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Sensors: Theory, Algorithms, and Applications

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Aims and Scope
Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics, and other sciences.

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Sensors: Theory, Algorithms, and Applications

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Preface

In recent years, technological advances have resulted in the rapid development of a new exciting research direction – the interdisciplinary use of sensors for data collection, systems analysis, and monitoring. Application areas include military surveillance, environmental screening, computational neuroscience, seismic detection, transportation, along with many other important fields.

Broadly speaking, a sensor is a device that responds to a physical stimulus (e.g., heat, light, sound, pressure, magnetism, or motion) and collects and measures data regarding some property of a phenomenon, object, or material. Typical types of sensors include cameras, scanners, radiometers, radio frequency receivers, radars, sonars, thermal devices, etc.

The amount of data collected by sensors is enormous; moreover, this data is heterogeneous by nature. The fundamental problems of utilizing the collected data for efficient system operation and decision making encompass multiple research areas, including applied mathematics, optimization, and signal/image processing, to name a few. Therefore, the task of crucial importance is not only developing the knowledge in each particular research field, but also bringing together the expertise from many diverse areas in order to unify the process of collecting, processing, and analyzing sensor data. This process includes theoretical, algorithmic, and application-related aspects, all of which constitute essential steps in advancing the interdisciplinary knowledge in this area.

Besides individual sensors, *interconnected systems of sensors*, referred to as *sensor networks*, are receiving increased attention nowadays. The importance of rigorous studies of sensor networks stems from the fact that these systems of multiple sensors not only acquire individual (possibly complimentary) pieces of information, but also effectively exchange the obtained information. Sensor networks may operate in static (the locations of individual sensor nodes are fixed) or dynamic (sensor nodes may be mobile) settings.

Due to the increasing significance of sensor networks in a variety of applications, a substantial part of this volume is devoted to theoretical and algorithmic aspects of problems arising in this area. In particular, the problems of information fusion are especially important in this context, for instance, in the situations when the data collected from multiple sensors is synthesized in order to ensure effective operation of the underlying systems (i.e., transportation, navigation systems, etc.). On the other hand, the reliability and efficiency of the sensor network itself (i.e., the ability of the network to withstand possible failures of nodes, optimal design of the network in terms of node placement, as well as the ability of sensor nodes to obtain location coordinates based on their relative locations – known as *sensor network localization* problems) constitutes another broad class of problems related to sensor networks. In recent years, these problems have been addressed from rigorous mathematical modeling and optimization perspective, and several chapters in this volume present new results in these areas.

From another theoretical viewpoint, an interesting related research direction deals with investigating information patterns (possibly limited or incomplete) that are obtained by sensor measurements. Rigorous mathematical approaches that encompass dynamical systems, control theory, game theory, and statistical techniques, have been proposed in this diverse field.

Finally, in addition to theoretical and algorithmic aspects, application-specific approaches are also of substantial importance in many areas. Although it is impossible to cover all sensor-related applications in one volume, we have included the chapters describing a few interesting application areas, such as navigation systems, transportation systems, and medicine.

This volume contains a collection of chapters that present recent developments and trends in the aforementioned areas. Although the list of topics is clearly not intended to be exhaustive, we attempted to compile contributions from different research fields, such as mathematics, electrical engineering, computer science, and operations research/optimization. We believe that the book will be of interest to both theoreticians and practitioners working in the fields related to sensor networks, mathematical modeling/optimization, and information theory; moreover, it can also be helpful to graduate students majoring in engineering and/or mathematics, who are looking for new research directions.

We would like to take the opportunity to thank the authors of the chapters for their valuable contributions, as well as Springer staff for their assistance in producing this book.

Gainesville, FL, USA

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Part III Sensors in Real-World Applications

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Part I
Models and Algorithms for Ensuring Efficient Performance of Sensor Networks

On Enhancing Fault Tolerance of Virtual Backbone in a Wireless Sensor Network with Unidirectional Links

Ravi Tiwari and My T. Thai

Abstract A wireless sensor network (WSN) is a collection of energy constrained sensor node forming a network which lacks infrastructure or any kind of centralized management. In such networks, virtual backbone has been proposed as the routing infrastructure which can alleviate the broadcasting storm problem occurring due to consistent flooding performed by the sensor node, to communicate their sensed information. As the virtual backbone nodes needs to carry other nodes' traffic, they are more subject to failure. Hence, it is desirable to construct a fault tolerant virtual backbone. Most of recent research has studied this problem in homogeneous networks. In this chapter, we propose solutions for efficient construction of a fault tolerant virtual backbone in a WSN where the sensor nodes have different transmission ranges. Such a network can be modeled as a disk graph (DG), where link between the two nodes is either unidirectional or bidirectional. We formulate the fault tolerant virtual backbone problem as a k-Strongly Connected m-Dominating and Absorbing Set (k, m) SCDAS problem. As the problem is NP-hard, we propose an approximation algorithm along with the theoretical analysis and conjectured its approximation ratio.

1 Introduction

A wireless sensor network (WSN) is a collection of power constrained sensors nodes with a base station. The sensors are supposed to sense some phenomena and collect information, which is required to be sent to the base station for further forwarding or processing. As the sensors are power constraint, their transmission

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ranges are small. Hence, the sensed information may be relayed on multiple intermediate sensor nodes before reaching the base station. As there is no fixed or predefined infrastructure, in order to enable data transfer in such networks, all the sensor nodes frequently flood control messages, thus causing a lot of redundancy, contentions, and collisions [20]. As a result, a virtual backbone has been proposed as the routing infrastructure of such networks for designing efficient protocols for routing, broadcasting, and collision avoidance [1]. With virtual backbone, routing messages are only exchanged between the sensor nodes in the virtual backbone, instead of being flooded to all the sensor nodes. With the help of virtual backbone, routing is easier and can adapt quickly to network topology changes. It has been seen that the virtual backbones could dramatically reduce routing overhead [18]. Furthermore, using virtual backbone as relay nodes can efficiently reduce the energy consumption, which is one of the critical issues in WSNs to maximize the sensor network lifetime.

However, transmission range of all the sensor nodes in the WSN are not necessarily equal. As the transmission range depends upon the energy level of a sensor node which can be different for different sensor nodes, this may result in sensor nodes having different transmission range. The sensor nodes can also tune their transmission ranges depending upon their functionality, or they may perform some power control to alleviate collisions or to achieve some level of connectivity. In some topology controlled sensor networks, sensor nodes may adjust their transmission ranges differently to obtain certain optimization goals. All these scenarios result into the WSN with different transmission ranges. Such a network can be modeled as a Disk Graph (DG) G. Note that G is a *directed* graph, consisting both bidirectional and unidirectional links.

Since the virtual backbone nodes in the WSN need to relay other sensor node's traffic, so, due to heavy load often they are vulnerable to frequent node or link failure which is inherent in WSNs. Hence, it is very important to study the fault tolerance of the virtual backbone in wireless sensor networks. Therefore, constructing a fault tolerant virtual backbone that continues to function during node or link failure is an important research problem, which has been not studied sufficiently. In [6, 7], the authors considered this problem in Unit Disk Graph (UDG) [2], in which all nodes have the same transmission ranges. When a wireless network has nodes with same transmission ranges then it will only have bidirectional links. In such a case the virtual backbone is represented by the connected dominating set (CDS) of the graph representing the wireless network. Whereas, when the wireless network has nodes with different transmission range then it will have both unidirectional and bidirectional links. In this case the virtual backbone is represented by a strongly connected dominating and absorbing set (SCDAS) [12], here, a node not in virtual backbone has at least one virtual backbone node as its incoming and outgoing neighbor, respectively.

Although the virtual backbone problem has been extensively studied in general undirected graphs and UDGs [3, 13, 16, 17, 19, 21–23], the construction of virtual backbone in wireless networks with different transmission ranges is explored to a little extent. In [8] and [5], the authors extended their marking process to networks with unidirectional links to find a SCDAS. However, the paper does not present any approximation ratio. Recently, we proposed a constant approximation algorithms for SCDAS problem [4, 10, 12, 14]. The construction of fault tolerant virtual backbone in general undirected graphs is also one of the newly studied problems. Dai et al. addressed the problem of constructing k-connected k dominating set ((k, k) CDS) [6] in UDG. In Feng et al. [7] introduced the problem of constructing $(2, 1)$ CDS in UDGs and proposed a constant approximation ratio. Note that the solutions of these two papers are applicable only to undirected graphs. In addition, the authors just considered a special case of the general problem, where $k = m$ or $k = 2$ and $m = 1$. In [24] Wu et al. studied the construction of (k, m) CDS but they considered undirected graph. Recently, we have considered the fault tolerant virtual backbone in heterogeneous networks with only bidirectional links [15]. We proposed a constant approximation algorithm for any value of k and m. In summary, no work has studied the (k, m) SCDAS in heterogeneous networks with unidirectional and bidirectional links for any value of k and m.

In this chapter we study the enhancing of fault tolerance of virtual backbone in WSN represented by a directed disk graph (DG). The virtual backbone in this case is represented as a strongly connected dominating and absorbing set (SCDAS). The fault tolerance of a virtual backbone can be enhanced in two aspects. Firstly, by increasing the dominance and absorption of the virtual backbone nodes, i.e., by increasing the number of virtual backbone nodes in the incoming and outgoing neighborhood of a non-virtual backbone nodes. Secondly, by increasing the connectivity of the virtual backbone, by ensuring the nodes in virtual backbone has multiple paths to each other in the subgraph induced by them. In order to generate a fault tolerant virtual backbone, we formulate the (k, m) SCDAS problem. The (k, m) SCDAS problem is to find an SCDAS of a directed graph $G = (V, E)$ such that the graph induced by the (k, m) SCDAS nodes is k-strongly node connected and any node not in (k, m) SCDAS has at least m nodes in its incoming and outgoing neighborhood, respectively.

The rest of this chapter is organized as follows. Section 2 describes the preliminaries, network model, and problem definition. The enhancement of fault tolerance of virtual backbone in terms of dominance and absorption is studied in Sect. 3. In Sect. 4 we conclude the chapter with a brief summary.

2 Network Model and Problem Definition

2.1 Preliminaries

Let a directed graph $G = (V, E)$ represent a network where V consists of all nodes in a network and E represents all the communication links.

For any vertex $v \in V$, the **incoming neighborhood** of v is defined as $N^-(v) = \{u \in V \mid (u,v) \in E\}$, and the **outgoing neighborhood** of v is defined as $N^+(v) = \{u \in V \mid (v,u) \in E\}$.

Likewise, for any vertex $v \in V$, the **closed incoming neighborhood** of v is defined as $N^-[v] = N^-(v) \cup \{v\}$, and the **closed outgoing neighborhood** of v is defined as $N^+[v] = N^+(v) \cup \{v\}$.

A subset $S \subseteq V$ is called a **dominating set** (DS) of G iff $S \cup N^+(S) = V$ where $N^+(S) = \bigcup_{u \in S} N^+(u)$ and $\forall v \in N^+(S), N^-(v) \cap S \neq \emptyset$. If $|N^-(v) \cap S| \geq m$, then S is said to be a m dominating set.

A subset $A \subseteq V$ is called an **absorbing set** (AS) of G iff $A \cup N^-(S) = V$ where $N^-(S) = \bigcup_{u \in S} N^-(u)$ and $\forall v \in N^-(S), N^+(v) \cap S \neq \emptyset$. If $|N^+(v) \cap S| \geq m$, then A is said to be a m absorbing set.

A subset $S \subseteq V$ is called an **independent set** (IS) of G iff $S \cup N^+(S) = V$ and $S \cap N^+(S) = \emptyset$.

A subset SI $\subseteq V$ is called a **Semi-independent Set** (SI) of G iff $u, v \in$ SI, then, $\{(u,v),(v,u)\} \notin E$, or if $(u,v) \in E$ then $(v,u) \notin E$ and vice-versa. Nodes u and v are said to be **Semi-independent** to each other.

Given a subset $S \subseteq V$, an **induced subgraph of** S, denoted as $G[S]$, obtained by deleting all vertices in the set $V - S$ from G.

A directed graph G is said to be **strongly connected** if for every pair of nodes $u, v \in V$, there exists a directed node disjoint path. Likewise, a subset $S \subseteq V$ is called a **strongly connected set** if $G[S]$ is strongly connected. If for every pair of nodes $u, v \in V$, there exists at least k directed node disjoint paths then the graph G is said to be ***k* strongly connected**, similarly a subset $S \subseteq V$ is called a ***k* strongly connected set** if $G[S]$ is k strongly connected.

A subset $S \subseteq V$ is called a **Strongly Connected Dominating Set** (SCDS) if S is a DS and $G[S]$ is strongly connected. S is called a **Strongly Connected Dominating and Absorbing Set** (SCDAS) if S is an SCDS and for all nodes $u \notin S$, $N^+(u) \cap S \neq \emptyset$ and $N^-(u) \cap S \neq \emptyset$. S is a (k,m) SCDAS if it is k strongly connected and m dominating and m absorbing.

2.2 Network Model and Problem Definition

In this chapter, we study the fault tolerant virtual backbone in wireless sensor networks with different transmission ranges. In this case, the WSN can be modeled as a directed graph $G = (V, E)$. The sensor nodes in V are located in the two dimensional Euclidean plane and each sensor node $v_i \in V$ has a transmission range $r_i \in [r_{\min}, r_{\max}]$. A directed edge $(v_i, v_j) \in E$ if and only if $d(v_i, v_j) \leq r_i$, where $d(v_i, v_j)$ denotes the Euclidean distance between v_i and v_j. Such a directed graphs G is called *Disk Graphs* (DG). An edge (v_i, v_j) is bidirectional if both (v_i, v_j) and (v_j, v_i) are in E, i.e., $d(v_i, v_j) \leq \min\{r_i, r_j\}$. Otherwise, it is a unidirectional edge. Figure 1 shows a disk graph (DG), here the black dots represents the sensor nodes and the dotted circles around them represents their transmission disks. The directed

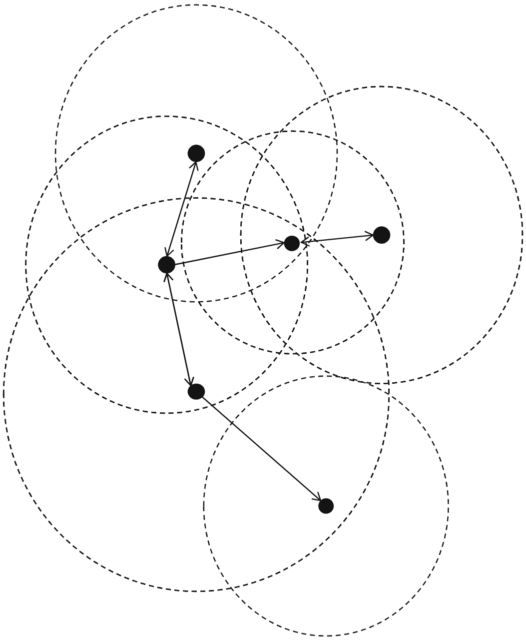

Fig. 1 A disk graph (DG) with unidirectional and bidirectional links

arrows represent the unidirectional links whereas the bidirected edge represents bidirectional links. In our network model, we consider both unidirectional and bidirectional edges.

The virtual backbone in a WSN that can be represented by the connected dominating set (CDS). In this chapter we studied the fault tolerance of the virtual backbone in the WSN modeled as a disk graph (DG). In this case the virtual backbone can be represented by a strongly connected dominating and absorbing set (SCDAS). There can be two kinds of faults occurring in WSN. A sensor node can become faulty or a link between two sensor nodes might go down. Hence, the fault tolerance of virtual backbone in WSN can be enhanced in two ways. Firstly, by enhancing the dominance and the absorption of the SCDAS representing the virtual backbone by ensuring more SCDAS nodes are there as incoming and outgoing neighbors to a non-SCDAS node. This ensures that a non-virtual backbone node has other options to forward its data, if one of its virtual backbone neighbor goes down due to some failure. Secondly, by ensuring there are multiple paths between the virtual backbone nodes in the subgraph generated by the virtual backbone nodes, so that if a link between two virtual backbone goes down it would not affect the connectivity of the virtual backbone. This can be achieved by increasing the connectivity of the SCDAS representing the virtual backbone.

Under such a model and requirements, we formulate the fault tolerant virtual backbone problem as follows:

***k*-Strongly Connected *m* Dominating and Absorbing Set problem ((k, m) SCDAS):** Given a directed graph $G = (V, E)$ representing a sensor network and two positive integers k and m, find a subset $C \subseteq V$ with a minimum size and satisfying the following conditions:

- C is an SCDAS
- The subgraph $G(C)$ is k-connected
- Each node not in C is dominated and absorbed by at least m nodes in C

3 Enhancing Domination and Absorption of the Virtual Backbone

In this section we study enhancing fault tolerance of the virtual backbone in the WSN represented by a directed disk graph $G = (V, E)$. The fault tolerance of a virtual backbone needs to be enhanced in two aspects, firstly in terms of domination and absorbtion, secondly, in terms of connectivity of the subgraph induced by the virtual backbone nodes. As a fault tolerant virtual backbone in the WSN with unidirectional and bidirectional links can be represented by a (k, m) SCDAS, hence, we propose an approximation algorithm for constructing a (k, m) SCDAS for a directed disk graph $G = (V, E)$ representing the WSN for any value of k and m. We also provide the theoretical analysis of our algorithm and conjectured its approximation ratio. The (k, m) SCDAS of graph G represents the virtual backbone of the WSN such that any node v not in virtual backbone has at least m virtual backbone nodes in $N^+(v)$ and $N^-(v)$, respectively, and the graph induced by the virtual backbone nodes is k-strongly connected. This ensures that the virtual backbone can sustain $m - 1$ virtual backbone nodes failure without isolating any non-virtual backbone node from the virtual backbone and it can sustain $k - 1$ virtual backbone nodes failure without disconnecting the virtual backbone. In order to generate a (k, m) SCDAS we first generate an $(1, m)$ SCDAS, which is a special case of (k, m) SCDAS where $k = 1$. We then enhance the connectivity of the subgraph induced by the $(1, m)$ SCDAS nodes to make it k-node connected, which results in a (k, m) SCDAS.

3.1 An Approximation Algorithm for (k, m) *SCDAS Problem*

The algorithm for generating the (k, m) SCDAS is illustrated in Algorithm 1. In order to generate a (k, m) SCDAS we first generate a $(1, m)$ SCDAS, which is a special case of (k, m) SCDAS where $k = 1$. The algorithm for generating a $(1, m)$ SCDAS is illustrated in Algorithm 2.

The construction of $(1, m)$ SCDAS is divided into two phases. In the first phase a strongly connected dominating and absorbing set (SCDAS) is generated and then in the second phase extra nodes are iteratively added to make it m dominating. In the first phase a strongly connected dominating and absorbing set is generated by

Algorithm 1 Approximation Algorithm for (k, m)-Strongly Connected Dominating and Absorbing Set

INPUT: An m-connected directed graph $G = (V, E)$, here $m \geq k$
OUTPUT: A (k, m) SCDAS C of G
Run Algorithm 2 on G to generated an $(1, m)$ SCDAS C.
for Every pair of black nodes $v_i, v_j \in C$ **do**
    $C = C \cup \text{Find k Path}(G, i, j, k)$
end for
Return C as the k-m-SCDAS

Algorithm 2 Algorithm for $(1, m)$ Strongly Connected Dominating and Absorbing Set

INPUT: An m connected directed graph $G = (V, E)$
OUTPUT: A $(1, m)$ SCDAS C of G
Generate a directed graph G' by reversing the edges of graph G
Select a nodes s as a root.
$C = \emptyset$
$C = C \cup \text{Find DS1 (G,s)}$;
$C = C \cup \text{Find DS1}(G',\text{s})$;
for $i = 1; i \leq m - 1; i++$ **do**
    Color all the Gray nodes in G and G' White
    $C = C \cup \text{Find DS2 (G)}$
    $C = C \cup \text{Find DS2}(G')$
end for
The set C is the $(1, m)$ SCDAS

calling Algorithm 3 twice. When Algorithm 3 terminates there are three different color nodes in the graph; the black nodes, the blue nodes, and the gray nodes. In first call to Algorithm 3 the graph G and a node s are passed as the parameter and it returns a set of black and blue nodes forming a directed dominating tree for G rooted at s. The black nodes in the tree form the dominating set of G and they are semi-independent to each other, they dominate all the gray nodes in the graph. The blue nodes act as connectors: they connect the black nodes in a way to form a directed tree rooted at s, as shown in Fig. 2a. In the second call to Algorithm 3 the inverse graph G' and the node s are passed as parameters. Similarly it returns a set of blue and black nodes forming a directed dominating tree for G' rooted at s. As the graph G' is the inverse graph of G, hence, the set of blue and black nodes forming a directed dominating tree for G' equivalently forms a directed absorbing tree in G, as shown in Fig. 2b. For all the gray nodes in G' the corresponding nodes in G are absorbed by the nodes in G corresponding to all the black nodes in G'. The union of the set of blue and black nodes returned for G and the set of nodes in G corresponding to the set nodes returned for G' forms a strongly connected dominating and absorbing set for G.

In the second phase extra nodes are added to enhance the dominance and the absorption of the strongly connected dominated and absorbing set to m. In order to do this $m - 1$ iterations are performed and in each iteration the Algorithm 4

Algorithm 3 Find DS1 (G,s)

```
Set S = ∅
S = S ∪ s
BLACK = ∅; BLUE = ∅
while There is a white node in G do
    Select a White node u ∈ S having maximum number of white nodes in N+(u)
    Color u black and remove it from S
    BLACK = BLACK ∪ u
    if u! = s then
        Color the Parent(u) Blue if it is Gray
        BLUE = BLUE ∪ Parent(u)
    end if
    Color all the nodes v ∈ N+(u) Gray
    for All the White node w ∈ N+(v) do
        if w ∉ S then
            S = S ∪ w
        end if
        Mark v as the parent of w
    end for
end while
Return BLACK ∪ BLUE
```

Algorithm 4 Find DS2(G)

```
BLACK = ∅
while There is a White node in G do
    Select a White node u having maximum number of white nodes in N+(u)
    Color u Black and all the White node v ∈ N+(u) Gray
    BLACK = BLACK ∪ u
end while
Return BLACK
```

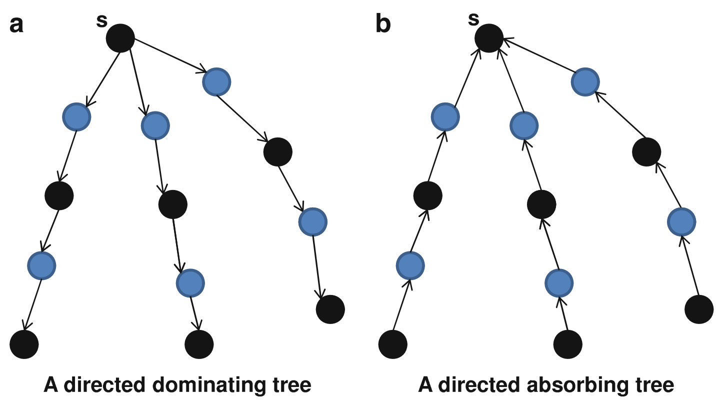

Fig. 2 A directed dominating and absorbing tree

Algorithm 5 Find k-Path(G, i, j, k)

Keeping vertex v_i as source and v_j as destination, construct a flow network G_f of G with $2 * |V - 2| + 2$ vertices and $|E| + |V - 2|$ edges
$S = \phi$
$G_r \leftarrow G_f$
flow$\leftarrow 0$
for $l = 1; l < k; l + +$ **do**
    Find an augmented path from v_i to v_j in G_r by increasing the *flow* by 1 unit.
    For all the saturated edges (v_{in}, v_{out}) on this augmented path color the corresponding nodes v in G blue if they are white and add them to S
    Update the residual network G_r
end for
Return S

is called twice. In the first call G is passed as a parameter, this results in the enhancement of the dominance of the SCDAS by one. In the second call G' is passed as a parameter, which results in the enhancement of the absorption of the SCDAS by one. After $m - 1$ the SCDAS becomes $(1, m)$ SCDAS.

Once the $(1, m)$ SCDAS is formed, for each ordered pair of nodes in $(1, m)$ SCDAS, $k - 1$ node disjoint paths are identified by running Find k-Path algorithm given in Algorithm 5. All white nodes on these paths are colored blue and are included in the virtual backbone. These nodes are called the connector nodes. Now, as the domination and the absorption of the virtual backbone is m and the connectivity of the subgraph generated by the virtual backbone nodes is k, hence, it forms a (k, m) SCDAS. One important thing to be noticed here is that to ensure that the subgraph $G((k, m)$ SCDAS) is k-connected and the graph G should be at least m-connected and $m \geq k$.

The Find k-Path Algorithm: The Find k-path Algorithm is illustrated in Algorithm 5. Given a k-connected directed graph $G = (V, E)$, and a pair of vertices $v_i, v_j \in V$, the algorithm finds the set of nodes on k node disjoint paths from v_i to v_j in graph G. The algorithm first generates a flow network G_f by partitioning each node $v \in V \setminus \{v_i, v_j\}$ into two nodes v_{in} and v_{out}. Then connecting v_{in} and v_{out} through a unidirectional edge $(v_{\text{in}}, v_{\text{out}})$ and assign this edge a capacity of 1 unit. All the incoming edges directed towards v in G are set as incoming edges to v_{in} in G_f whereas all the outgoing edges emanating from v in G are set as outgoing edges from v_{out} in G_f assign infinite capacity to these edges, this results in G_f having $2|V - 2| + 2$ nodes and $|E| + |V - 2|$ edges. Once the flow network G_f of graph G is formed, we run k iterations and in each iteration an augmented path in G_f from v_i to v_j is determined by increasing the flow by 1 unit. We consider that a unit flow is indivisible. On each newly found augmented path, for any saturated edge $(v_{\text{in}}, v_{\text{out}})$ we select the corresponding vertex v in G as a node on the k disjoint path from v_i to v_j. Figure 3 show the iterations of finding augmented paths between v_i and v_j for $k = 2$.

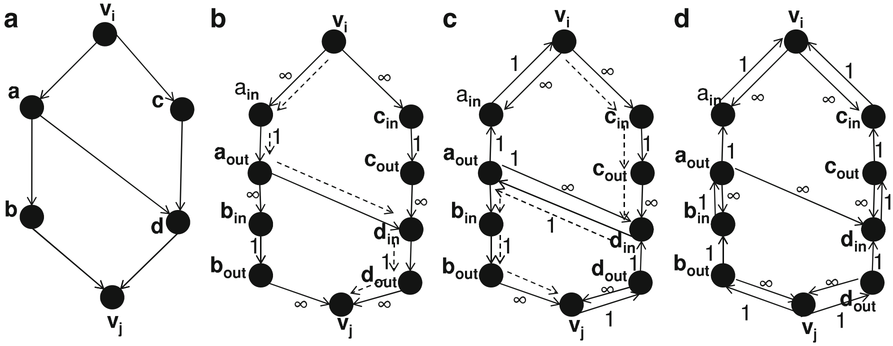

Fig. 3 Iterations for finding augmented paths for $k = 2$

3.1.1 Theoretical Analysis

Lemma 1. *The Algorithm 2 is correct and produces a virtual backbone which is* $(1, m)$ *SCDAS.*

Proof. In order to prove this lemma we need to show that the virtual backbone formed is strongly connected and m dominating and absorbing. The algorithm works in two phases, in the first phase it generates a dominating tree and an absorbing tree for the graph G. Let the set of blue and black nodes forming the dominating tree represented as D and the set of blue and black nodes forming the absorbing tree be represented as A. Now let the set of black nodes in D and A be represented as *Black*(D) and *Black*(A) respectively. The node s has a directed path using blue and black nodes to all the nodes in *Black*(D), and all the nodes in *Black*(A) has a directed path using blue and black nodes to s. Hence, the black nodes in *Black*(D)$\cap$ *Black*(A) has a path using blue and black nodes to all the other blue and black node in $D \cup A$. The black nodes in *Black*(A) $\setminus$ *Black*(D) have a directed blue–black path to s and as this node must be dominated by some black node in $Black(D)$ it must also have a directed blue–black path from s to it through its dominator. Similarly the nodes in *Black*(D) $\setminus$ *Black*(A) will have a blue–black path from and to the root node s. As all the black nodes in *Black*(D)$\cup$ *Black*(A) have a directed blue–black path from and to the root s, hence, all the nodes in $D \cup A$ are strongly connected and forms a SCDAS.

In the second phase extra nodes are added to enhance the domination and the absorption of the virtual backbone by $m - 1$. As all these extra nodes are dominated and absorbed by black nodes, hence, the extended virtual backbone will still be strongly connected. □

Lemma 2. *The number of Semi-Independent neighbors* K_{SI} *of any node u can be bounded by* $(2R + 1)^2$. *Here* $R = \frac{r_{max}}{r_{min}}$.

Proof. Let u be the node with transmission range $r_{\max}$. The number of semi-independent neighbors of u, i.e., $N^+(u) \cap SI$ can be bounded by $K_{\rm SI}$. It can be noticed that the distance between any two nodes v and $w \in SI$, i.e., $d(v, w) > r_{\min}$. Hence, the size of $N^+(u) \cap SI$, i.e., K_{SI} is bounded by the number of disjoint disks with radius $r_{\min}/2$ packing in the disk centered at u with radius of $r_{\max} + r_{\min}/2$. So, we have:

$$\left|N^+(v) \cap S\right| \leq \frac{\pi(r_{\max} + r_{\min}/2)^2}{\pi(r_{\min}/2)^2} \leq (2R + 1)^2. \tag{1}$$

□

Lemma 3. *Let $G = (V, E)$ be any directed graph (DG) with bounded transmission range ratio R, then the number of black nodes in a dominating set of G obtained on calling Algorithm 3 or the number of black nodes added on every call to Algorithm 4 in order to enhance the domination by 1 is bounded by: $|DS| \leq \left(\frac{K_{SI}}{m} + 1\right)|DS_m^*|$, here DS_m^* is the optimal solution for m dominating set of G.*

Proof. Let us consider DS and DS_m^*, there are two possible cases:

1. $\mathrm{DS} \subseteq \mathrm{DS}_m^*$
2. $\mathrm{DS} \subsetneq \mathrm{DS}_m^*$

Case (a): As $\mathrm{DS} \subseteq \mathrm{DS}_m^*$, we have $|\mathrm{DS}| \leq |\mathrm{DS}_m^*|$.
Case (b): $\forall u \in \mathrm{DS} \setminus \mathrm{DS}_m^*$, let $D_u = |\mathrm{DS}_m^* \cap N^-(u)|$. As DS_m^*, is an m dominating set of G, $D_u \geq m$ for each $u \in \mathrm{DS} \setminus \mathrm{DS}_m^*$ and we have:

$$\sum_{u \in \mathrm{DS}\setminus\mathrm{DS}_m^*} D_u \geq m\,|\mathrm{DS} \setminus \mathrm{DS}_m^*|\,. \tag{2}$$

For all $v \in \mathrm{DS}_m^*$, let

$$d_v = |(\mathrm{DS} \setminus \mathrm{DS}_m^*) \cap N^+(v)|. \tag{3}$$

As the black nodes in DS obtained on calling Algorithm 3 or obtained in every call to Algorithm 4 cannot have a bidirectional edge between each other, hence, they form a Semi-Independent set. From Lemma 2, we have $\forall v \in \mathrm{DS}_m^*$ there are at most K_{SI} Semi-independent nodes in its neighborhood, hence $d_v \leq K_{\mathrm{SI}}$. Therefore we have:

$$K_{SI}|\mathrm{DS}_m^*| \geq \sum_{v \in \mathrm{DS}_m^*} d_v. \tag{4}$$

However note that

$$\sum_{u \in \mathrm{DS}\setminus\mathrm{DS}_m^*} D_u = |\{(v, u) \in E | u \in \mathrm{DS} \setminus \mathrm{DS}_m^*, v \in \mathrm{DS}_m^*\}| = \sum_{u \in \mathrm{DS}_m^*} d_v. \tag{5}$$

From (2), (4) and (5) we have:

$$m|\mathrm{DS} \setminus \mathrm{DS}_m^*| \leq \sum_{u \in \mathrm{DS} \setminus \mathrm{DS}_m^*} D_u = \sum_{u \in \mathrm{DS}_m^*} d_v \leq K_{\mathrm{SI}} |\mathrm{DS}_m^*|. \tag{6}$$

Therefore,

$$m|\mathrm{DS} \setminus \mathrm{DS}_m^*| \leq K_{\mathrm{SI}} |\mathrm{DS}_m^*|. \tag{7}$$

Thus it follows that,

$$|\mathrm{DS}| \leq \left(\frac{K_{\mathrm{SI}}}{m} + 1 \right) |\mathrm{DS}_m^*|. \tag{8}$$

Therefore from the two cases (a) and (b), we conclude that

$$|\mathrm{DS}| \leq \left(\frac{K_{\mathrm{SI}}}{m} + 1 \right) |\mathrm{DS}_m^*|. \tag{9}$$

□

Lemma 4. *The number of nodes in m dominating set of G is at most* $(K_{SI} + m)$ $|DS_{1,m}^*|$*, here* $DS_{1,m}^*$ *is the optimal solution for* $(1, m)$ *SCDS.*

Proof. The number of nodes in m dominating set are $|\mathrm{DS}^1 \cup \mathrm{DS}^2 \cdots \cup \mathrm{DS}^n|$. Here DS^i is the set of nodes added in the ith iteration. Let $|\mathrm{DS}| = \max_i \{|\mathrm{DS}^i|\}$, then from Lemma 3, we have:

$$\left|\mathrm{DS}^1 \cup \mathrm{DS}^2 \ldots \cup \mathrm{DS}^n\right| \leq m|\mathrm{DS}| \leq m \left(\frac{K_{\mathrm{SI}}}{m} + 1 \right) |\mathrm{DS}_m^*| \leq (K_{\mathrm{SI}} + m)|\mathrm{DS}_{1,m}^*|. \tag{10}$$

□

Theorem 1. *The Algorithm 2 produces a* $(1, m)$ *SCDAS with the size bounded by* $2\left(K_{SI}\left(1 + \frac{1}{m}\right) + m + 1\right)|DAS_{1,m}^*|$*, here* $DAS_{1,m}^*$ *is the optimal solution for* $(1, m)$ *SCDAS.*

Proof. Let C denotes our solution to the $(1, m)$ SCDS. Let $BLUE$ and $BLACK$ be the set of blue and black nodes in G and $BLUE'$ and $BLACK'$ be the set of blue and black nodes in G' respectively. Then we have:

$$|C| = |BLUE| + |BLACK| + \left|BLUE'\right| + \left|BLACK'\right|. \tag{11}$$

When the Algorithm 3 runs on G and G' it results in a dominating tree for each of them, respectively. For both G and G' the dominating tree is rooted at same node s. The dominating tree for G' is equivalent to the absorbing tree for G. On every