the most impressively successful achievements of 20th-century physics: Dirac's equation for the electron, the general framework of quantum mechanics, and Einstein's general relativity. But in all these cases, physical considerations—ultimately observational ones—have provided the overriding criteria for acceptance. In many of the modern ideas for fundamentally advancing our understanding of the laws of the universe, adequate physical criteria—i.e. experimental data, or even the possibility of experimental investigation—are not available. Thus we may question whether the accessible mathematical desiderata are sufficient to enable us to estimate the chances of success of these ideas. The question is a delicate one, and I shall try to raise issues here that I do not believe have been sufficiently discussed elsewhere.

Although, in places, I shall present opinions that may be regarded as contentious, I have taken pains to make it clear to the reader when I am actually taking such liberties. Accordingly, this book may indeed be used as a genuine guide to the central ideas (and wonders) of modern physics.

It is appropriate to use it in educational classes as an honest introduction to modern physics—as that subject is understood, as we move forward into the early years of the third millennium.

1 This is called an 'equivalence class' because it actually is a class of entities (the entities, in this particular case, being pairs of whole numbers), each member of which is deemed to be equivalent, in a specified sense, to each of the other members.

Acknowledgements

It is inevitable, for a book of this length, which has taken me about eight years to complete, that there will be a great many to whom I owe my thanks. It is almost as inevitable that there will be a number among them, whose valuable contributions will go unattributed, owing to congenital disorganization and forgetfulness on my part. Let me first express my special thanks—and also apologies—to such people: who have given me their generous help but whose names do not now come to mind. But for various specific pieces of information and assistance that I can more clearly pinpoint, I thank Michael Atiyah, John Baez, Michael Berry, Dorje Brody, Robert Bryant, Hong-Mo Chan, Joy Christian, Andrew Duggins, Maciej Dunajski, Freeman Dyson, Artur Ekert, David Fowler, Margaret Gleason, Jeremy Gray, Stuart Hameroff, Keith Hannabuss, Lucien Hardy, Jim Hartle, Tom Hawkins, Nigel Hitchin, Andrew Hodges, Dipankar Home, Jim Howie, Chris Isham, Ted Jacobson, Bernard Kay, William Marshall, Lionel Mason, Charles Misner, Tristan Needham, Stelios Negre-pontis, Sarah Jones Nelson, Ezra (Ted) Newman, Charles Oakley, Daniel Oi, Robert Osserman, Don Page, Oliver Penrose, Alan Rendall, Wolfgang Rindler, Engelbert Schücking, Bernard Schutz, Joseph Silk, Christoph Simon, George Sparling, John Stachel, Henry Stapp, Richard Thomas, Gerard 't Hooft, Paul Tod, James Vickers, Robert Wald, Rainer Weiss, Ronny Wells, Gerald Westheimer, John Wheeler, Nick Woodhouse, and Anton Zeilinger. Particular thanks go to Lee Smolin, Kelly Stelle, and Lane Hughston for numerous and varied points of assistance. I am especially indebted to Florence Tsou (Sheung Tsun) for immense help on matters of particle physics, to Fay Dowker for her