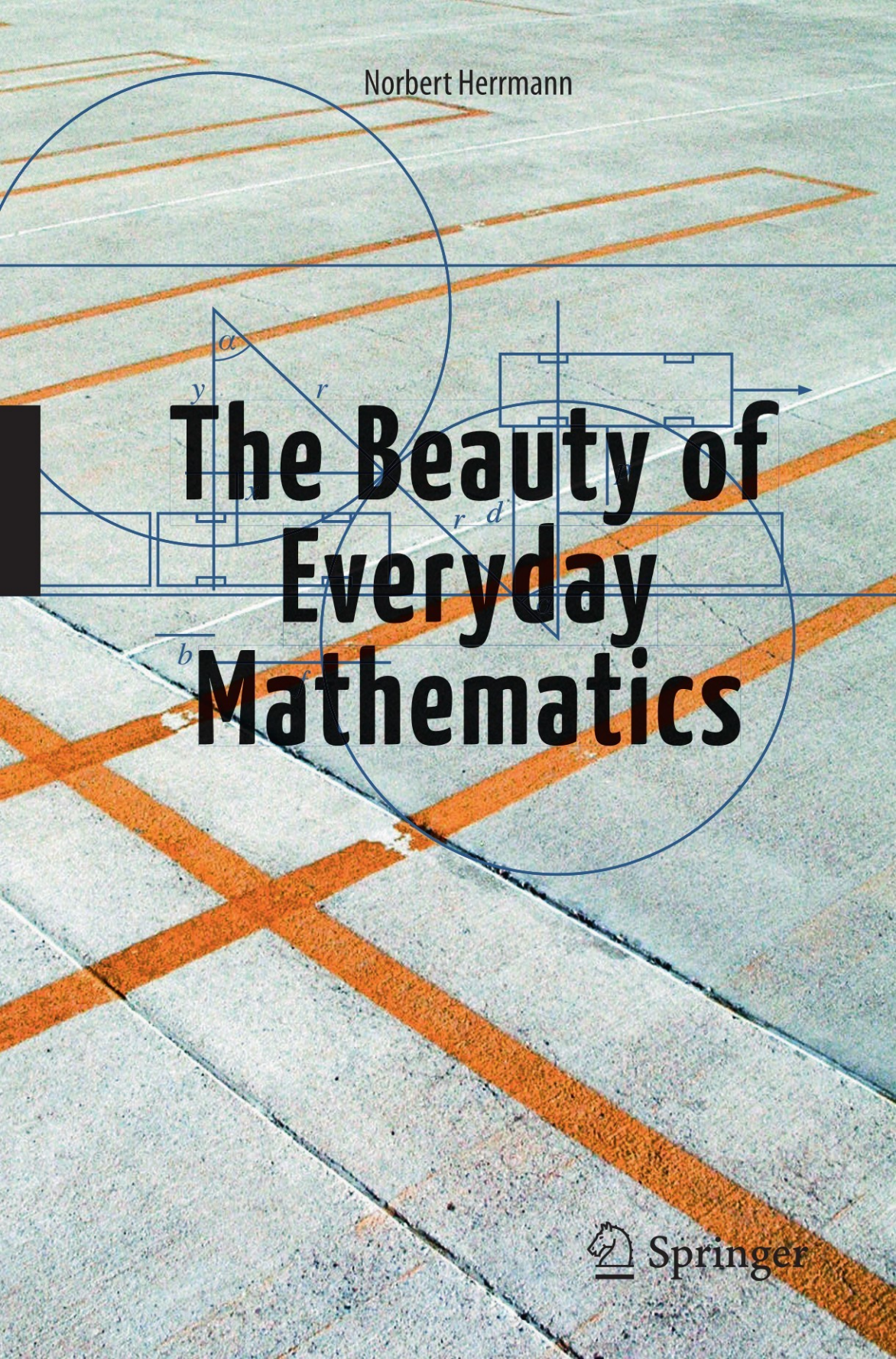

Norbert Herrmann

The Beauty of Everyday Mathematics

Springer

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Preface

Once upon a time, there was a group of representatives from the State of Utah in the United States of America around the year 1875. One of them was James A. Garfield. During a break, they were sitting in the congressional cafeteria. To pass the time, one of them, namely Mr. Garfield, suggested that they take a look at the Pythagorean Theorem. Even though this famous theorem had already been studied and proven 2000 years ago, he wanted to come up with a new proof. Together with his colleagues, he worked for a little while, and discovered the following construction:

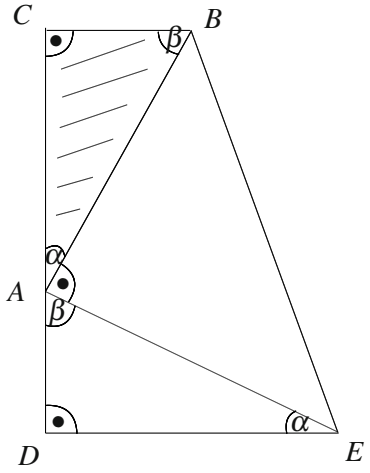

Fig. 0.1 Sketch proving the Pythagorean Theorem.

Here, we ave the crosshatched right triangle $\triangle ABC$. We sketch this triangle once more below it, though this time turned slightly so that side $\overline{AD}$ lies exactly on the extension of side $\overline{AC}$. The connecting line $\overline{EB}$ completes the figure, turning it into a trapezoid because the bottom side is parallel to the top side thanks to the right angles. The two triangles meet, with their angles α and β, at A. Because the triangles are right triangles, the two angles add up to 90°, from which we conclude immediately that the remaining angle at A is also a right angle. After all, three angles equal 180° when added together.

Now, only the little task remains of comparing the area of the trapezoid (central line times height, where the central line equals (base line + top line)/2 with the sum of the areas of the three right triangles:

$$\frac{a+b}{2} \cdot (b+a) = 2 \cdot \frac{a \cdot b}{2} + \frac{c^2}{2}.$$

The simple solution of this equation provides the formula of Mr. Pythagoras:

$$a^2 + b^2 = c^2.$$

The sum of the areas of the squares on the two legs equals the area of the square on the hypotenuse.

Mr. Garfield submitted this proof for publication. And, sure enough, the proof was actually published in the New England Journal of Education. The mere fact that there had been some representatives who had occupied their spare time during a break with mathematics would have been worth mentioning.

But now comes the most extraordinary aspect. The spokesman of these math fans was the James A. Garfield who, a little later, became the President of the United States.

You just have to savor the moment. A long, long time ago, there was actually once a president of the United States who published a new proof of Pythagoras, in the nineteenth century. He not only could recite this famous theorem, but also understood it completely and even proved it.

We wouldn't dare claim that many politicians today probably consider the Pythagorean Theorem to be a new collection of bed linen. But what is so remarkable is the fact that representatives whiled away their spare time with mathematical problems. Today, any mathematician who openly proclaims his or her profession is immediately confronted with the merry message that their listener has always been bad at math.

Mr. Garfield was only President for less than a year, because he was shot with a pistol by a crazy person in Washington's train station. He died soon after the attack. Is this maybe a reason why today's presidents, kings, chancellors, etc. avoid mathematics?

I truly hope that this little book will make a small contribution towards conveying the beauty of mathematics to everyone.

I would like to thank specifically my editor, Mr. Clemens Heine. His enthusiastic response to the idea of writing this book was very helpful. Many thanks to the Assistent Editor Mathematics, Mrs. Agnes Herrmann, for her cooperation during the preparation of this edition.

Last but not least, I would like to thank my wife, who cleaned my desk at home out of desperation while I was spreading chaos elsewhere in the house.

Meissen, November 2011 *Norbert Herrmann*

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