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Variance-Constrained Multi-Objective Stochastic Control and Filtering

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Preface

Nonlinearity and stochasticity are arguably two of the main resources in reality that have resulted in considerable system complexity. Therefore, recently, control and filtering of nonlinear stochastic systems have been an active branch within the general research area of nonlinear control problems. In engineering practice, it is always desirable to design systems capable of simultaneously guaranteeing various performance requirements to meet the ever-increasing practical demands toward the simultaneous satisfaction of performances such as stability, robustness, precision, and reliability, among which the system covariance plays a vital role in system analysis and synthesis due to the fact that several design objectives, such as stability, time-domain and frequency-domain performance specifications, robustness, and reliability, can be directly related to steady-state covariance of the closed-loop systems.

In this book, we discuss the multi-objective control and filtering problems for a class of nonlinear stochastic systems with variance constraints. The stochastic nonlinearities taken into consideration are quite general and could cover several classes of well-studied nonlinear stochastic systems. The content of this book is divided mainly into two parts. In the first part, we focus on the variance-constrained control and filtering problems for time-invariant nonlinear stochastic systems subject to different kinds of complex situations, including measurements missing, actuator failures, output degradation, etc. Some sufficient conditions are derived for the existence of the desired controllers and filters in terms of the linear matrix inequalities (LMIs). The control and filtering problems with multiple performance specifications are considered in the second part for time-varying nonlinear stochastic systems. In this part, several design techniques including recursive linear matrix inequalities (RLMIs), game theory, and gradient method have been employed to develop the desired controllers and filters capable of simultaneously achieving multiple pre-specified performance requirements.

The compendious frame and description of the book are given as follows: Chapter 1 introduces the recent advances on variance-constrained multi-objective control and filtering problems for nonlinear stochastic systems and the outline of the book. Chapter 2 is concerned with the H_∞ control problem for a class of nonlinear stochastic systems with variance constraints. Chapter 3 deals with the mixed H_2/H_∞ filtering problem for a type of time-invariant nonlinear stochastic systems.

In Chapter 4, the variance-constrained filtering problem is solved in the case of missing measurements. Chapter 5 discusses the controller design problem with variance constraints when the actuator is confronted with possible failures. The sliding mode control problem is investigated in Chapter 6 for a class of nonlinear discrete-time stochastic systems with H_2 specification. In Chapter 7, the dissipativity performance is taken into consideration with variance performance and the desired control scheme is given. For a special type of nonlinear stochastic system, namely, systems with multiplicative noises, Chapter 8 deals with the robust controller design problem with simultaneous consideration of variance constraints and H_∞ requirement. For time-varying nonlinear stochastic systems, Chapters 9 and 10 investigate the H_∞ control and filtering problems over a finite horizon, respectively. Chapters 11 and 12 discuss the mixed H_2/H_∞ control problems, taking the randomly occurring nonlinearities (RONs) and Markovian jump parameters into consideration, respectively. Chapters 13 and 14 give the solutions to the multi-objective control problems for time-varying nonlinear stochastic systems in the presence of sensor and actuator failures, respectively. Chapter 15 gives the conclusions and some possible future research topics.

This book is a research monograph whose intended audience is graduate and postgraduate students as well as researchers.

Series Preface

Electromechanical Systems permeate the engineering and technology fields in aerospace, automotive, mechanical, biomedical, civil/structural, electrical, environmental, and industrial systems. The Wiley Book Series on dynamics and control of electromechanical systems covers a broad range of engineering and technology in these fields. As demand increases for innovation in these areas, feedback control of these systems is becoming essential for increased productivity, precision operation, load mitigation, and safe operation. Furthermore, new applications in these areas require a reevaluation of existing control methodologies to meet evolving technological requirements. An example involves distributed control of energy systems. The basics of distributed control systems are well documented in several textbooks, but the nuances of its use for future applications in the evolving area of energy system applications, such as wind turbines and wind farm operations, solar energy systems, smart grids, and energy generation, storage and distribution, require an amelioration of existing distributed control theory to specific energy system needs. The book series serves two main purposes: (1) a delineation and explication of theoretical advancements in electromechanical system dynamics and control and (2) a presentation of application driven technologies in evolving electromechanical systems.

This book series embraces the full spectrum of dynamics and control of electromechanical systems from theoretical foundations to real world applications. The level of the presentation should be accessible to senior undergraduate and first-year graduate students, and should prove especially well suited as a self-study guide for practicing professionals in the fields of mechanical, aerospace, automotive, biomedical, and civil/structural engineering. The aim is to provide an interdisciplinary series ranging from high-level undergraduate/graduate texts, explanation and dissemination of science and technology and good practice, through to important research that is immediately relevant to industrial development and practical applications. It is hoped that this new and unique perspective will be of perennial interest to students, scholars, and employees in these engineering disciplines. Suggestions for new topics and authors for the series are always welcome.

This book, *Variance-Constrained Multi-Objective Stochastic Control and Filtering*, has the objective of providing a theoretical foundation as well as practical insights on the topic at hand. It is broken down into two essential parts: (1) variance-constrained control and filtering problems for time-invariant nonlinear stochastic systems and (2) designing controllers and filters capable of simultaneously achieving multiple pre-specified performance requirements. The book is accessible to readers who have a basic understanding of stochastic processes, control, and filtering theory. It provides detailed derivations from first principles to allow the reader to thoroughly understand the particular topic. It also provides several illustrative examples to bridge the gap between theory and practice. This book is a welcome addition to the Wiley Electromechanical Systems Series because no other book is focused on the topic of stochastic control and filtering with a specific emphasis on variance-constrained multi-objective systems.

Mark J. Balas, John L. Crassidis, and Florian Holzapfel

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List of Abbreviations

$\mathbb{R}^n$	The n-dimensional Euclidean space.
$\mathbb{I}^+$	The set of non-negative integers.
$\mathbb{R}^{n\times m}$	The set of all $n \times m$ real matrices.
$\lvert \cdot \rvert$	The Euclidean norm in $\mathbb{R}^n$.
$L_2[0, \infty)$	The space of square-integrable vector functions over $[0, \infty)$.
$\rho(A)$	The spectral radius of matrix A.
$\lambda(A)$	The eigenvalue of matrix A.
$\mathrm{tr}(A)$	The trace of matrix A.
$\otimes$	The Kronecker product of matrices.
$\mathrm{st}(A)$	The stack that forms a vector out of the columns of matrix A.
$\lVert a\rVert_A^2$	Equal to $a^{\mathrm{T}}Aa$.
$\mathrm{Prob}\{\cdot\}$	The occurrence probability of the event "·".
$\mathbb{E}\{x\}$	The expectation of stochastic variable x.
$\mathbb{E}\{x \mid y\}$	The expectation of x conditional on y, x, and y are both stochastic variables.
I	The identity matrix of compatible dimension.
$X > Y$	The $X - Y$ is positive definite, where X and Y are symmetric matrices.
$X \geq Y$	The $X - Y$ is positive semi-definite, where X and Y are symmetric matrices.
M^{T}	The transpose matrix of M.
$\mathrm{diag}\{M_1, \dots, M_n\}$	The block diagonal matrix with diagonal blocks being the matrices $M_1, \dots, M_n$.
$*$	The ellipsis for terms induced by symmetry, in symmetric block matrices.

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1

Introduction

It is widely recognized that in almost all engineering applications, nonlinearities are inevitable and could not be eliminated thoroughly. Hence, the nonlinear systems have gained more and more research attention, and many results have been published. On the other hand, due to the wide appearance of stochastic phenomena in almost every aspect of our daily lives, stochastic systems that have found successful applications in many branches of science and engineering practice have stirred quite a lot of research interest during the past few decades. Therefore, control and filtering problems for nonlinear stochastic systems have been studied extensively in order to meet an ever-increasing demand toward systems with both nonlinearities and stochasticity.

In many engineering control/filtering problems, the performance requirements are naturally expressed by the upper bounds on the steady-state covariance, which is usually applied to scale the control/estimation precision, one of the most important performance indices of stochastic design problems. As a result, a large number of control and filtering methodologies have been developed to seek a convenient way to solve the variance-constrained design problems, among which the linear quadratic Gaussian(LQG) control and Kalman filtering are two representative minimum variance design algorithms.

On the other hand, in addition to the variance constraints, real-world engineering practice also desires the simultaneous satisfaction of many other frequently seen performance requirements, including stability, robustness, reliability, energy constraints, to name but a few key ones. This gives rise to the so-called multi-objective design problems, in which multiple cost functions or performance requirements are simultaneously considered with constraints being imposed on the system. An example of multi-objective control design would be to minimize the system steady-state variance indicating the performance of control precision, subject to a pre-specified external disturbance attenuation level evaluating system robustness. Obviously, multi-objective design methods have the ability to provide more flexibility in dealing with the trade-offs and constraints in a much more explicit manner on the pre-specified performance

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requirements than those conventional optimization methodologies like the LQG control scheme or H_∞ design technique, which do not seem to have the ability of handling multiple performance specifications.

When coping with the multi-objective design problem with variance constraints for stochastic systems, the well-known covariance control theory provides us with a useful tool for system analysis and synthesis. For linear stochastic systems, it has been shown that multi-objective control/filtering problems can be formulated using linear matrix inequalities (LMIs), due to their ability to include desirable performance objectives such as variance constraints, H_2 performance, H_∞ performance, and pole placement as convex constraints. However, as nonlinear stochastic systems are concerned, the relevant progress so far has been very slow due primarily to the difficulties in dealing with the variance-related problems resulting from the complexity of the nonlinear dynamics. A key issue for the nonlinear covariance control study is the existence of the covariance of nonlinear stochastic systems and its mathematical expression, which is extremely difficult to investigate because of the complex coupling of nonlinearities and stochasticity. Therefore, it is not surprising that the multi-objective control and filtering problems for nonlinear stochastic systems with variance constraints have not been adequately investigated despite the clear engineering insights and good application prospect.

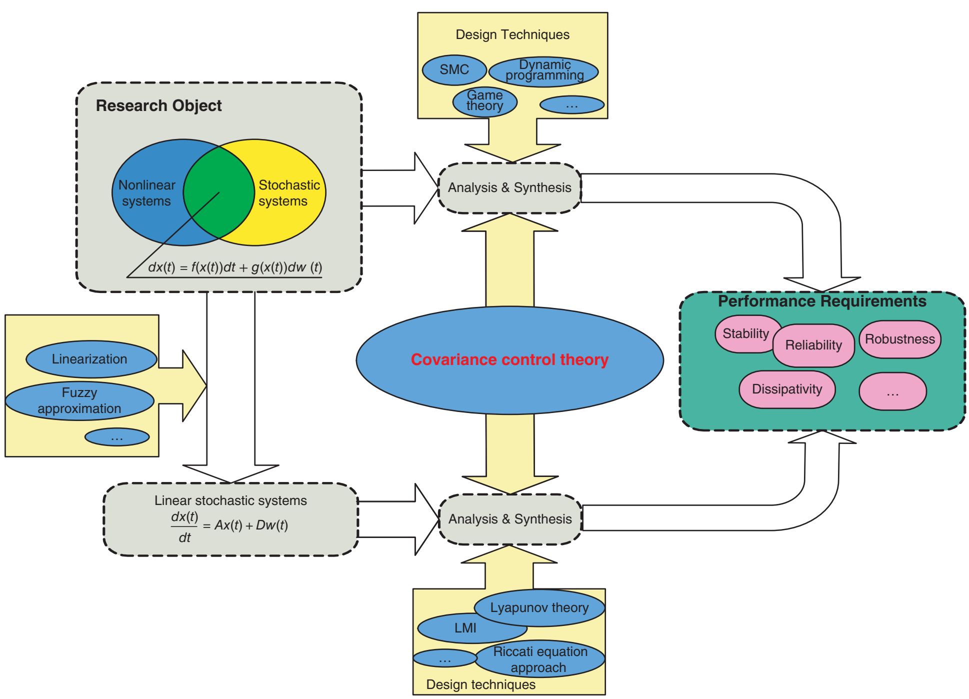

Figure 1.1 Architecture of surveyed contents.

In this chapter, we focus mainly on the multi-objective control and filtering problems for nonlinear systems with variance constraints and aim to give a survey on some recent advances in this area. We shall give a comprehensive discussion from three aspects, i.e., design objects (nonlinear stochastic system), design requirements (multiple performance specifications including variance constraints), and several design techniques. Then, as a special case of the addressed problem, mixed H_2/H_∞ design problems have been discussed in great detail with some recent advances. Subsequently, the outline of this book is given. The contents that are reviewed in this chapter and the architecture are shown in Figure. 1.1.

1.1 Analysis and Synthesis of Nonlinear Stochastic Systems

For several decades, nonlinear stochastic systems have been attracting increasing attention in the system and control community due to their extensive applications in a variety of areas ranging from communication and transportation to manufacturing, building automation, computing, automotive, and chemical industries, to mention just a few key areas. In this section, the analysis and synthesis problems for nonlinear systems and stochastic systems are recalled respectively, and some recent advances in these areas are also given.

1.1.1 Nonlinear Systems

It is well recognized that in almost all engineering applications, nonlinearities are inevitable and could not be eliminated thoroughly. Hence, nonlinear systems have gained more and more research attention, and many results have been reported; see, for example, Refs [1–3]. When analyzing and designing nonlinear dynamical systems, there are a wide range of nonlinear analysis tools, among which the most common and widely used is linearization because of the powerful tools we know for linear systems. It should be pointed out that, however, there are two basic limitations of linearization [4]. (1) As is well known, linearization is an approximation in the neighborhood of certain operating points. Thus, the resulting linearized system can only show the local behavior of the nonlinear system in the vicinity of those points. Neither nonlocal behavior of the original nonlinear system far away from those operating points nor global behavior throughout the entire state space can be correctly revealed after linearization. (2) The dynamics of a nonlinear system are much richer than that of a linear system. There are essentially nonlinear phenomena, like finite escape time, multiple isolated equilibria, subharmonic, harmonic or almost periodic oscillations, to name just a few key ones that can take place only in the presence of nonlinearity; hence, they cannot be described by linear models [5–8]. Therefore, as a compromise, during the past few decades, there has been tremendous interest in studying nonlinear systems, with nonlinearities being taken as the exogenous disturbance input to a linear system, since it could better illustrate the dynamics of the original nonlinear system than the linearized one with less sacrifice of the convenience on the application of existing mathematical tools. The nonlinearities emerging in such systems may arise from the linearization process of an originally highly nonlinear plant or may be an external nonlinear input, which would drastically degrade the system performance or even cause instability; see, for example, Refs [9–11].

On the other hand, in real-world applications, one of the most inevitable and physically important features of some sensors and actuators is that they are always corrupted by different kinds of nonlinearities, either from within the device themselves or from the external disturbances. Such nonlinearities generally result from equipment limitations as well as the harsh environments such as uncontrollable elements (e.g., variations in flow rates, temperature, etc.) and aggressive conditions (e.g., corrosion, erosion, and fouling, etc.) [12]. Since the sensor/actuator nonlinearity cannot be simply ignored and often leads to poor performance of the controlled system, a great deal of effort in investigating the analysis and synthesis problems has been devoted by many researchers to the study of various systems with sensor/actuator nonlinearities; see Refs [13–18].

Recently, the systems with randomly occurring nonlinearities (RONs) have started to stir quite a lot of research interest as it reveals an appealing fact that, instead of occurring in a deterministic way, a large amount of nonlinearities in real-world systems would probably take place in a random way. Some of the representative publications can be discussed as follows. The problem of randomly occurring nonlinearities was raised in Ref. [19], where an iterative filtering algorithm has been proposed for the stochastic nonlinear system in the presence of both RONs and output quantization effects. The filter parameters can be obtained by resorting to solving certain recursive linear matrix inequalities. The obtained results have quickly been extended to the case of multiple randomly occurring nonlinearities [20]. Such a breakthrough on how to deal with nonlinear systems with RONs has been well recognized and quickly followed by other researchers in the area. Using similar techniques, the filtering as well as control problems have been solved for a wide range of systems containing nonlinearities that are occurring randomly, like Markovian jump systems [21, 22], sliding mode control systems [23], discrete-time complex networks [24], sensor networks [25], time-delay systems [26], and other types of nonlinear systems [27–29], which therefore has proven that the method developed in Ref. [19] is quite general and is applicable to the analysis and synthesis of many different kinds of nonlinear systems.

It should be emphasized that, for nonlinearities, there are many different constraint conditions for certain aims, such as Lipschitz conditions, among which the kind of stochastic nonlinearities described by statistical means has drawn particular research focus since it covers several well-studied nonlinearities in stochastic systems; see Refs [27, 30–33] and the references therein. Several techniques for analysis and synthesis of this type of nonlinear system have been exploited, including the linear matrix inequality approach [30], the Riccati equation method [31], the recursive matrix inequality approach [32], gradient method [33], sliding mode control scheme [34], and the game theory approach [27].

1.1.2 Stochastic Systems

As is well known, in the past few decades there have been extensive studies and applications of stochastic systems because the stochastic phenomenon is inevitable and cannot be avoided in real-world systems. When modeling such kinds of systems, the way of neglecting the stochastic disturbances, which is a conventional technique in traditional control theory for deterministic systems, is no longer suitable. Having realized the necessity of introducing more realistic models, nowadays a great number of real-world systems such as physical systems, financial systems, ecological systems, as well as social systems are more suitable to be modeled by stochastic systems. Therefore, the stochastic control problem, which deals with dynamical systems described by difference or differential equations, and subject to disturbances characterized as stochastic processes, has drawn much research attention; see Ref. [35] and the references therein. It is worth mentioning that a kind of stochastic system represented as a deterministic system adding a stochastic disturbance characterized as white noise has gained special research interest and found extensive applications in engineering based on the fact that it is possible to generate stochastic processes with covariance functions belonging to a large class simply by sending white noise through a linear system; hence a large class of problems can be reduced to the analysis of linear systems with white noise inputs; see Refs [36–40] for examples.

Parallel to the control problems, the filtering and prediction theory for stochastic systems, which aims to extract a signal from observations of signal and disturbances, has been well studied and found to be widely applied in many engineering fields. It also plays a very important role in the solution of the stochastic optimal control problem. Research on the filtering problem originated in Ref. [41], where the well-known Wiener–Kolmogorov filter has been proposed. However, the Wiener–Kolmogorov filtering theory has not been widely applied mainly because it requires the solution of an integral equation (the Wiener–Hopf equation), which is not easy to solve either analytically or numerically. In Refs [42, 43], Kalman and Bucy gave a significant contribution to the filtering problem by giving the celebrated Kalman–Bucy filter, which could solve the filtering problem recursively. The Kalman–Bucy filter (also known as the H_2 filter) has been extensively adopted and widely used in many branches of stochastic control theory, due to the fast development of digital computers recently; see Refs [44–47] and the references therein.

1.2 Multi-Objective Control and Filtering with Variance Constraints

In this section, we first review the covariance control theory, which provides us with a powerful tool in variance-constrained design problems with multiple requirements specified by engineering practice. Then, we discuss several important performance specifications including robustness, reliability, and dissipativity. Two common techniques for solving the addressed problems for nonlinear stochastic systems are introduced. The mixed H_2/H_∞ design problem is reviewed in great detail as a special case of the multi-objective control/filtering problem with variance constraints.

1.2.1 Covariance Control Theory

As we have stated in the previous section, engineering control problems always require upper bounds on the steady-state covariances [39, 48, 49]. However, many control design techniques used in both theoretical analysis and engineering practice, such as LQG and H_∞ design, do not seem to give a direct solution to this kind of design problem since they lack a convenient avenue for imposing design objectives stated in terms of upper bounds on the variance values. For example, the LQG controllers minimize a linear quadratic performance index without guaranteeing the variance constraints with respect to individual system states. The covariance control theory [50] developed in late 1980s has provided a more direct methodology for achieving the individual variance constraints than the LQG control theory. The covariance control theory aims to solve the variance-constrained control problems while satisfying other performance indices [38, 45, 50, 51]. It has been shown that the covariance control approach is capable of solving multi-objective design problems, which has found applications in dealing with transient responses, round-off errors in digital control, residence time/probability in aiming control problems, and stability and robustness in the presence of parameter perturbations [51]. Such an advantage is based on the fact that several control design objectives, such as stability, time-domain and frequency-domain performance specifications, robustness and pole location, can be directly related to steady-state covariances of the closed-loop systems. Therefore, covariance control theory serves as a practical method for multi-objective control design as well as a foundation for linear system theory.

On the other hand, it is always the case in real-world applications such as the tracking of a maneuvering target that the filtering precision is characterized by the error variance of estimation [51, 52]. Considering its clear engineering insights, in the past few years the filtering problem with error variance constraints has received much interest and a great many research findings have been reported in the literature [42, 43, 53, 54]. The celebrated Kalman filtering approach is a typical method that aims to obtain state information based on the minimization of the variance of the estimation error [42, 43]. Nevertheless, the strict request of a highly accurate model seriously impedes the application of Kalman filtering as in many cases only an approximate model of the system is available. It therefore has brought about remarkable research interest to the robust filtering method, which aims to minimize the error variance of estimation against the system uncertainties or external unknown disturbances [55, 56]. Despite certain merits and successful applications, as in the case of the LQG control problem, the traditional minimum variance filtering techniques cannot directly impose the designing objectives stated in terms of upper bounds on the error variance values, by which we mean that those techniques try to minimize the filtering error variance in a mean-square sense rather than to constrain it within a pre-specified bound, which is obviously better able to meet the requirements of practical engineering. Motivated by the covariance control theory, in Ref. [57] the authors have proposed a more direct design procedure for achieving the individual variance constraint in filtering problems. Due to its design flexibility, the covariance control theory is capable of solving the error variance-constrained filtering problem while guaranteeing other multiple designing objectives [58]. Therefore, it always serves as one of the most powerful tools in dealing with multi-objective filtering as well as control problems [59].

It should be pointed out that most available literature regarding covariance control theory has been concerned with *linear time invariant* stochastic systems using the linear matrix inequality (LMI) approach. Moreover, when it comes to the variance-constrained controller/filter design problems for much more complicated systems such as *time-varying systems, nonlinear systems, Markovian Jump systems*, etc., unfortunately, the relevant results have been very few due primarily to the difficulties in dealing with the existence problem of the steady-state covariances and their mathematical expressions for those above-mentioned systems with complex dynamics. With the hope of resolving such difficulties, in recent years, special effort has been devoted to studying variance-constrained multi-objective design problems for systems of complex dynamics, and several methodologies for analysis and synthesis have been developed. For example, in Ref. [45], a Riccati equation method has been proposed to solve the filtering problem for linear time-varying stochastic systems with pre-specified error variance bounds. In Refs [60–62], by means of the technique of sliding mode control (SMC), the robust controller design problem has been solved for linear parameter perturbed systems, since SMC has strong robustness to matched disturbances or parameter perturbations. We shall return to this SMC problem later and more details will be discussed in the following section.

When it comes to nonlinear stochastic systems, limited work has been done in the covariance-constrained analysis and design problems, just as we have anticipated. A multi-objective filter has been designed in Ref. [63] for systems with Lipschitz nonlinearity, but the variance bounds cannot be pre-specified. Strictly speaking, such an algorithm cannot be referred to as variance-constrained filtering in view of the lack of capability for directly imposing specified constraints on variance. An LMI approach has been proposed in Ref. [30] to cope with robust filtering problems for a class of stochastic systems with nonlinearities characterized by statistical means, attaining an assignable H_2 performance index. In Ref. [59], for a special class of nonlinear stochastic systems, namely, systems with multiplicative noises (also called bilinear systems or systems with state/control dependent noises), a state feedback controller has been put forward in a unified LMI framework in order to ensure that the multiple objectives, including stability, H_∞ specification, and variance constraints, are simultaneously satisfied. This paper is always regarded as the origination of covariance control theory for nonlinear systems, for within the established theoretical framework quite a lot of performance requirements can be taken into consideration simultaneously. Furthermore, with the developed techniques, the obtained elegant results could be easily extended to a wide range of nonlinear stochastic systems; see, for example, Refs [27, 33, 64–66].

We shall return to such types of nonlinear stochastic systems later to present more details of recent progress in Section 1.2.3.

1.2.2 Multiple Performance Requirements

In the following, several performance indices originating from engineering practice and frequently applied in multi-objective design problems are introduced.

1.2.2.1 Robustness

In real-world engineering practice, various reasons, such as variations of the operating point, aging of devices, identification errors, etc., would lead to the parameter uncertainties that result in the perturbations of the elements of a system matrix when modeling the system in a state-space form. Such a perturbation in system parameters cannot be avoided and would cause degradation (sometimes even instability) to the system. Therefore, in the past decade, considerable attention has been devoted to different issues for linear or nonlinear uncertain systems, and a great number of papers have been published; see Refs [2, 46, 67–70] for some recent results.

On another research frontier of robust control, the H_∞ design method, which is used to design controller/filter with guaranteed performances with respect to the external disturbances as well as internal perturbations, has received an appealing research interest during the past decades; see Refs [71–74] for instance. Since Zames' original work [71], significant advances have been made in the research area of H_∞ control and filtering. The standard H_∞ control problem has been completely solved by Doyle *et al.* for linear systems by deriving simple state-space formulas for all controllers [72]. For nonlinear systems, the H_∞ performance evaluation can be conducted through analyzing the L_2 gain of the relationship from the external disturbance to the system output, which is a necessary step when deciding whether further controller design is needed. In the past years, the nonlinear H_∞ control problem has also received considerable research attention, and many results have been available in the literature [73–77]. On the other hand, the H_∞ filtering problem has also gained considerable research interest along with the development of H_∞ control theory; see Refs [75, 78–82]. It is well known that the existence of a solution to the H_∞ filtering problem is in fact associated with the solvability of an appropriate algebraic Riccati equality (for linear cases) or a so-called Hamilton–Jacobi equation (for nonlinear ones). So far, there have been several approaches for providing solutions to nonlinear H_∞ filtering problems, few of which, however, is capable of handling multiple performance requirements in an H_∞ optimization framework.

It is worth mentioning that, in contrast to the H_∞ design framework within which multiple requirements can hardly be under simultaneous consideration, the covariance control theory has provided a convenient avenue for the robustness specifications to be perfectly integrated into the multi-objective design procedure; see Refs [59, 76] for example. For nonlinear stochastic systems, control and filtering problems have been