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Modeling, Control and Coordination of Helicopter Systems

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For the subject under study:

by modeling, we have a deeper and better understanding;

by control, we make our course of action steady and solid;

by coordination, we appreciate our friends for their supports in our mission.

Preface

Unmanned aerial vehicles (UAVs) are finding wider applications including surveillance, rescue, navigation, formation, coordination, among others. Helicopter-type UAVs offer the best option in many open and/or built-up areas for their maneuverability through narrow alleys and sharp beds, and their ability to hover in place if, for example, there is a need to have a close look at a place of interest.

As with any vehicular design, control design for helicopters is non-trivial. For example, ensuring stability in helicopter flight is a challenging problem for nonlinear control design and development. Unlike many classes of mechanical systems that naturally possess desirable structural properties such as passivity or dissipativity, helicopter systems are inherently unstable without closed-loop control, especially when hovering. In addition, helicopter dynamics are highly nonlinear and strongly coupled such that disturbances along a single degree of freedom can easily propagate to the other degrees of freedom and lead to loss of performance, even destabilization.

The fundamental requirement for control system design is to guarantee the stability for helicopter systems. Many techniques have been proposed in the literature for the motion control of helicopter systems, ranging from feedback linearization to model reference adaptive control and dynamic inversion, and these techniques typically require reasonably precise knowledge of the dynamic models in order to achieve satisfactory performance. An important concern when designing controllers for helicopters is the manner on how to deal with unknown perturbations to the nominal model, in the form of parametric and functional uncertainties, unmodelled dynamics, and disturbances from the environment. Helicopter control applications are characterized by time-varying aerodynamical disturbances, which are generally difficult to model accurately. The presence of uncertainties and disturbances could disrupt the function of the feedback controller and lead to degradation of performance.

While autonomous vehicles performing solo missions can yield significant benefits, greater efficiency and operational capability can be realized from teams of autonomous vehicles operating in a coordinated fashion. Potential applications for multi-vehicle systems include space-based interferometers, future combat systems, surveillance and reconnaissance, hazardous material handling, and distributed reconfigurable sensor networks. To realize the potential of a multi-helicopter team, coordination techniques between helicopters are needed. One fundamental problem in multi-helicopter cooperation is formation control, in which the helicopters are required to keep a desired formation configuration to complete the assigned tasks.

The proposed book is meant to provide a comprehensive treatment of helicopter systems, ranging from related nonlinear flight dynamic modeling and stability analysis, to advanced control design for a single UAV, to the coordination and formation control of multiple helicopters to achieve high performance tasks. This book can be a good reference and introduction to the modeling, control and coordination of helicopter systems.

The book starts with a brief introduction to the development of helicopter systems modeling, control and coordination in Chap. 1.

Chapter 2 focuses on nonlinear fight dynamic modeling for a general class of rotary-wing aircraft. Centred around the general rigid body equations of motion, the modeling process is based on modules that follow closely with physical subsystems on the rotary-wing aircraft. These modules may include main and tail rotors, propeller, horizontal and vertical tails, wings, fuselage, and the flight control system. Aerodynamic interference among different modules can also be established. Besides the general modeling, performance calculation is also illustrated as it is a critical index in understanding the aircraft.

Following the flight dynamic modeling, trim study and stability analysis are conducted in Chap. 3. Trim is a state of the aircraft in which forces and moments reach equilibrium. Trim is the basis of many fundamental analysis, including stability analysis, control system design, handling qualities assessment, and simulation. A numerical procedure on how to obtain trim is described. With the trim condition established, a linearization process can be carried out to obtain a linearized model for the rotary-wing aircraft. Elements in the linearized model are called stability and control derivatives. A detailed description on the derivatives is provided, with the emphasis of their physical meanings. To illustrate, three examples are also presented in the chapter, which include a simplified single helicopter system and two hobby helicopters at hover.

Linearized models are not only useful for stability analysis, but also for control design, by blending them over different operating points with gain scheduling. However, this requires extensive modeling, which is expensive and time-consuming. There is a need for control systems that can operate with minimal model information and handle nonlinearities over the entire flight regime. Chapter 4 aims to address this need, by presenting a robust adaptive neural network (NN) control for helicopters. In particular, we focus on vertical flight, which can be represented in the single-input–single-output (SISO) nonlinear nonaffine form, because the coupling between longitudinal and lateral directional equations in this flight regime is weak. Although a nonaffine system can be rendered affine by adding an integrator to the control input, allowing many control methods for affine nonlinear system to be used, the disadvantage of this approach is that the dimension of the system is increased, and control efforts are not direct and immediate either. Subsequently, effective control for the system may not be achieved. In this chapter, we focus on control design for the nonaffine system directly, without adding any integrators to the input. Differing from the approaches in the literature, which were based on approximate dynamic inversion with augmented NNs, we utilize the Mean Value Theorem and the Implicit Function Theorem as mathematical tools to handle the nonaffine nonlinearities in the helicopter dynamics. In cases where reasonably accurate knowledge of the dynamic inversion model is available, the method using approximate dynamic inversion has been shown to provide an effective solution to the problem. However, the construction of the dynamic inversion for a nonaffine system may not be an easy task in general. For such cases, our approach offers a feasible means of tackling the problem, since a priori knowledge of the inversion is not required.

In Chap. 5, neural network control is redesigned to track both altitude and yaw angle at the same time. We consider a scale model helicopter, mounted on an experimental platform, in the presence of model uncertainties, which may be caused by unmodelled dynamics, or aerodynamical disturbances from the environment. Two different types of NN, namely a multilayer neural network (MNN) and a radial basis function neural network (RBFNN) are adopted in control design and stability analysis. Based on Lyapunov synthesis, the proposed adaptive NN control ensures that both the altitude and the yaw angle track the given bounded reference signals to a small neighborhood of zero, and guarantees the Semi-Globally Uniformly Ultimate Boundedness (SGUUB) of all the closed-loop signals at the same time. The effectiveness of the proposed control is illustrated through extensive simulations. Compared with the model-based control, approximation-based control yields better tracking performance in the presence of model uncertainties.

Unlike previous chapters, which consider actuators with instantaneous response, Chap. 6 deals with a more realistic scenario where actuator dynamics are present. Backstepping technique, combined with NNs, is employed to design the robust attitude control for uncertain multi-input multi-output (MIMO) nonlinear helicopter dynamics. To the best of our knowledge, there are few works in the literature that take the actuator dynamics into account in the helicopter control, which is practically relevant but more challenging as well. In this chapter, helicopter models are considered as MIMO nonlinear dynamic systems, where the actuator dynamics in the first-order low-pass filter form are considered. Secondly, the possible singularity problem of the control coefficient matrix for the model-based attitude control case is tackled effectively by introducing a design matrix. Thirdly, approximation-based attitude control is developed to handle the model uncertainties (e.g. unknown moment coefficients and mass) and external disturbances. Rigorous stability analysis and extensive simulations results show the effectiveness and robustness of the proposed attitude control.

Beyond single-helicopter control, this book also touches on formation control of multiple helicopters. In Chap. 7, the concept of the Q-structure is introduced as a novel and flexible methodology to define and support a large variety of formations. The Q-structure allows automatic scaling of formations according to changes in the overall size of the helicopter team. The chapter begins by exploring the use of the Q-structure for formation control where perfect communication is present between all members of the team. The second part of the chapter focuses on how the Q-structure can be adapted and used for teams where communication is imperfect. In this chapter, we examine the properties of Q-structures in relation to other formation representation schemes, and look at the ways Q-structures can be used with artificial potential trenches to improve the scalability of the formations and support a large number of different formations. In particular, the Q-structure does not require explicit representation of every single node of the formation and is able to ensure the formation maintenance of a large number of helicopters. The formation is also robust against possible communication breakdown and/or limited wireless communication ranges.

The Q-structure formation control is useful for motion planning at the kinematics level. For formation control that takes into account the dynamics of helicopters, a different approach is proposed. Chapter 8 presents synchronized altitude tracking control of helicopters with unknown dynamics by graph theory, while the desired trajectory is available to a portion of the team. Since only the neighbors' information is available to the helicopter, we use the weighted average of neighbors' states as the reference state of the helicopter in the control design. We prove that if the extended communication graph contains a spanning tree with the virtual vehicle as its root, then its Laplacian will be positive definite. This property is essential for the stability proven and it also makes the proof of the stability for the results easy and direct. The mathematical stability proof, which makes use of the positive definite property of the graph Laplacian, is provided for both full-state and output feedback cases.

This book is primarily intended for academics, researchers and engineers who are interested in modeling, control and coordination of helicopter systems, and autonomous systems at large because many of the techniques and concepts could be extended further and applied directly to many other systems of interest. It can also serve as complementary reading for nonlinear systems, robotics, and adaptive control at the postgraduate level.

The book summarizes the research works carried out by the authors and a number of their close collaborators including Keng Peng Tee, Mou Chen and Rongxin Cui. For the final completion of the book, we gratefully acknowledge the unreserved support, constructive comments, and fruitful discussions from Miroslav krstic, University of California, San Diego; Frank L. Lewis, University of Texas at Arlington; Masayoshi Tomizuka, University of California at Berkeley; Ian Postlethwaite, University of Northumbria; J.V.R. Prasad, Daniel Schrage, Lakshmi N. Sankar and George Vachtsevanos, Georgia Institute of Technology; Ben M. Chen, National Univesity of Singapore; Xianbin Cao and Jun Zhang, Beijing University of Aeronautics and Astronautics; Lei Guo and Yiguang Hong, Chinese Academy of Sciences; Trung T. Han and Rui Li, University of Electronic Science and Technology of China and Hailong Pei and Cong Wang, South China University of Technology.

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Chapter 1
Introduction

1.1 Background

Unmanned helicopters pose a challenge to nonlinear control design and development. Unmanned helicopters are inherently unstable without closed loop control, setting it apart from many other classes of mechanical systems that possess desirable structural properties such as passivity or dissipativity. In addition, helicopter dynamics are highly nonlinear and strongly coupled, and therefore disturbances along a single degree of freedom can easily propagate to the other degrees of freedom, leading to a loss of performance or even destabilization. Furthermore, unrestrained helicopter motion is governed by underactuated configuration, that is, the number of control inputs is less than the number of degrees of freedom to be stabilized, which makes it difficult to apply the conventional approach (usually in the field of robotics) for controlling Euler–Lagrange systems. Due to these reasons, substantial research efforts have focused on control design that guarantee stability and robustness for unmanned helicopters.

Most helicopters systems are intrinsically nonlinear and their dynamics are described by nonlinear differential equations. However, for the purpose of analysis, researchers commonly derive linearized models of helicopter systems. More specifically, a linear model that approximates a particular nonlinear helicopter system can be obtained if this system operates around an operating point and the signals involved are small. Many techniques for the design of controllers and analysis techniques for linear systems have been proposed in literature, for example, linear H_∞ control theory has been applied to a linear helicopter model in [10, 82, 95, 105]. However, control laws based on the linearized helicopter dynamics do not have global applicability as they only exhibit desirable behavior around an operating point. Because of this, many researchers have applied nonlinear control techniques to dynamic models of helicopter systems. Based on a reduced order subsystem of a Lagrangian helicopter model, feedback linearization control was applied for altitude tracking [104]. Dynamic sliding mode control was proposed for altitude regulation in [93]. By modifying the internal dynamics to remove the nonhyperbolicity

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followed by stable inversion, the problem of near nonhyperbolic internal dynamics in helicopter control was addressed in [19]. In [55], approximate input–output linearization was employed to obtain a dynamically linearizable helicopter system without zero dynamics, and which possessed the desirable property of differential flatness. Internal model-based control was suitably applied to the problem of helicopter landing on an oscillating ship deck in [40]. In [45, 47], a novel method is presented for the design of a fuzzy flight controller for the unmanned APID-MK3 helicopter.

The control techniques proposed in the research work mentioned above require reasonably precise knowledge of dynamic models involved in order to achieve satisfactory performance. In the control of helicopters, an important concern is how one can deal with unknown perturbations to the nominal model. Such unknown perturbations include parametric and functional uncertainties, unmodelled dynamics, and disturbances from the environment. The presence of uncertainties and disturbances could disrupt the function of the feedback controller and lead to degradation of performance. To deal with the presence of model uncertainties, approximation-based techniques using neural networks (NNs) have been proposed. Approximate dynamic inversion with augmented NNs was proposed to handle unmodelled dynamics in [38, 54], while neural dynamic programming was shown to be effective for the tracking and trimming control of helicopters in [21].

A significant portion of this book is dedicated to a comprehensive treatment of the control of a single helicopter. However, multi-agent systems, consisting of huge numbers of interacting agents and autonomous unmanned vehicles, linked together in complex networks, are becoming increasingly prevalent. The advancement of technology in robotics, vehicular control, computer science and communications, has made possible the deployment of large teams of mobile UAVs in real life scenarios. These systems have been applied to a wide variety of areas, from manufacturing and warehouse automation, to construction and shipping industries, to autonomous UAV humanitarian demining, and to surveillance and urban search-and-rescue with UAVs/helicopters. A comprehensive overview of issues in multi-vehicle cooperation may be found in [5].

Formation control of UAV teams in dynamic and uncertain environments has been intensively studied in recent years. The amount of interest in this topic can be attributed to at least two reasons: (1) it is a good platform (and task) for studying multi-agent cooperation techniques and (2) there are practical applications for formation control techniques (e.g., in autonomous demining where a team of demining-vehicles need to systematically "sweep" through a plot of land). The need to use UAVs in real applications means considering real-world constraints, such as imperfect communications between agents, in the design of coordination techniques.

A host of different techniques for formation control in multi-UAV teams is available in the literature, for example, [3, 17, 18, 22, 59]. In particular, many of these techniques deal, in some way or other, with the robust operation of UAVs. For example, Balch and Hybinette [4] used the concept of social potentials that allow UAVs to "snap" into the correct relative positions, based on a predetermined set of "attachment sites" around each UAV. This method however offers no control over the geometry of the formation, and more dispersed formations, like the wedge and diamond formations, cannot be achieved. Carpin and Parker [11] described a distributed approach, based on explicit communication between UAVs, for coordinated motion in a linear pattern. The framework is also able to handle the emergence of unexpected obstacles within the formation. Kostelnik et al. [56] solved the problem by using a communication network between the UAVs, so that each UAV may be assigned social roles by a dynamically chosen leader. Kang et al. [49, 50] proposed general methods for the controller design for the formation maintenance of multiple vehicles tracking a desired path. It was assumed that the desired trajectories are known, and each UAV is associated with a specific node in the formation. The formation controllers can be easily altered by the designer when vehicles are added or removed from the system.

Another pressing concern in formation control is that of ensuring convergence and stability within the proposed schemes. Many researchers have focused on this aspect, and have derived provably stable convergent algorithms for formation control. In the work by Song and Kumar [96], specific artificial potential fields for each UAV relative to the other UAVs are calculated to produce stable formations in equilibrium. The concept of Leader-to-Formation Stability is introduced in [100], and the paper examined how errors propagate from the leader and influence the stability of formations (those based on leader following). Navigation functions, first proposed in [86], have also been used for centralized control of formations [98]. The problem of ensuring formation stability in a UAV team while it moves in formation through an obstacle field between two points was investigated in [75].

The later chapters in this book dealing with the coordination within UAV teams present control techniques that are specifically designed to take into account heterogeneity within teams and the variability of the environments these teams are likely to operate in. Specifically, they will focus on improving the scalability and robustness of agent teams. These chapters will complement earlier chapters in the book by providing an overarching coordination framework within which agents (UAVs) will operate, where each agent is subject to control laws described in earlier chapters of this book.

The main objectives of the book is to present and discuss various control strategies for helicopters and algorithms for coordinating groups of helicopters.

1.2 Outline of the Book

The book starts with a brief introduction of development of helicopter systems modeling, control and coordination in Chap. 1.

Chapter 2 focuses on nonlinear fight dynamic modeling for a general class of rotary-wing aircraft. Centered around the general rigid body equations of motion, the modeling process is based on modules that follow closely with physical subsystems on the rotary-wing aircraft. These modules may include main and tail rotors, propeller, horizontal and vertical tails, wings, fuselage, and the flight control system. Aerodynamic interference among different modules can also be established. Besides the general modeling, performance calculation is also illustrated as it is a critical index in understanding the aircraft.

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Linearized models are not only useful for stability analysis, but also for control design, by blending them over different operating points with gain scheduling. However, this requires extensive modeling, which is expensive and time-consuming. There is a need for controllers that can operate with minimal model information and handle nonlinearities over the entire flight regime. Chapter 4 aims to address this need, by presenting a robust adaptive NN control for helicopters. In particular, we focus on vertical flight, which can be represented in the single-input–single-output (SISO) nonlinear nonaffine form, because the coupling between longitudinal and lateral directional equations in this flight regime is weak. Although a nonaffine system can be rendered affine by adding an integrator to the control input, thus allowing many control methods for affine nonlinear system to be used, the disadvantage of this approach is that the dimension of the system is increased, and control efforts are not direct and immediate either. Subsequently, effective control for the system may not be achieved. In this chapter, we focus on control design for the nonaffine system directly, without adding any integrators to the input. Differing from the approaches in the literature, which were based on approximate dynamic inversion with augmented NNs, we utilize the Mean Value Theorem and the Implicit Function Theorem as mathematical tools to handle the nonaffine nonlinearities in the helicopter dynamics. In cases where reasonably accurate knowledge of the dynamic inversion model is available, the method using approximate dynamic inversion has been shown to provide an effective solution to the problem. However, the construction of the dynamic inversion for a nonaffine system may not be an easy task in general. For such cases, our approach offers a feasible means of tackling the problem, since a priori knowledge of the inversion is not required.

In Chap. 5, NN control is redesigned to track both altitude and yaw angle at the same time. We consider a scale model helicopter, mounted on an experimental platform, in the presence of model uncertainties, which may be caused by unmodelled dynamics, or aerodynamical disturbances from the environment. Two different types of NN, namely a multilayer neural network (MNN) and a radial basis function neural network (RBFNN) are adopted in control design and stability analysis. Based on Lyapunov synthesis, the proposed adaptive NN control ensures that both the altitude and the yaw angle track the given bounded reference signals to a small neighborhood of zero, and guarantees the Semi-Globally Uniformly Ultimate boundedness (SGUUB)of all the closed-loop signals at the same time. The effectiveness of the proposed control is illustrated through extensive simulations. Compared with the model-based control, approximation-based control yields better tracking performance in the presence of model uncertainties.

Unlike previous chapters which consider actuators with instantaneous response, Chap. 6 deals with a more realistic scenario where actuator dynamics are present. Backstepping technique, combined with NNs, is employed to design the robust attitude control for uncertain multi-input–multi-output (MIMO) nonlinear helicopter dynamics. To the best of our knowledge, there are few works in the literature that take the actuator dynamics into account in the helicopter control, which is practically relevant but more challenging as well. In this chapter, helicopter models are considered as MIMO nonlinear dynamic systems, where the actuator dynamics in the first-order low-pass filter form are considered. Secondly, the possible singularity problem of the control coefficient matrix for the model-based attitude control case has been tackled effectively by introducing a design matrix. Thirdly, approximation-based attitude control is developed to handle the model uncertainties (e.g., unknown moment coefficients and mass) and external disturbances. Rigorous stability analysis and extensive simulations results show the effectiveness and robustness of the proposed attitude control.

Beyond single-helicopter control, this monograph also touches on formation control of multiple helicopters. In Chap. 7, the concept of the Q-structure is introduced as a novel and flexible methodology to define and support a large variety of formations. The Q-structure allows automatic scaling of formations according to changes in the overall size of the helicopter team. The chapter begins by exploring the use of the Q-structure for formation control where perfect communication is present between all members of the team. The second part of the chapter focuses on how the Q-structure can be adapted and used for teams where communication is imperfect. In this chapter, we examine the properties of Q-structures in relation to other formation representation schemes, and look at the ways Q-structures can be used with artificial potential trenches to improve the scalability of the formations and support a large number of different formations. In particular, the Q-structure does not require explicit representation of every single node of the formation and is able to ensure the formation maintenance of a large number of helicopters. The formation is also robust against possible communication breakdown and/or limited wireless communication ranges.

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Chapter 2
Building a Nonlinear Rotary-Wing Aircraft Model

2.1 Introduction

This chapter focuses on nonlinear flight dynamic modeling of rotary-wing aircraft. The intention is to establish a general modeling framework, that applies to not only traditional single main-rotor helicopters, but also unconventional rotary-wing aircraft. Apart from general equations of motion, the modeling framework relies heavily on and is extended from the results in [97]. Differing from a traditional modeling description, the inflow dynamics of a rotor module include latest result from [13] to handle a rotor in descent motion. In addition, a propeller thrust and torque calculation procedure is illustrated using the typical nomenclature in helicopter theory books. For the purposes of easy implementation and simple writing format, vector description of equations is utilized whenever possible.

2.2 General Equations of Motion

Assumptions associated with the classical six degree-of-freedom (DOF) equations of motion include rigid body dynamics and insignificant effects from Earth's curvature and rotation. These two assumptions are usually satisfied for applications in performance study, stability analysis, dynamic simulation, control system design, and handling qualities assessment.

Typically, the general equations of motion include four sets of equations: force equation, moment equation, kinematic equation, and navigation equation.

2.2.1 Force Equation

Figure 2.1 shows typical coordinate systems for both aircraft body and the main rotor hub. Let u_{B}, v_{B}, and w_{B} be three velocity components and p_{B}, q_{B}, and r_{B}

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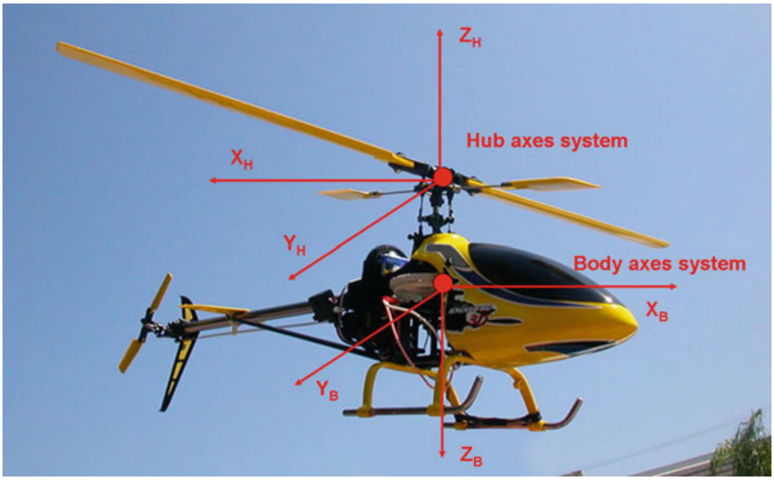

Fig. 2.1 Definitions of body axes and hub axes system (Image with courtesy of http://cheaprchelicopters.net)

be three angular rate components along the x-, y-, z-axis in the body axes system, respectively. Furthermore, define the Euler angle representation as roll (ϕ), pitch (θ), and yaw (ψ). The force equation is listed as follows:

$$\begin{bmatrix} \dot{u}_{\mathrm{B}} \\ \dot{v}_{\mathrm{B}} \\ \dot{w}_{\mathrm{B}} \end{bmatrix} = -\begin{bmatrix} p_{\mathrm{B}} \\ q_{\mathrm{B}} \\ r_{\mathrm{B}} \end{bmatrix} \times \begin{bmatrix} u_{\mathrm{B}} \\ v_{\mathrm{B}} \\ w_{\mathrm{B}} \end{bmatrix} + \left(\begin{bmatrix} -g\sin\theta \\ g\sin\phi\cos\theta \\ g\cos\phi\cos\theta \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \right) \tag{2.1}$$

The cross-product term is the Coriolis acceleration due to body rotation. There are two contributions to the force term of the equation: one is the gravitational force resolved in the body axes system, and another is all other forces in the body axes system, represented as F_x, F_y, and F_z.

2.2.2 *Moment Equation*

Define L, M, and N as the resultant rolling, pitching, and yawing moments along the x-, y-, z-axis in the body axes system, respectively. The moment equation is:

$$\begin{bmatrix} \dot{p}_B \\ \dot{q}_B \\ \dot{r}_B \end{bmatrix} = -I^{-1} \left(\begin{bmatrix} p_{\mathrm{B}} \\ q_{\mathrm{B}} \\ r_{\mathrm{B}} \end{bmatrix} \times \left(I \begin{bmatrix} p_{\mathrm{B}} \\ q_{\mathrm{B}} \\ r_{\mathrm{B}} \end{bmatrix} \right) \right) + I^{-1} \begin{bmatrix} L \\ M \\ N \end{bmatrix} \tag{2.2}$$