

Andrew Liddle



An Introduction to

MODERN COSMOLOGY

Third Edition

WILEY

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Andrew Liddle

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To my grandmothers

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Preface

The development of cosmology will no doubt be seen as one of the scientific triumphs of the twentieth century. At its beginning, cosmology hardly existed as a scientific discipline. By its end, the Hot Big Bang cosmology stood secure as the accepted description of the Universe as a whole. Telescopes such as the Hubble Space Telescope are capable of seeing light from galaxies so distant that the light has been travelling towards us for most of the lifetime of the Universe. The cosmic microwave background, a fossil relic of a time when the Universe was both denser and hotter, is routinely detected and its properties examined. That our Universe is presently expanding is established without doubt, and its material composition is accurately determined.

We are now in an era where understanding of cosmology has shifted from the qualitative to the quantitative. The turn of the millennium saw the establishment of what has come to be known as the Standard Cosmological Model, representing an almost universal consensus amongst cosmologists as to the best description of our Universe. Nevertheless, it is a model with a major surprise – the belief that our Universe is presently experiencing accelerated expansion. Add to that ongoing mysteries such as the properties of the so-called dark matter, which is believed to be the dominant form of matter in the Universe, and it is clear that we have some way to go before we can say that a full picture of the physics of the Universe is in our grasp.

Such a bold endeavour as cosmology easily captures the imagination, and there has been increasing demand for cosmology to be taught at university in an accessible manner. Traditionally, cosmology was taught, as it was to me, as the tail end of a general relativity course, with a derivation of the metric for an expanding universe and a few solutions. Such a course fails to capture the flavour of modern cosmology, which takes classic physical sciences like thermodynamics, atomic physics and gravitation and applies them on a grand scale.

In fact, introductory modern cosmology can be tackled in a different way, by avoiding general relativity altogether. By a lucky chance, and a subtle bit of cheating, the correct equations describing an expanding universe can be obtained from Newtonian gravity. From this basis, one can study all the triumphs of the Hot Big Bang cosmology – the expansion of the Universe, the prediction of its age, the existence of the cosmic microwave background, and the abundances of light elements such as helium and deuterium – and even go on to discuss more speculative ideas such as the inflationary cosmology.

The origin of this book, first published in 1998, was a short lecture course at the University of Sussex, around 20 lectures, taught to students in the final year of a bachelor's degree or the penultimate year of a master's degree. The prerequisites are all very standard

physics, and the emphasis is aimed at physical intuition rather than mathematical rigour. It has been further developed through my experiences of teaching cosmology at Imperial College London, the University of Hawai‘i, and at the University of Edinburgh, plus much useful feedback from readers.

The structure of the book is a central ‘spine’, the main chapters from 1 to 15, which provide a self-contained introduction to modern cosmology. In addition there are six Advanced Topic chapters, each with prerequisites, which can be added to extend the course as desired. Ordinarily the best time to tackle those Advanced Topics is immediately after their prerequisites have been attained, though they could also be included at any later stage.

Cosmology is an interesting course to teach, as it is not like most of the other subjects taught in undergraduate physics courses. There is no perceived wisdom, built up over a century or more, which provides an unquestionable foundation, as in thermodynamics, electromagnetism, and even quantum mechanics and general relativity. Each successive edition of this book has introduced new discoveries, and lecturers can expect opportunities to crop up during their course to discuss new results which impact on cosmologists’ views of the Universe.

You can follow my own evolving views by checking out this book’s WWW Home Page at

<http://www.roe.ac.uk/~arl/cosbook.html>

There you may find some updates on observations, and also a list of any errors in the book that I am aware of. If you are confident you’ve found one yourself, and it’s not on the list, I’d be very pleased to hear of it. Full-colour versions of many of the images can also be found there.

Andrew R. Liddle
Edinburgh
September 2014



Supplementary material, including full-colour images, updates and links for students and instructors, are available on the author's website: <http://www.roe.ac.uk/~arl/cosbook.html>

Some fundamental constants

Newton’s constant	G	$6.672 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-2}$
Speed of light	c	$2.998 \times 10^8 \text{ m sec}^{-1}$
		or $3.076 \times 10^{-7} \text{ Mpc yr}^{-1}$
Reduced Planck constant	$\hbar = h/2\pi$	$1.055 \times 10^{-34} \text{ m}^2 \text{ kg sec}^{-1}$
Boltzmann constant	k_B	$1.381 \times 10^{-23} \text{ J K}^{-1}$
		or $8.619 \times 10^{-5} \text{ eV K}^{-1}$
Radiation constant	$\alpha = \pi^2 k_B^4/15\hbar^3 c^3$	$7.565 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$
Electron mass–energy	$m_e c^2$	0.511 MeV
Proton mass–energy	$m_p c^2$	938.3 MeV
Neutron mass–energy	$m_n c^2$	939.6 MeV
Thomson cross-section	σ_e	$6.652 \times 10^{-29} \text{ m}^2$
Free neutron half-life	t_{half}	610 sec

Some conversion factors

$1 \text{ pc} = 3.261 \text{ light years} = 3.086 \times 10^{16} \text{ m}$
$1 \text{ yr} = 3.156 \times 10^7 \text{ sec}$
$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
$1 M_\odot = 1.989 \times 10^{30} \text{ kg}$
$1 \text{ J} = 1 \text{ kg m}^2 \text{ sec}^{-2}$
$1 \text{ Hz} = 1 \text{ sec}^{-1}$

Commonly-used symbols

z	redshift	<i>defined on page</i>	11, 39
H_0	Hubble constant		12, 49
r	physical distance		12
v	velocity		12
f	frequency		14
T	temperature		16
k_B	Boltzmann constant		16
ϵ	energy density		17
α	radiation constant		18
G	Newton's gravitational constant		21
ρ	mass density		22
a	scale factor		23
x	comoving distance		23
k	curvature		24
p	pressure		26
	(or occasionally momentum)		13)
H	Hubble parameter		38
n, N	number density		43
h	Hubble constant		50
	(or Planck's constant)		14)
Ω_0	present density parameter		51
ρ_c	critical density		51
Ω	density parameter		51
Ω_k	curvature 'density parameter'		52
q_0	deceleration parameter		52
Λ	cosmological constant		55
Ω_Λ	cosmological constant density parameter		56
t	time		61
t_0	present age		61
Ω_B	baryon density parameter		68
Y_4	helium abundance		95
d_{lum}	luminosity distance		131
d_{diam}	angular diameter distance		135
$\Delta T/T, C_\ell$	cosmic microwave background anisotropies		154, 155

Chapter 1

A Brief History of Cosmological Ideas

The cornerstone of modern cosmology is the belief that the place that we occupy in the Universe is in no way special. This is known as the **cosmological principle**, and it is an idea which is both powerful and simple. It is intriguing, then, that for the bulk of the history of civilization it was believed that we occupy a very special location, usually the centre, in the scheme of things.

The ancient Greeks, in a model further developed by the Alexandrian Ptolemy, believed that the Earth must lie at the centre of the cosmos. It would be circled by the Moon, the Sun and the planets, and then the ‘fixed’ stars would be yet further away. A complex combination of circular motions, Ptolemy’s Epicycles, was devised in order to explain the motions of the planets, especially the phenomenon of retrograde motion where planets appear to temporarily reverse their direction of motion. It was not until the early 1500s that Copernicus stated forcefully the view, initiated nearly two thousand years before by Aristarchus, that one should regard the Earth, and the other planets, as going around the Sun. By ensuring that the planets moved at different speeds, retrograde motion could easily be explained by this theory. However, although Copernicus is credited with removing the anthropocentric view of the Universe, which placed the Earth at its centre, he in fact believed that the Sun was at the centre.

Newton’s theory of gravity put what had been an empirical science (Kepler’s discovery that the planets moved on elliptical orbits) on a solid footing, and it appears that Newton believed that the stars were also suns pretty much like our own, distributed evenly throughout infinite space, in a static configuration. However it seems that Newton was aware that such a static configuration is unstable.

Over the next two hundred years, it became increasingly understood that the nearby stars are not evenly distributed, but rather are located in a disk-shaped assembly which we now know as the Milky Way galaxy. The Herschels were able to identify the disk structure in the late 1700s, but their observations were not perfect and they wrongly concluded that the Solar System lay at its centre. Only in the early 1900s was this convincingly overturned,

by Shapley, who realized that we are some two-thirds of the radius away from the centre of the Galaxy. Even then, he apparently still believed our galaxy to be at the centre of the Universe. Only in 1952 was it finally demonstrated, by Baade, that the Milky Way is a fairly typical galaxy, leading to the modern view, known as the **cosmological principle** (or sometimes the Copernican principle), that the Universe looks the same whoever and wherever you are.

It is important to stress that the cosmological principle isn't exact. For example, no one thinks that sitting in a lecture theatre is exactly the same as sitting in a bar, and the interior of the Sun is a very different environment from the interstellar regions. Rather, it is an approximation that we believe holds better and better the larger the length scales we consider. Even on the scale of individual galaxies it is not very good, but once we take very large regions (though still much smaller than the Universe itself), containing say a million galaxies, we expect every such region to look more or less like every other one. The cosmological principle is therefore a property of the global Universe, breaking down if one looks at local phenomena.

The cosmological principle is the basis of the Big Bang Cosmology. The Big Bang is the best description we have of our Universe, and the aim of this book is to explain why. The Big Bang is a picture of our Universe as an evolving entity, which was very different in the past as compared to the present. Originally, it was forced to compete with a rival idea, the Steady State Universe, which holds that the Universe does not evolve but rather has looked the same forever, with new material being created to fill the gaps as the Universe expands. However, the observations I will describe now support the Big Bang so strongly that the Steady State theory is almost never considered.