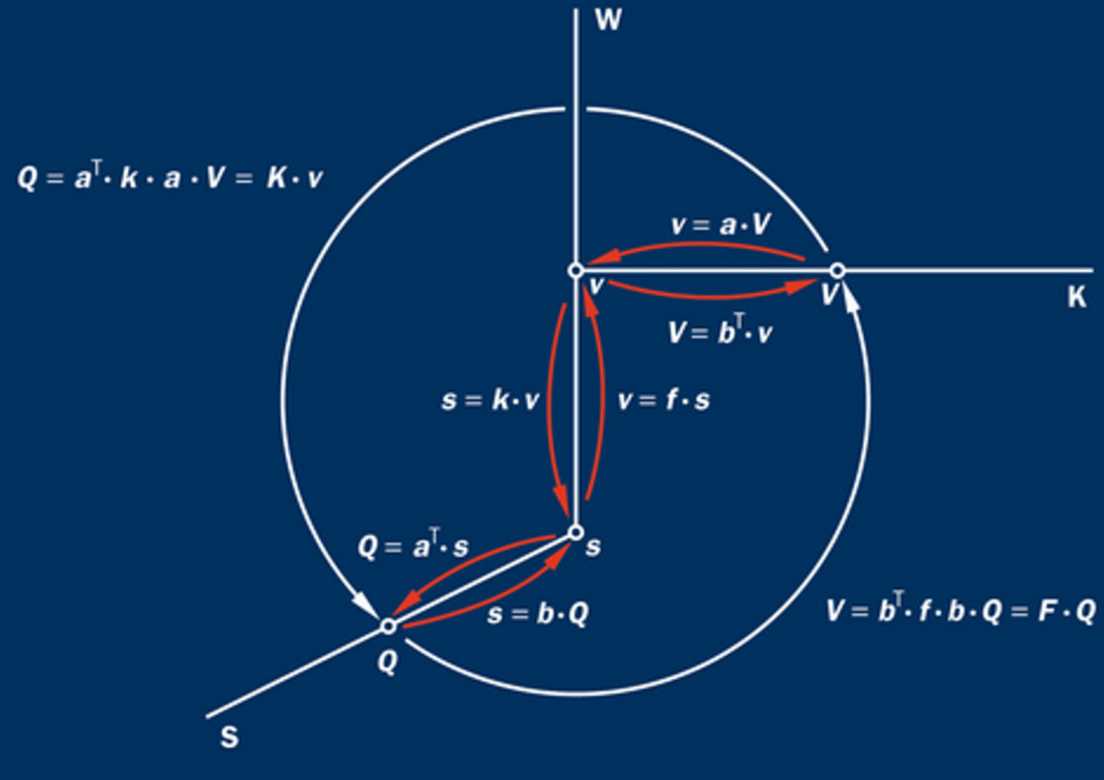
PETER MARTI

THEORY OF STRUCTURES

FUNDAMENTALS
FRAMED STRUCTURES
PLATES AND SHELLS



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Prof. Dr. Peter Marti
ETH Zurich
Institute of Structural Engineering (IBK)
8093 Zurich
Switzerland
marti@ibk.baug.ethz.ch

Translated by Philip Thrift, German2English Language Services, Hanover, Germany

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PREFACE

This book grew out of the lectures I gave at the University of Toronto between 1982 and 1987 and those I have been giving at the Swiss Federal Institute of Technology Zurich (ETH Zurich) since 1990. The lectures in Toronto were entitled "Energy methods in structural engineering" and "Structural stability", those in Zurich "Theory of structures I-III" and "Plate and shell structures". In addition, the book contains material from my lectures on "Applied mechanics" and "Plasticity in reinforced concrete" (Toronto) as well as "Conceptual design", "Bridge design", "Building structures" and "Structural concrete I-III" (Zurich).

The book is aimed at students and teaching staff as well as practising civil and structural engineers. Its purpose is to enable readers to model and handle structures sensibly, and to provide support for the planning and checking of structures.

These days, most structural calculations are carried out by computers on the basis of the finite element method. This book provides only an introduction to that topic. It concentrates on the fundamentals of the theory of structures, the goal being to convey appropriate insights into and knowledge about structural behaviour. Framed structures and plate and shell structures are treated according to elastic theory and plastic theory. There are many examples and also a number of exercises for the reader to solve independently. On the whole, the aim is to provide the necessary support so that the reader, through skilful modelling, can achieve meaningful results just adequate for the respective engineering issue, using the simplest means possible. In particular, such an approach will enable the reader to check computer calculations critically and efficiently – an activity that is always necessary, but unfortunately often neglected. Moreover, the broader basis of more in-depth knowledge focuses attention on the essentials and creates favourable conditions for the synthesis of the structural, constructional, practical realisation and creative issues so necessary in structural design.

Chapters 3 and 4, which deal with the general principles of structural engineering, have been heavily influenced by my work as the head of the "Swisscodes" project of the Swiss Engineers & Architects Association (SIA). The purpose of this project, carried out between 1998 and 2003, was to revise fully the structures standards of the SIA, which were subsequently republished as Swiss standards SIA 260 to 267. I am grateful to the SIA for granting permission to reproduce Fig. 1 and Tab. 1 from SIA 260 "Basis of structural design" as Fig. 3.1 and Tab. 4.1 in this book. Further, I would also like to thank the SIA for consenting to the use of the service criteria agreement and basis of design examples, which formed part of my contribution to the introduction of SIA 260 in document SIA D 0181, as examples 3.1 and 3.2 here.

In essence, the account of the theory of structures given in this book is based on my civil engineering studies at ETH Zurich. Hans Ziegler, professor of mechanics, and Bruno Thürlimann, professor of theory of structures and structural concrete, and also my dissertation advisor and predecessor, had the greatest influence on me. Prof. Thürlimann was a staunch advocate of the use of plastic theory in structural engineering and enjoyed support from Prof. Ziegler for his endeavours in this respect. I am also grateful to the keen insights provided by Pierre Dubas, professor of theory of structures and structural steelwork, and Christian Menn, professor of theory of structures and design, especially with regard to the transfer of theory into practice. Many

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examples and forms of presentation used in this book can be attributed to all four of these teachers, whom I hold in high esteem, and the Zurich school of theory of structures, which they have shaped to such a great extent.

During my many years as a lecturer in Toronto and Zurich, students gave me many valuable suggestions for improving my lectures; I am deeply obliged to all of them. Grateful thanks also go to my current and former assistants at ETH Zurich. Their great dedication to supervising students and all their other duties connected with teaching have contributed greatly to the ongoing evolution of the Zurich school of theory of structures.

Susanna Schenkel, dipl. Ing. ETH, and Matthias Schmidlin, dipl. Arch. ETH/dipl. Ing. ETH, provided invaluable help during the preparation of the manuscript. Mr. Schmidlin produced all the figures and Mrs. Schenkel coordinated the work, maintained contact with the publisher and wrote all the equations and large sections of the text; I am very grateful to both for their precise and careful work. Furthermore, I would like to thank Maya Stacey for her typing services. A great vote of thanks also goes to my personal assistant, Regina Nöthiger, for her help during the preparations for this book project and for always relieving me from administrative tasks very effectively. Philip Thrift translated the text from German into English. I should like to thank him for the care he has taken and also for his helpful suggestions backed up by practical experience. Finally, I would like to thank the publisher, Ernst & Sohn, for the pleasant cooperation and the meticulous presentation of this book.

Zurich, February 2013 Peter Marti

CONTENTS

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EXAMPLECOLLECTION

1 THE PURPOSE AND SCOPE OF THEORY OF STRUCTURES

1.1 General

Without doubt, many are convinced that the calculations should determine the dimensions unequivocally and conclusively. However, in the light of the impossibility of taking into account all secondary circumstances, every calculation constitutes only a basis for the design engineer, who thus has to grapple with those secondary circumstances...

A totally simple form of calculation alone is therefore possible and sufficient.

Robert MAILLART (1938)

Theory of structures is a subdiscipline of applied mechanics which is configured to suit the needs of civil and structural engineers. The purpose of theory of structures is to present systematically the knowledge about the behaviour of structures at rest, to expand that knowledge and to prepare it for practical applications. It forms the basis for the design of every new structure and the examination of every existing one.

The terms and methods used in the theory of structures enable the engineer to adopt a uniform approach not tied to any particular type of construction (concrete, steel, composite, timber or masonry). With the advent of the computer in the third quarter of the 20th century, this approach gradually became *structural mechanics*, the discipline to which theory of structures belongs today.

At the heart of every theory of structures exercise there is a *structural model*, which is obtained through isolation and idealisation and takes into account the geometry of the structure, the properties of the construction materials and the possible actions. Determining the action effects, i. e. the structure's responses to the actions, is carried out with the help of *analytical models* that link the governing force and deformation variables via equilibrium and compatibility conditions plus constitutive equations.

1.2 The basis of theory of structures

Structural behaviour is expressed in the form of *internal* and *external force* and *deformation variables* (loads and stresses plus displacements and strains). Static relationships (equilibrium conditions and static boundary conditions, see chapter 5) link the force variables, kinematic relationships (kinematic relationships and boundary conditions, see chapter 6) link the deformation variables, and constitutive relationships (see chapter 7) link the internal force and deformation variables. The most general statements within the scope of theory of structures are obtained when the internal and external force and deformation variables are rigorously associated in the form of *work-associated variables* (see chapter 8) [1].

Statics is based on three fundamental principles of mechanics. According to the *principle of virtual work*, a (statically admissible) force state (equilibrium set of force variables) fulfilling the static relationships in conjunction with a (kinematically admissible) deformation state (compatibility set of deformation variables) fulfilling the kinematic relationships does not perform any work. Added to this are the *reaction principle* (for every force there is a equal and opposite reaction with the same direction of action) and the *free-body principle* (every part removed from a system in equilibrium undergoing compatible deformation is itself in equilibrium and undergoes compatible deformation).

Looking beyond its link with mechanics, theory of structures has a special significance for *structural engineering* (see chapters 3 and 4). It is a tool for assessing the stability, strength and stiffness of a structure that either exists or is being designed. This application of theory of structures manifests itself in specific methods developed for ascertaining structural behaviour in general and (numerical) treatment in individual cases.

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1.3 Methods of theory of structures

The principle of virtual work can be expressed as the principle of virtual deformations or the principle of virtual forces. The systematic application of these two principles leads to a series of *dual* kinematic or static *methods*. On the kinematic side it is important to mention LAND's method for determining influence lines (section 12.3), the displacement method for solving statically indeterminate framed structures (chapter 17 and section 19.3) and the kinematic method of limit analysis (sections 21.3 and 21.7). On the static side we have the work theorem for determining single deformations (section 14.2), the force method for solving statically indeterminate framed structures (chapter 16 and section 19.2) and the static method of limit analysis (sections 21.3 and 21.7).

Assuming linear elastic behaviour and small deformations leads to linear statics, in which all the force and deformation variables may be superposed. This possibility of superposition is used extensively in theory of structures, especially in the force and displacement methods. Introducing unknown force or deformation variables and superposing their effects on those of external actions results in sets of linear equations for the unknowns.

However, the *superposition law* no longer applies in the case of non-linear materials problems (chapters 20 and 21) and non-linear geometrical problems (chapter 22). In such instances an (incremental) *iterative procedure* is generally necessary. Errors caused by simplifications at the beginning are evaluated step by step and successively reduced through appropriate corrections.

Analogies can often be used to make complex situations more accessible, or to reduce them to simpler, known situations. Examples of this are the membrane analogy (section 13.4.2) and the sand hill analogy (section 21.4.4) for dealing with elastic or plastic torsion problems, and MOHR's analogy for determining deformation diagrams (section 15.3.2). Combined warping and pure torsion problems (section 13.4.4) can be approached in a similar way to combined shear and bending problems (section 18.5.2) or bending problems in beams with tension (section 18.9). Edge disturbance problems in cylindrical shells (sections 18.7.4 and 26.5) can be reduced to the theory of beams on elastic foundation (section 18.4.4); this theory is also useful for approximating edge disturbance problems in spherical (section 26.7.3) and other shells (section 26.7.4). Furthermore, plates (chapter 23) can be idealised as plane trusses, slabs (chapter 24) as grillages, and folded plates (chapter 25) and shells (chapter 26) as space trusses or spatial frameworks

The development of powerful numerical methods has led to the methods of *graphical statics* (section 10.1) gradually losing the importance they had in the past. However, graphical aids still represent an unbeatable way of illustrating the flow of the forces in structures, e. g. with thrust lines (section 5.3.2, Figs. 17.19 and 21.7) or truss models (section 23.4.2). They represent an indispensable foundation for conceptual design (section 3.2) and the detailing of structural members and their connections.

The development of numerical methods has also brought about a change in the significance of *experimental statics*. From the 1920s through to the 1970s, loading tests on scale models made from celluloid, acrylic sheet and other materials were central to understanding the elastic loadbearing behaviour of complex structures. Such tests are no longer significant today. What continues to be important, however, is scientific testing to verify theoretical models, primarily in conjunction with non-linear phenomena, new materials or forms of construction and accidental actions. In structural design, physical models are not only useful for form-finding and detailing, but also very helpful when assessing the quality of the structural behaviour of the design. During the dimensioning, tests are a sensible backup if, for example, there are no appropriate analytical models available or a large number of identical structural members is required.

Finally, specific measurements during and after execution enable extremely valuable comparisons with the predicted behaviour of a structure – a source of experience that is all too often neglected.

In the *numerical methods* of theory of structures, it is the *finite element method* (FEM) that plays the leading role (section 19.3). These days FEM is the basis of almost all structural calculations. Users have extremely powerful tools at their disposal in the shape of appropriate modern computer programs. But to be able to deploy such programs responsibly, designers should at least understand the basics of the algorithms on which they are based. First and foremost, however, the engineer's knowledge of theory of structures should enable him or her to check the computer output critically. The crucial thing here is the ability to be able to approximate complex issues by reducing them to simple, understandable problems. Adequate training in the classical methods of theory of structures, which this book aims at, will supply the foundation for that ability.

1.4 Statics and structural dynamics

When it comes to dynamic problems, the principle of virtual work has to be formulated taking into account *inertial forces* (proportional to acceleration): the motion in a system is such that at any point in time the internal, external and inertial forces are in equilibrium. Appropriate additional terms in the equilibrium conditions turn them into *equations of motion*, and can be included, for example, within the scope of the finite element method by way of local and global *mass matrices*. Instead of a set of linear equations, this leads to a set of simultaneous ordinary second-order differential equations for the (time-dependent) node displacement parameters. Assuming constant coefficients, the differential equations can be decoupled according to the method of *modal analysis*. The associated eigenvalue problem leads to a solution in the form of superposed *natural vibrations*.

Generally, damping forces must also be taken into account in the equations of motion. In order that the differential equations remain linear, it is usual to assume that these forces are proportional to velocity. And so that a modal analysis remains possible with decoupled natural vibrations, we use a so-called *modal damping* for simplicity.

Structural dynamics is essentially readily accessible via statics. However, adding the dimension of time makes a more in-depth examination necessary so that dynamic processes become just as familiar as static phenomena. In the end, engineers prepared to make the effort obtain a broader view of theory of structures.

1.5 Theory of structures and structural engineering

For *structural engineering*, theory of structures is an ancillary discipline, like materials science. The knowledge and experience of practising design engineers in this and other relevant special subjects, e. g. geotechnics and construction technology, must be adequate for the complexity and significance of the jobs to which they are assigned. Furthermore, appropriate practical experience with the respective types of construction is an essential requirement for managing the design and execution of construction projects.

Theory of structures plays a role in all phases of conventional project development, from the preliminary design and tender design to the detail design, but in different ways, to suit the particular phase. Whereas for the conceptual design rough structural calculations are adequate, the subsequent phases require analyses of structural safety and serviceability that can be verified by others – and not just for the final condition of the structure, but especially for critical conditions during construction.

Besides new-build projects, the conservation and often the deconstruction of structures also throw up their share of interesting theory of structures problems. Frequently such tasks are far more demanding than those of new structures because fewer, if any, standards are available to help the engineer, and appraising the current condition of a structure is often difficult and associated with considerable uncertainties. The development of appropriate structural and actions models in such cases can be extremely tricky yet fascinating.

Looking beyond the immediate uses of structural design, there are various applications that can be handled with the methods of theory of structures, especially in mechanical engineering, shipbuilding and automotive manufacture, aerospace engineering, too. We are thus part of the great interdisciplinary field of *structural mechanics*.

2 BRIEF HISTORICAL BACKGROUND

Apart from a few minor modifications, this chapter is based on an earlier essay by the author [19]. Readers who wish to find out more should consult references [10], [16], [32] and [33].

Until well into the 19th century, the practical experience of architects, builders and engineers far exceeded their theoretical knowledge. The scientifically founded knowledge of structural behaviour that prevails today had its beginnings in antiquity and the Middle Ages and evolved with the development of mechanics. However, it was not until the 18th century that the first attempts were made to use the new findings in practical construction.

We have to thank the Greek mathematician ARCHIMEDES (*c.* 287 – *c.* 212 BC) for the discovery of hydrostatic buoyancy and for formulating the lever principle for unequal straight levers subjected to vertical forces. Besides formulating theories for the functions of the "simple machines" lever, wedge, screw, pulley and wheel and axle, Archimedes is also credited with inventing technical artefacts such as the screw pump.

Jordanus DE NEMORE (*c.* 1200) is thought to have written various treatises that draw on the works of Greek scholars. But he also added new observations on the cranked lever and the inclined plane.

Leonardo DA VINCI (1452 – 1519) recognised the principle of resolving a force into two components, and also applied the term "moment" (force × lever arm) to skew forces. He also investigated the breakage of a rope due to its own weight (specific strength), the bending of beams and columns and the equilibrium and failure mechanisms of arches. His extremely imaginative and diverse, yet unsystematic, insights went apparently largely unnoticed during his lifetime.

Simon STEVIN's (1548 – 1620) approach to the concept of moments and the resolution of forces into components cannot be faulted. He worked on many practical applications and provided very vivid descriptions, e. g. the funicular polygon and the "wreath of spheres" experiment to prove the law of the inclined plane.

Pierre VARIGNON (1654 – 1722) identified the connection between the force and funicular polygons and formulated the theorem of the summability of moments.

Giovanni POLENI (1683 – 1761) analysed the load transfer of the 42 m span of the dome to St. Peter's in Rome by constructing the funicular polygon for the weights corresponding to the individual segments of the vaulting. He selected the funicular polygon that passed through the centres of the springing and crown joints and established that the inverted funicular polygon must lie within the arch profile. In 1743 POLENI was appointed to investigate the damage to the dome of St. Peter's, just as one year before the three mathematicians Ruggiero Giuseppe BOŠCOVIĆ (1711 – 1787), Thomas LE SEUR (1703 – 1770) and François JACQUIER (1711 – 1788) had been commissioned to do. Based on the crack pattern observed, the three mathematicians analysed an assumed mechanism and hence determined a deficit in the resistance with respect to the thrust in the arch. They recommended adding further horizontal iron hoops (to resist the tension) around the dome to the three already in place. Although POLENI did not agree with the cause of the damage

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as described by the mathematicians, he did agree with the proposed strengthening measures.

GALILEO Galilei (1564 – 1642) founded the discipline of strength of materials through his studies of the failure of the cantilever beam. Starting with the tensile test as a “thought experiment” and the associated question of the specific strength, he analysed the equilibrium of a cantilever beam as a cranked lever with its fulcrum at the bottom edge of the fixed-end cross-section. Applying similitude theory, he determined the failure load relationships of simple beam structures with different geometries. He realised that no structure can exceed a certain given size (maximum span) determined by the limits of strength and remarked that hollow cross-sections and cross-sections that vary over the length of the beam can make better use of the strength than prismatic, solid cross-sections.

Edmé MARIOTTE (1620 – 1684) and Pieter van MUSSCHENBROEK (1692 – 1761) carried out tensile and bending strength tests on various materials, the latter also buckling strength tests. Applying similitude theory, it became possible to design a beam. In the bending failure problem, MARIOTTE, like GALILEO, initially assumed that the cantilever beam rotates about the bottom edge of the fixed-end cross-section, but presumed a triangular distribution of the tensile force over the depth of the cross-section. In a further step, he introduced the “axe d’équilibre” (neutral axis) in the middle of the depth of the cross-section and distinguished between zones in tension and compression, with triangular distributions of the tensile and compressive forces above and below this axis. Instead of the theoretically correct reduction factor of 3 of GALILEO’s strength studies, he mistakenly arrived at a value of 1.5; his tests resulted in a reduction factor of about 2.

Antoine PARENT (1666 – 1716) recognised that the tensile and compressive forces due to bending must be equal in magnitude and that there are also shear forces acting on the cross-section. Based on MARIOTTE’s tests, PARENT positioned the neutral axis somewhat below the middle, i. e. at 45% of the depth of the cross-section, which when compared with GALILEO’s work leads to a reduction factor of 2.73 for an equal tensile strength.

Robert HOOKE (1635 – 1703) undertook experiments with springs and reached the conclusion that the forces in elastic bodies are proportional to the corresponding displacements. He also recognised that some of the fibres in a beam subjected to bending are pulled and hence extended and some are compressed and hence shortened. Further, he recommended giving arches the form of an inverted catenary.

Jacob BERNOULLI (1654 – 1705) investigated the deformation of elastic bars with the help of the infinitesimal calculus introduced by Isaac NEWTON (1643 – 1727) and Gottfried Wilhelm LEIBNIZ (1646 – 1716). He assumed that the cross-sections of the bar remain plane during the deformation and discovered that the change in curvature is proportional to the bending forces. However, as he was not yet aware of the stress concept, the integration of the internal forces over the cross-section, which is taken for granted today, is missing from his deductions.

The principle of virtual displacements, already used in a simple form by DE NEMORE, STEVIN and GALILEO, was stated in general form in 1717 by Johann BERNOULLI (1667 – 1748).

Following a proposal by Daniel BERNOULLI (1700 – 1782), Leonhard EULER (1707 – 1783) showed that Jacob BERNOULLI’s differential equation of the elastic curve corresponds to a variational problem. According to this, the integral of the squares of the curvatures over the length of the bar is a minimum; for homogeneous prismatic bars, this integral is proportional to the elastically stored deformation work. EULER’s detailed treatises on elastic curves led to the solution of the eigenvalue problems of buckling and laterally vibrating bars. Apart from the concept of hydrostatic stress, we also have EULER to thank for the free-body principle at the root of all mechanics. This principle states that every free body separated with an imaginary cut from a body in equilibrium is itself in equilibrium; internal forces are thus externalised and can therefore be dealt with. Starting by considering the individual mass elements of a body, EULER formulated NEWTON's law of motion in the form of the theorem of linear momentum, and also postulated the theorem of angular momentum. Therefore, equilibrium conditions for forces and moments became special cases of the equations of motion.

The designation "engineer" had already been used in isolated cases in the Middle Ages to describe the builders of military apparatus and fortifications. The direct predecessors of civil engineers as we know them today were French engineering officers who were called upon to carry out civil as well as military tasks. At the suggestion of the most outstanding of these engineering officers, Sébastien le Prêtre de VAUBAN (1633 – 1707), the "Corps des ingénieurs du génie militaire" was set up in 1675. The "Corps des ingénieurs des ponts et chaussées" followed around 1720.

The French engineering officers received scientific, primarily mathematical, training at state schools. The "Ecole des ponts et chaussées" in Paris, founded in 1747 by Daniel Charles TRUDAINE (1703 – 1769) and reorganised in 1760 by Jean Rodolphe PERRONET (1708 – 1794), was at that time unique in Europe. The "Ecole polytechnique", which opened in Paris in 1794, was followed by the polytechnic schools of Prague (1806), Vienna (1815), Karlsruhe (1825) and other cities.

PERRONET was primarily active as a builder of stone bridges. He reduced the widths of the piers in order to improve the flow cross-section, employed very shallow three-centred arches and introduced various other new ideas into the design and construction of such bridges.

Charles Augustin de COULOMB (1736 – 1806) was another French engineering officer. He set down his practical experience in the building of fortifications in the "Essai sur une application des règles de maximis et minimis à quelques problèmes de statique relatifs à l'architecture", which was published in 1776. Based on the tensile tests of samples of stone, he determined the resistance to cleavage fracture per unit area, a property that he termed "cohesion". Although shearing-off tests gave a somewhat larger resistance, COULOMB ignored this difference and, considering possible failure planes in masonry piers, introduced a friction resistance proportional to the normal compression on the failure plane. By varying the inclination of the failure plane, he discovered the smallest possible and hence critical ratio between compressive strength and cohesion. He proceeded in a similar way when investigating active and passive earth pressure problems and when determining the upper and lower limits for arch thrust. COULOMB also concluded the strength problem of the beam in bending. Using the example of the cantilever beam, he distinguished between internal forces normal to and parallel with the cross-section and formulated the equilibrium conditions for the free body separated by the cross-section being studied. In doing so, he assumed a generally non-linear distribution of the internal forces over the depth of the beam. For the special case of the rectangular cross-section with linear force distribution, as with GALILEO's strength studies, he obtained the right result with a reduction factor of 3.

Claude Louis Marie Henri NAVIER (1785 – 1836) was appointed professor at the "Ecole des ponts et chaussées" in 1819 and the "Ecole polytechnique" in 1831. It is him we have to thank for today's form of the differential equation for the beam in bending, with the modulus of elasticity of the construction material and the principal moment of inertia of the cross-section. His published lecture notes bring together the scattered knowledge of his predecessors in a form suitable for practical building applications. He solved numerous problems of static indeterminacy, investigated the buckling of elastic bars subjected to eccentric loads and also became involved with suspension bridges and many other issues. As a design engineer, NAVIER also had to cope with setbacks: his Pont des Invalides in Paris, spanning 160 m across the Seine, was abandoned shortly before completion (1826) because of various difficulties encountered during construction.

Augustin Louis CAUCHY (1789 – 1857) abandoned the notion that the stress vector must be orthogonal to the surface of the section, which applies in hydrostatics, and established the concept of the stress tensor. He also introduced the strain tensor and recognised that the linear elastic theory of homogeneous isotropic materials requires two material constants. Important contributions to the ongoing expansion of elastic theory were supplied by Siméon Denis POISSON (1781 – 1840), Gabriel LAMÉ (1795 – 1870), Benoît Pierre Emile CLAPEYRON (1799 – 1864), Adhémar Jean Claude Barré de SAINT-VENANT (1797 – 1886) and others.

Karl CULMANN (1821 – 1881), a professor at Zurich Polytechnic, which had opened in 1855, established graphical statics, i. e. the geometric/graphic treatment of theory of structures problems which is especially suitable for trusses. The rigorous application of force and funicular polygons enabled him to reduce beam statics to cable statics and obtain a universally applicable method of integration by adding the closing line to the funicular polygon. Antonio Luigi Gaudenzio Giuseppe CREMONA (1830 – 1903), Maurice LÉVY (1838 – 1910) and Karl Wilhelm RITTER (1847 – 1906) were firm advocates of the use of graphical statics.

Emil WINKLER (1835 – 1888) made important contributions to the elastic theory foundations of theory of structures. He introduced the axial and shear stiffnesses of elastic bars, investigated thermal deformations, analysed the arch fixed on both sides, studied beams on elastic foundation and worked on how "stress curves" indicate the effects of travelling loads, for which Johann Jacob WEYRAUCH (1845 – 1917) coined the term influence line.

Otto Christian MOHR (1835 – 1918) discovered the analogy between line loads and bending moments on the one hand and curvatures and deflections of beams on the other, thus paving the way for the graphical determination of deflection curves. He introduced his circle diagrams for presenting general stress and strain conditions and proposed a universal failure hypothesis based on COULOMB's approach. His studies of the secondary stresses in trusses, which are due to the fact that the connections between the members are actually rigid and not hinged as assumed in theory, gave him the idea of considering joint and bar rotations as unknowns. It was not until the first decades of the 20th century that this idea was exploited, in the form of the slope-deflection method for dealing with statically indeterminate systems.

James Clerk MAXWELL (1831 – 1879) regarded elastic trusses as machines working without energy losses and discovered that the displacement caused by a first unit force at the position and in the direction of a second unit force is equal to the displacement caused by the second unit force at the position and in the direction of the first unit force. This reciprocal theorem is a special case of the interaction relationship for linear elastic systems named after Enrico BETTI (1823 – 1892). According to this relationship, a first force system does the same work on the displacements of a second force system as the second system does on the displacements of the first. It is Carlo Alberto CASTIGLIANO (1847 – 1884) we have to thank for the theorem that the force variables in an elastic system are equal to the derivatives of the deformation work with respect to the corresponding deformation variables. Mathias KOENEN (1849 – 1924) transferred the work theorem for the displacement calculation, introduced by MOHR for trusses, to beams in bending. Friedrich ENGESSER (1848 – 1931) highlighted the difference between deformation work and complementary work and thus paved the way for the treatment of non-linear elastic systems in the theory of structures.

Heinrich Franz Bernhard MÜLLER-BRESLAU (1851 – 1925) placed the concept of work at the focus of the formulation of structural analysis theories and developed the force method for dealing with statically indeterminate systems. Robert LAND (1857 – 1899) created a method for determining influence lines based on a unit displacement imposed on the structural system at the position and in the direction of the relevant force variable. The development of the deformation method by Asger Skovgaard OSTENFELD (1866 – 1931) concluded the theory of elastic framed structures with small deformations.

The further evolution of the theory of structures in the 20th century primarily concerned plate and shell structures, stability theory, plastic theory and the development of computer-aided methods for analysing structures by means of discretised structural models.

3 DESIGN OF STRUCTURES

3.1 General

Fig. 3.1 [31] summarises the relationships between various design elements. The terminology in the figure is defined in appendix A1 (together with further specialist terminology that, generally, is highlighted in italics the first time it is used or explained in the text).

Fig. 3.1 applies to all *construction works* or their *structures* erected in the natural and built environments, i. e. all the structural members and all the subsoils that are necessary for their equilibrium and for retaining their form. The figure refers to the total life cycle of the construction works, which extends from *design* to *execution*, *use* and *conservation* right up to *deconstruction*. *Construction works documents* corresponding to the individual phases are listed in a separate column.

Fig. 3.1 and the associated terminology assist in understanding the subject and enable a uniform, systematic approach to theory and practice for all design, site management and construction work specialists engaged in the areas of structures and geotechnics. The figure is not a flow diagram, nor does it refer directly to the conventional course of a project from *preliminary design* to *tender design* and *detail design*. Rather, it gives an order to the steps in the process and the relationships between various design elements, and can be used to understand the connections between and the categorisation of the specialist terminology used.

The design of a structure encompasses the *conceptual design*, the *structural analysis* and the *dimensioning*. The conceptual design is all the activities and developments, and the outcomes thereof, that lead from the service criteria to the structural concept. The structural analysis uses structural models to determine action effects, i. e. the responses of the structure to potential actions as a result of execution and use as well as environmental influences. Dimensioning establishes the sizes, construction materials and detailing of the structure; the basis for this are structural and construction technology considerations plus numerical verifications.

The quality of a structure primarily depends on its conceptual design, its detailing and its execution. The importance of structural analyses and numerical verifications is often overrated; they are merely tools for guaranteeing an appropriate reliability, i. e. the behaviour of a structure with respect to structural safety and serviceability within specified limits.

Key aspects of conceptual design and the associated construction works documents (service criteria agreement and basis of design) are described below. Structural analysis and dimensioning are covered in chapter 4.

3.2 Conceptual design

The aim of the draft design is to develop a suitable *structural concept*, which specifies the structural system, the most important dimensions, construction material properties and construction details plus the intended method of construction. It is developed as part of the integrative planning of the construction works in consultation with all the specialists involved. The structural concept is based on the overall planning, the

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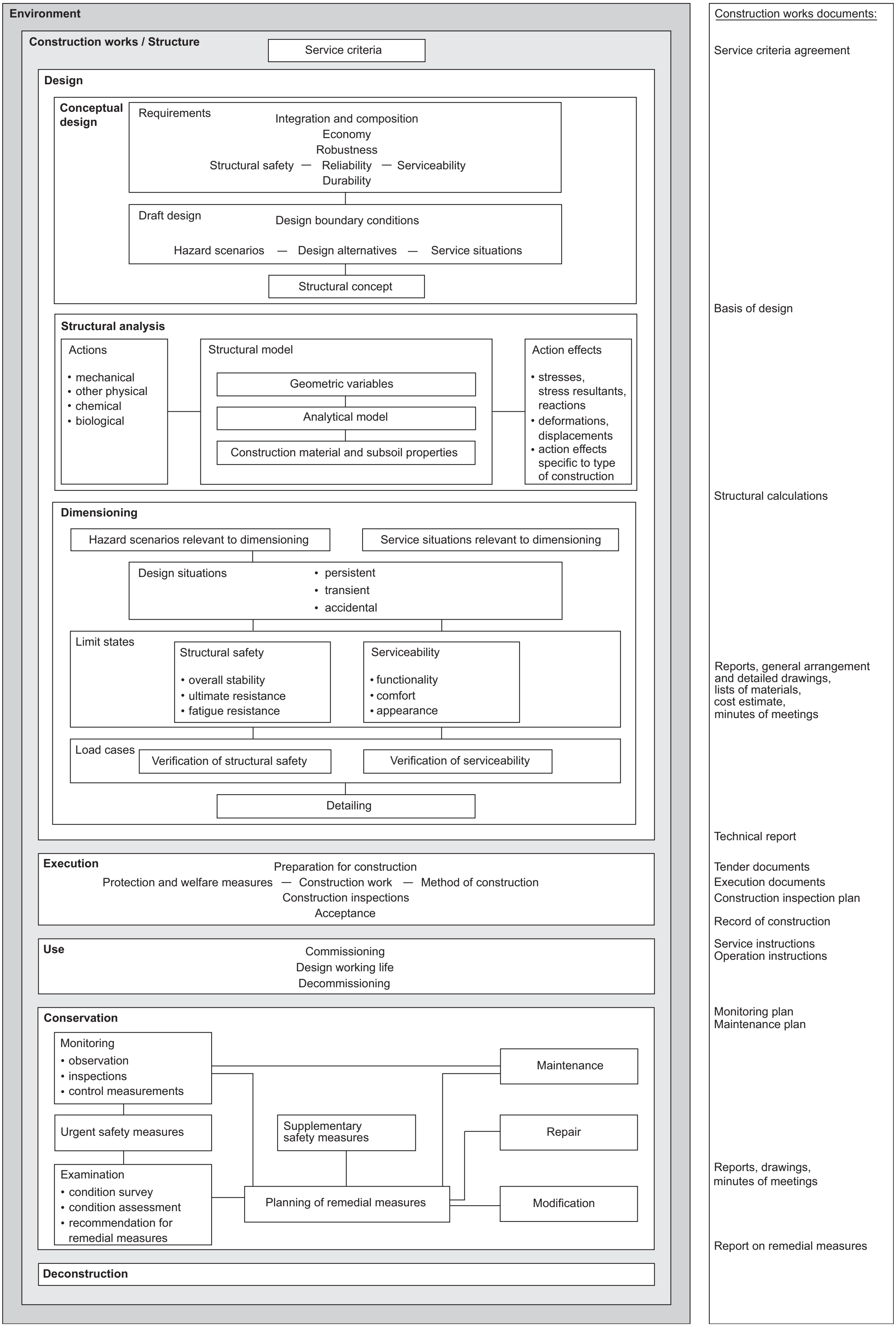

Fig. 3.1 Relationships between various design elements