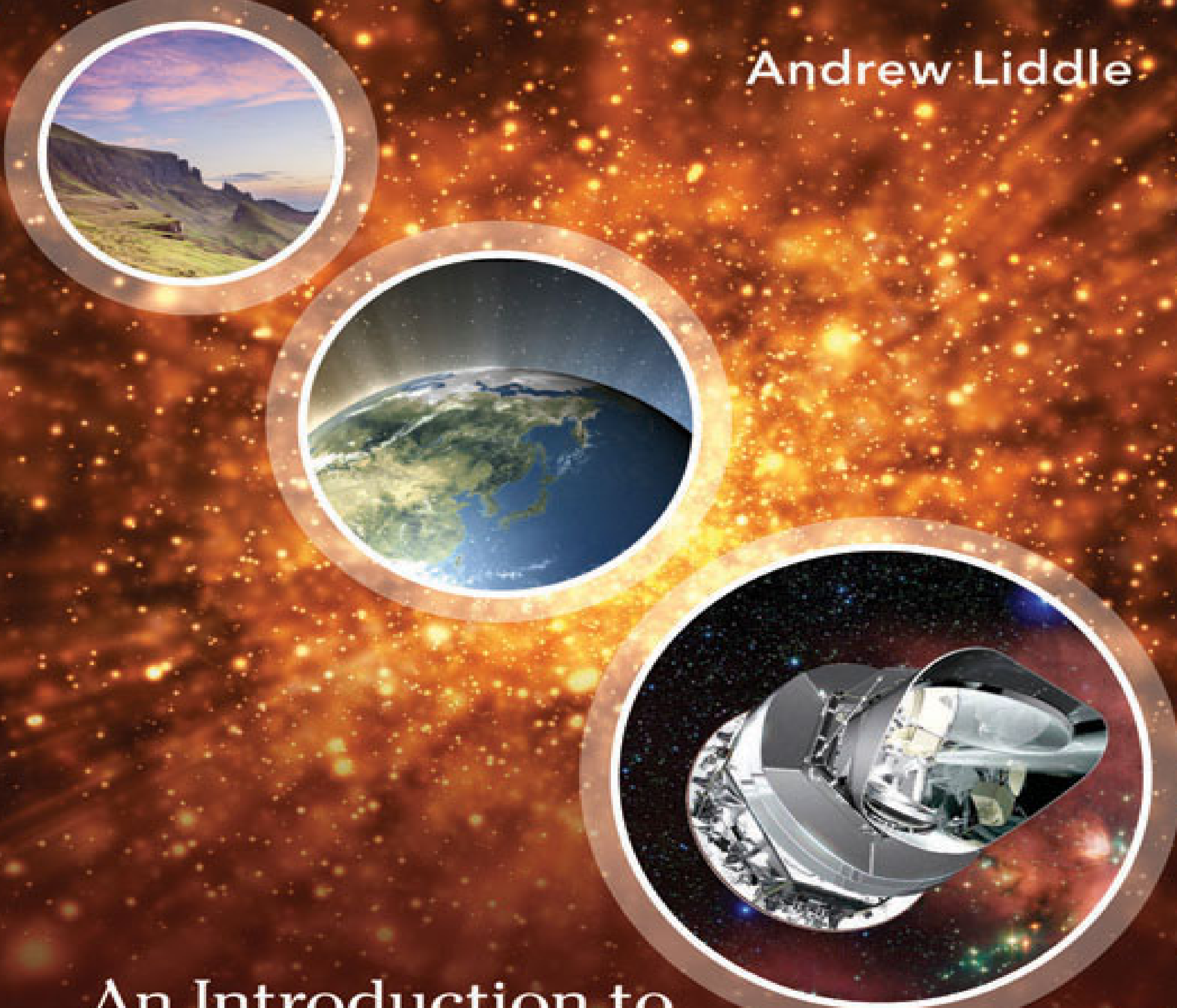
Andrew Liddle



An Introduction to MODERN COSMOLOGY

Third Edition

WILEY

An Introduction to Modern Cosmology

Third Edition

Andrew Liddle

Institute for Astronomy, University of Edinburgh, UK

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To my grandmothers

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Preface

The development of cosmology will no doubt be seen as one of the scientific triumphs of the twentieth century. At its beginning, cosmology hardly existed as a scientific discipline. By its end, the Hot Big Bang cosmology stood secure as the accepted description of the Universe as a whole. Telescopes such as the Hubble Space Telescope are capable of seeing light from galaxies so distant that the light has been travelling towards us for most of the lifetime of the Universe. The cosmic microwave background, a fossil relic of a time when the Universe was both denser and hotter, is routinely detected and its properties examined. That our Universe is presently expanding is established without doubt, and its material composition is accurately determined.

We are now in an era where understanding of cosmology has shifted from the qualitative to the quantitative. The turn of the millennium saw the establishment of what has come to be known as the Standard Cosmological Model, representing an almost universal consensus amongst cosmologists as to the best description of our Universe. Nevertheless, it is a model with a major surprise – the belief that our Universe is presently experiencing accelerated expansion. Add to that ongoing mysteries such as the properties of the so-called dark matter, which is believed to be the dominant form of matter in the Universe, and it is clear that we have some way to go before we can say that a full picture of the physics of the Universe is in our grasp.

Such a bold endeavour as cosmology easily captures the imagination, and there has been increasing demand for cosmology to be taught at university in an accessible manner. Traditionally, cosmology was taught, as it was to me, as the tail end of a general relativity course, with a derivation of the metric for an expanding universe and a few solutions. Such a course fails to capture the flavour of modern cosmology, which takes classic physical sciences like thermodynamics, atomic physics and gravitation and applies them on a grand scale.

In fact, introductory modern cosmology can be tackled in a different way, by avoiding general relativity altogether. By a lucky chance, and a subtle bit of cheating, the correct equations describing an expanding universe can be obtained from Newtonian gravity. From this basis, one can study all the triumphs of the Hot Big Bang cosmology – the expansion of the Universe, the prediction of its age, the existence of the cosmic microwave background, and the abundances of light elements such as helium and deuterium – and even go on to discuss more speculative ideas such as the inflationary cosmology.

The origin of this book, first published in 1998, was a short lecture course at the University of Sussex, around 20 lectures, taught to students in the final year of a bachelor’s degree or the penultimate year of a master’s degree. The prerequisites are all very standard physics, and the emphasis is aimed at physical intuition rather than mathematical rigour. It has been further developed through my experiences of teaching cosmology at Imperial College London, the University of Hawai‘i, and at the University of Edinburgh, plus much useful feedback from readers.

The structure of the book is a central ‘spine’, the main chapters from 1 to 15, which provide a self-contained introduction to modern cosmology. In addition there are six Advanced Topic chapters, each with prerequisites, which can be added to extend the course as desired. Ordinarily the best time to tackle those Advanced Topics is immediately after their prerequisites have been attained, though they could also be included at any later stage.

Cosmology is an interesting course to teach, as it is not like most of the other subjects taught in undergraduate physics courses. There is no perceived wisdom, built up over a century or more, which provides an unquestionable foundation, as in thermodynamics, electromagnetism, and even quantum mechanics and general relativity. Each successive edition of this book has introduced new discoveries, and lecturers can expect opportunities to crop up during their course to discuss new results which impact on cosmologists' views of the Universe.

You can follow my own evolving views by checking out this book's WWW Home Page at

http://www.roe.ac.uk/~arl/cosbook.html

There you may find some updates on observations, and also a list of any errors in the book that I am aware of. If you are confident you've found one yourself, and it's not on the list, I'd be very pleased to hear of it. Full-colour versions of many of the images can also be found there.

Andrew R. Liddle
Edinburgh
September 2014

Supplementary material, including full-colour images, updates and links for students and instructors, are available on the author's website:
http://www.roe.ac.uk/~arl/cosbook.html

Some fundamental constants

Newton's constant	G	6.672×10^{-11} m^3 kg^{-1} sec^{-2}
Speed of light	c	2.998×10^8 msec^{-1} or 3.076×10^{-7} Mpc yr^{-1}
Reduced Planck constant	$\hbar = h/2\pi$	1.055×10^{-34} m^2 kg sec^{-1}
Boltzmann constant	k_B	1.381×10^{-23} JK^{-1} or 8.619×10^{-5} eV K^{-1}
Radiation constant	$\alpha = \pi^2 k_B^4 / 15\hbar^3 c^3$	7.565×10^{-16} Jm^{-3} K^{-4}
Electron mass–energy	$m_e\, c^2$	0.511 MeV
Proton mass–energy	$m_p\, c^2$	938.3 MeV
Neutron mass–energy	$m_n\, c^2$	939.6 MeV
Thomson cross-section	σ_e	6.652×10^{-29} m^2
Free neutron half-life	t_{half}	610 sec

Some conversion factors

1 pc = 3.261 light years = 3.086×10^{16} m
1 yr = 3.156×10^7 sec
1 eV = 1.602×10^{-19} J
1 $M_\odot = 1.989 \times 10^{30}$ kg
1 J = 1 kg m^2 sec^{-2}
1 Hz = 1 sec^{-1}

Commonly-used symbols

z	redshift

H_0	Hubble constant
r	physical distance
v	velocity
f	frequency
T	temperature
k_B	Boltzmann constant
ϵ	energy density
α	radiation constant
G	Newton’s gravitational constant
ρ	mass density
a	scale factor
x	comoving distance
k	curvature
p	pressure
	(or occasionally momentum)
H	Hubble parameter
n, N	number density
h	Hubble constant
	(or Planck’s constant)
Ω_0	present density parameter
ρ_c	critical density
Ω	density parameter
Ω_k	curvature ‘density parameter’
q_0	deceleration parameter
Λ	cosmological constant
Ω_Λ	cosmological constant density parameter
t	time

t_0	present age
Ω_{B}	baryon density parameter
Y_4	helium abundance
d_{lum}	luminosity distance
d_{diam}	angular diameter distance
$\Delta T/T$, C_ℓ	cosmic microwave background anisotropies

Chapter 1
A Brief History of Cosmological Ideas

The cornerstone of modern cosmology is the belief that the place that we occupy in the Universe is in no way special. This is known as the **cosmological principle**, and it is an idea which is both powerful and simple. It is intriguing, then, that for the bulk of the history of civilization it was believed that we occupy a very special location, usually the centre, in the scheme of things.

The ancient Greeks, in a model further developed by the Alexandrian Ptolemy, believed that the Earth must lie at the centre of the cosmos. It would be circled by the Moon, the Sun and the planets, and then the ‘fixed’ stars would be yet further away. A complex combination of circular motions, Ptolemy’s Epicycles, was devised in order to explain the motions of the planets, especially the phenomenon of retrograde motion where planets appear to temporarily reverse their direction of motion. It was not until the early 1500s that Copernicus stated forcefully the view, initiated nearly two thousand years before by Aristarchus, that one should regard the Earth, and the other planets, as going around the Sun. By ensuring that the planets moved at different speeds, retrograde motion could easily be explained by this theory. However, although Copernicus is credited with removing the anthropocentric view of the Universe, which placed the Earth at its centre, he in fact believed that the Sun was at the centre.

Newton’s theory of gravity put what had been an empirical science (Kepler’s discovery that the planets moved on elliptical orbits) on a solid footing, and it appears that Newton believed that the stars were also suns pretty much like our own, distributed evenly throughout infinite space, in a static configuration. However it seems that Newton was aware that such a static configuration is unstable.

Over the next two hundred years, it became increasingly understood that the nearby stars are not evenly distributed, but rather are located in a disk-shaped assembly which we now know as the Milky Way galaxy. The Herschels were able to identify the disk structure in the late 1700s, but their observations were not perfect and they wrongly concluded that the Solar System lay at its centre. Only in the early 1900s was this convincingly overturned, by Shapley, who realized that we are some two-thirds of the radius away from the centre of the Galaxy. Even then, he apparently still believed our galaxy to be at the centre of the Universe. Only in 1952 was it finally demonstrated, by Baade, that the Milky Way is a fairly typical galaxy, leading to the modern view, known as the **cosmological principle** (or sometimes the Copernican principle), that the Universe looks the same whoever and wherever you are.

It is important to stress that the cosmological principle isn't exact. For example, no one thinks that sitting in a lecture theatre is exactly the same as sitting in a bar, and the interior of the Sun is a very different environment from the interstellar regions. Rather, it is an approximation that we believe holds better and better the larger the length scales we consider. Even on the scale of individual galaxies it is not very good, but once we take very large regions (though still much smaller than the Universe itself), containing say a million galaxies, we expect every such region to look more or less like every other one. The cosmological principle is therefore a property of the global Universe, breaking down if one looks at local phenomena.

The cosmological principle is the basis of the Big Bang Cosmology. The Big Bang is the best description we have of our Universe, and the aim of this book is to explain why. The Big Bang is a picture of our Universe as an evolving entity, which was very different in the past as compared to the present. Originally, it was forced to compete with a rival idea, the Steady State Universe, which holds that the Universe does not evolve but rather has looked the same forever, with new material being created to fill the gaps as the Universe expands. However, the observations I will describe now support the Big Bang so strongly that the Steady State theory is almost never considered.

Chapter 2
Observational Overview

For most of history, astronomers have had to rely on light in the visible part of the spectrum in order to study the Universe. One of the great astronomical achievements of the 20th century was the exploitation of the full electromagnetic spectrum for astronomical measurements. We now have instruments capable of making observations of radio waves, microwaves, infrared light, visible light, ultraviolet light, X-rays and gamma rays, which all correspond to light waves of different (in this case increasing) frequency. We are even entering an epoch where we can go beyond the electromagnetic spectrum and receive information of other types. A remarkable feature of observations of a nearby supernova in 1987 was that it was also seen through detection of neutrinos, an extraordinarily weakly interacting type of particle normally associated with radioactive decay. Very high energy cosmic rays, consisting of highly-relativistic elementary particles, are now routinely detected, though as yet there is no clear understanding of their astronomical origin. And as I write, experiments appear close to the goal of directly detecting gravitational waves, ripples in space–time itself, and ultimately of using them to observe astronomical events such as colliding stars.

The advent of large ground-based and satellite-based telescopes operating in all parts of the electromagnetic spectrum has revolutionized our picture of the Universe. While there are probably gaps in our knowledge, some of which may be important for all we know, we do seem to have a consistent picture, based on the cosmological principle, of how material is distributed in the Universe. My discussion here is brief; for a much more detailed discussion of the observed Universe, see Rowan-Robinson's book *Cosmology* (full reference in the Bibliography). A set of images, including full-colour versions of the figures in this chapter, can be found via the book's Home Page as given in the Preface.

2.1 In visible light

Historically, our picture of the Universe was built up through ever more careful observations using visible light.

Stars: The main source of visible light in the Universe is nuclear fusion within stars. The Sun is a fairly typical star, with a mass of about 2×10^{30} kilograms. This is known as a solar mass, indicated $M_{\odot}$, and is a convenient unit for measuring masses. The nearest stars to us are a few light years away, a light year being the distance (about 10^{16} metres) that light can travel in a year. For historical reasons, an alternative unit, known as the **parsec** and denoted 'pc',[1] is more commonly used in cosmology. A parsec equals 3.261 light years. In cosmology, one seldom considers individual stars, instead preferring to adopt as the smallest considered unit the conglomerations of stars known as ...

Galaxies: Our Solar System lies some way off-centre in a giant disk structure known as the Milky Way galaxy. It contains a staggering hundred thousand million (10^{11}) or so stars, with masses ranging from about a tenth that of our Sun to tens of times larger. It consists of a central bulge, plus a disk of radius 12.5 kiloparsecs (kpc, equal to 10^3 pc) and a thickness of only about 0.3 kpc. We are located in the disk about 8 kpc from the centre. The disk rotates slowly (and also differentially, with the outer edges moving more slowly, just as more distant planets in the Solar System orbit more slowly). At our radius, the galaxy rotates with a period of 200 million years. Because we are within it, we can't get an image of our own galaxy, but it probably looks not unlike the M100 galaxy shown in Figure 2.1.

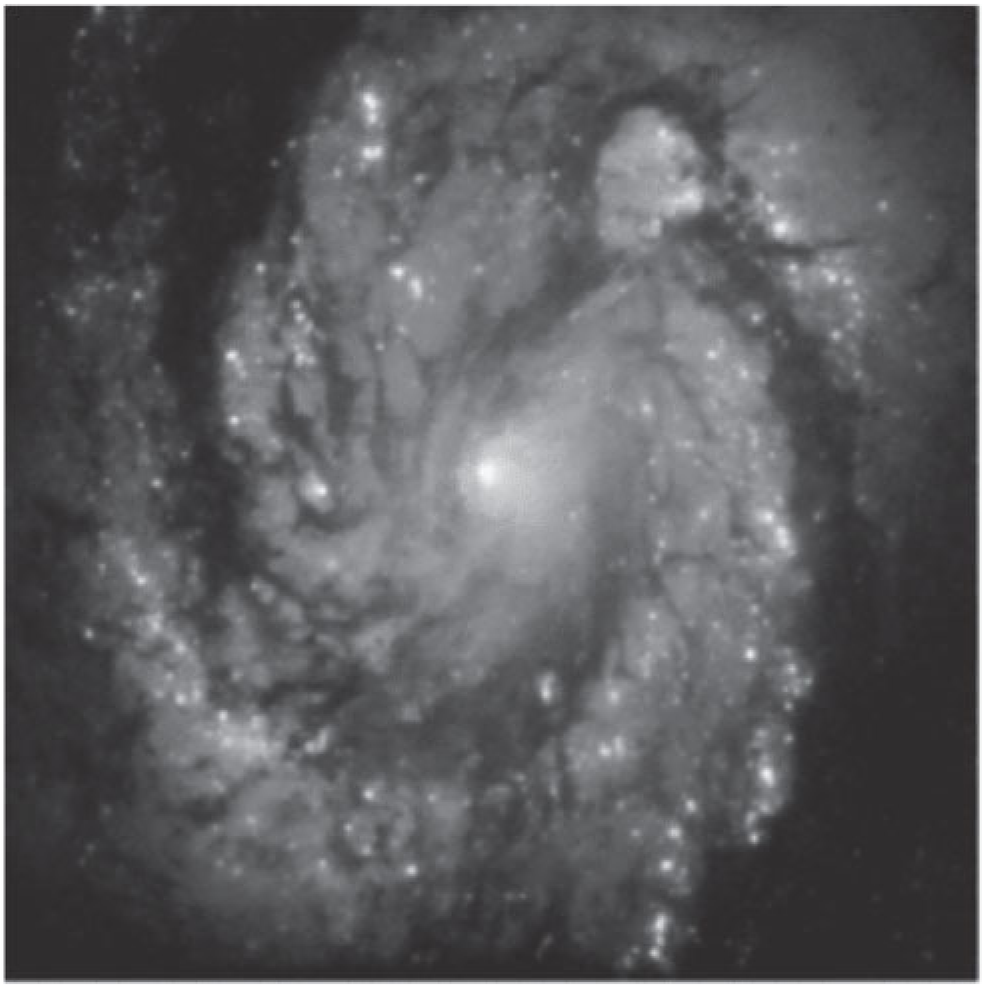

Figure 2.1 If viewed from above the disk, our own Milky Way galaxy would probably resemble the M100 galaxy, imaged here by the Hubble Space telescope. (Figure courtesy NASA.)

Our galaxy is surrounded by smaller collections of stars, known as globular clusters. These are distributed more or less symmetrically about the bulge, at distances of 5–30 kpc. Typically they contain a million stars, and are thought to be remnants of the formation of the galaxy. As we shall discuss later, it is believed that the entire disk and globular cluster system is embedded in a larger spherical structure known as the Galactic halo.

Galaxies are the most visually striking and beautiful astronomical objects in the Universe, exhibiting a wide range of properties. However, in cosmology the detailed structure of a galaxy is usually irrelevant, and galaxies are normally thought of as point-like objects emitting light, often broken into sub-classes according to colours, luminosities and morphologies.

The local group: Our galaxy resides within a small concentrated group of galaxies known as the local group. The nearest galaxy is a small irregular galaxy known as the Large Magellanic Cloud (LMC), which is 50 kpc away from the Sun. The nearest galaxy of similar size to our own is the Andromeda Galaxy, at a distance of 770 kpc. The Milky Way is one of the largest galaxies in the local group. A typical galaxy group occupies a volume of a few cubic **megaparsecs**. The megaparsec, denoted Mpc and equal to a million parsecs, is the cosmologist's favourite unit for measuring distances, because it is roughly the separation between neighbouring galaxies. It equals 3.086×10^{22} metres.

Clusters of galaxies, superclusters and voids: Surveying larger regions of the Universe, say on a scale of hundreds of Mpc, one sees a variety of large-scale structures, as shown in Figure 2.2. This figure is not a photograph, but rather a carefully constructed map of the nearby region of our Universe, on a scale of about $1{:}10^{27}$! In some places galaxies are clearly grouped into clusters of galaxies.

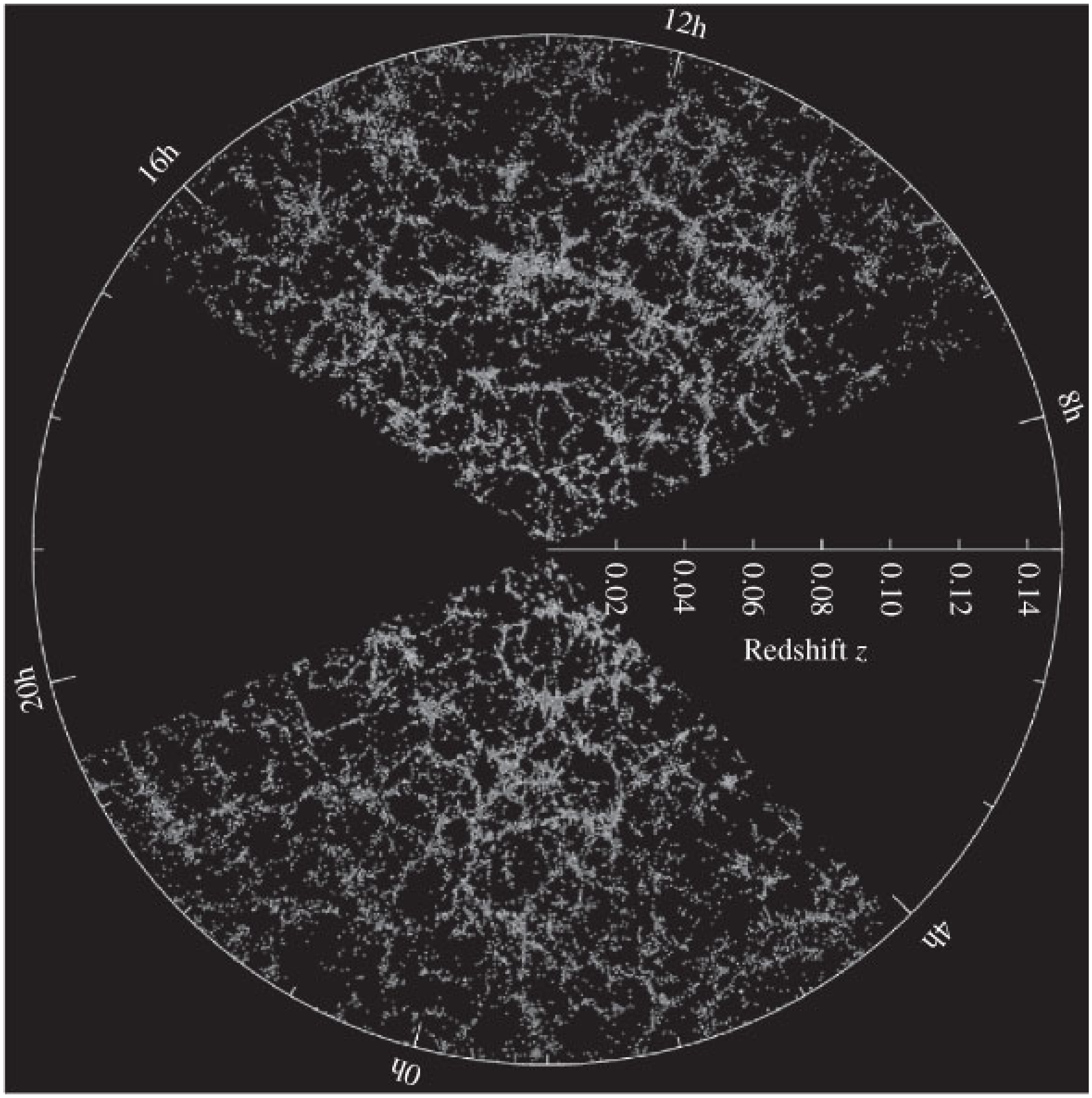

Figure 2.2 A map of galaxy positions in a narrow slice of the Universe, as measured by the Sloan Digital Sky Survey. Our galaxy is located at the centre, and the survey radius is around 600 Mpc. The galaxy positions were obtained by measurement of the shift of spectral lines, as described in Section 2.4. (Figure courtesy M. Blanton and the Sloan Digital Sky Survey, www.sdss3.org.)

A famous example of a cluster of galaxies is the Coma cluster, which is about 100 Mpc away from our own Galaxy. The upper panel of Figure 2.3 shows a combined optical/infrared image of Coma; although the image resembles a star field, almost every source is a distinct galaxy (the main exception being two bright stars in the upper right quadrant). Coma contains perhaps 10 000 galaxies, mostly too faint to show in this image, orbiting in their common gravitational field.

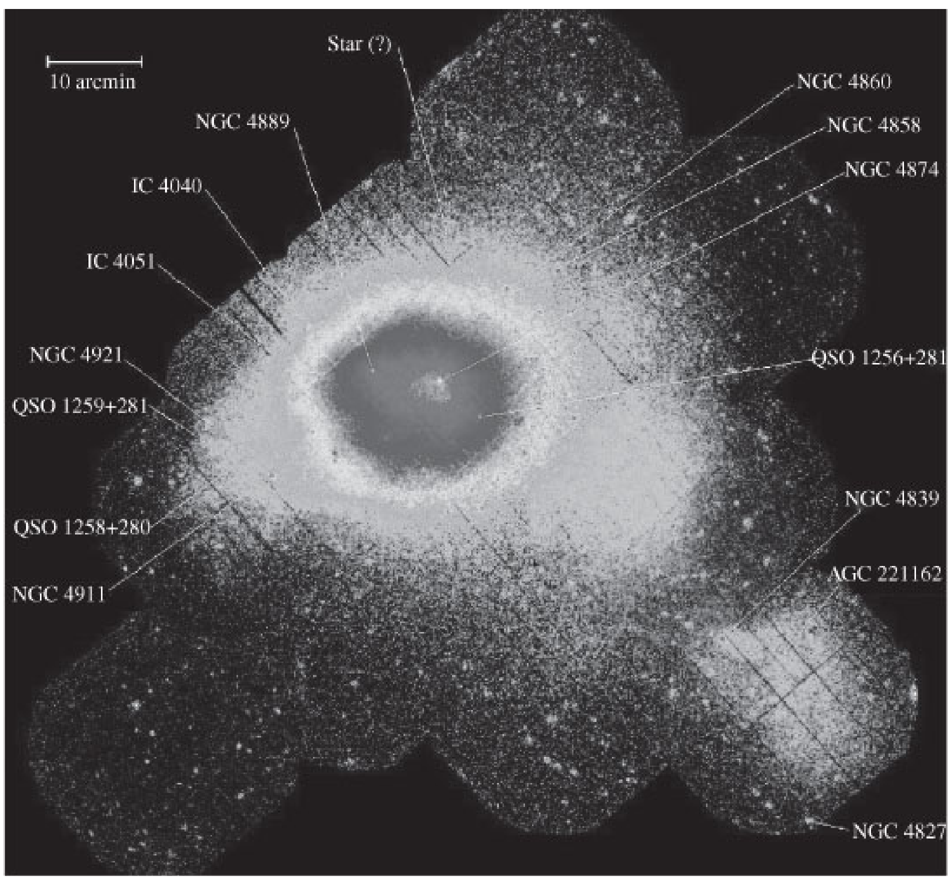
10 arcmin
Star (?)
NGC 4860
NGC 4858
NGC 4874
NGC 4889
IC 4040
IC 4051
NGC 4921
QSO 1256+281
QSO 1259+281
QSO 1258+280
NGC 4911
NGC 4839
AGC 221162
NGC 4827

Figure 2.3 Images of the Coma cluster of galaxies in visible/infrared light (top) and in X-rays (bottom), the latter being on a larger angular scale. Colour versions can be found on the book's WWW site. (Images courtesy of NASA/Spitzer satellite and ESA/U. Briel/MPE Garching/XMM-Newton satellite.)

However, most galaxies, sometimes called field galaxies, are not part of a cluster. Galaxy clusters are the largest gravitationally-collapsed objects in the Universe, and they themselves are grouped into superclusters, joined by filaments and walls of galaxies. In between this 'foamlike' structure lie large voids, some as large as 50 Mpc across. Structures in the Universe will be further described in Advanced Topic 5. Figure 2.4 shows an example of a computer simulation aiming to model the distribution of material within the Universe.