Nikolay Ivanov Kolev

Multiphase Flow Dynamics

4 Turbulence, Gas Absorption and Release, Diesel Fuel Properties

Second Edition

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Turbulence, Gas Adsorption and Release, Diesel Fuel Properties

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To Iva, Rali and Sonja with love

Balkan, Bulgaria (painting by Nikolay Ivanov Kolev, 2000)

My first physics teacher, my father Ivan Gutev (drawing by Nikolay Ivanov Kolev, 2000)

Родина

Че си нещо повече дълбоко в мене сещам,
отколкото земя в граници обгърната,
по тръпнещата болка, която често чувствам,
с косурите ти спорейки и с вярата невърната.

Намирам те в изгарящия порив
да те видя коваща мощни технологии
и новото под синия ти покрив,
да помита фразеологии и демагогии.

Намирам те в пулсиращата мисъл
от своя интелект нещо да ти дам,
че мускули и лакти животът е отписал
от средсвата градящи прогреса тъй желан.

В онези хора те намирам,
които с възрожденски дух горят
и с пламъка си бъдното трасират,
едничка полза те за теб да извлекат.

Те често си остават неразбрани,
понякога ги смазва простотия,
но макар и след години, лекувайки тез рани,
превръщаш делото им в светиня.

1983 Sofia

Nikolay Ivanov Kolev, PhD, DrSc
(born 1/8/1951, Gabrowo, Bulgaria)

Summary

This monograph contains theory, methods and practical experience for describing complex transient multi-phase processes in arbitrary geometrical configurations. It is intended to help applied scientists and practicing engineers to better understand natural and industrial processes containing dynamic evolutions of complex multi-phase flows. It is also intended to be a useful source of information for students in the high semesters and in PhD programs.

The monograph consists of five volumes:

- Volume 1, Fundamentals.
- Volume 2, Mechanical interactions.
- Volume 3, Thermal interactions.
- Volume 4, Turbulence, gas absorption and release, diesel fuel properties.
- Volume 5, Nuclear thermal hydraulics.

In Volume 1 the concept of three-fluid modeling is presented in detail "from origin to applications." This includes the derivation of local volume- and time-averaged equations and their working forms, the development of methods for their numerical integration and, finally, a variety of solutions for problems of practical interest.

Special attention is paid in Volume 1 to the link between the partial differential equations and the constitutive relations called closure laws, without providing too much information on the closure laws.

Volumes 2 and 3 are devoted to these important constitutive relations for the mathematical description of mechanical and thermal interactions. The structure of these two volumes is in fact a state-of-the-art review and selection of the best available approaches for describing interfacial transfer processes. In many cases the original author's contribution is incorporated in the overall presentation. The most important aspects of the presentation are that they stem from the author's long years of experience developing computer codes. The emphasis is on the practical use of these relationships: either as stand-alone estimation methods or within a framework of computer codes.

This book, Volume 4, is devoted to turbulence in multiphase flows as well as selected subjects in multiphase fluid dynamics that are very important for practical applications but could not find place in the first three volumes of this work. The state-of-the-art of turbulence modeling in multiphase flows is presented. First, some basics of single-phase boundary layer theory, including important scales and flow oscillation characteristics in pipes and rod bundles, are presented. Then the scales characterizing dispersed flow systems are presented. The description of the

turbulence is provided at different levels of complexity: simple algebraic models for eddy viscosity; algebraic models based on the *Boussinesq* hypothesis; modification of the boundary layer shear due to modification of the bulk turbulence; and modification of the boundary layer shear due to nucleate boiling. Then the role of the following forces on the mathematical description of turbulent flows is discussed: the lift force; the lubrication force in the wall boundary layer; and the dispersion force. A pragmatic generalization of the *k-eps* models for a continuous velocity field is proposed, covering flows in large volumes and flows in porous structures. Its large eddy simulation variant is also presented. A method of how to derive source and sink terms for multiphase *k-eps* models is also presented. A set of 13 single- and two-phase benchmarks for verification of *k-eps* models in system computer codes are provided and reproduced with the IVA computer code as an example of the application of the theory. This methodology is intended to help other engineers and scientists to introduce this technology step-by-step into their own engineering practice.

In many practical applications, gases are dissolved in liquids under certain conditions and released under other conditions, and therefore affect technical processes in many ways, both good and bad. There is almost no systematic description of this subject in the literature. That is why I decided to collect in Volume 3 useful information on the solubility of oxygen, nitrogen, hydrogen and carbon dioxide in water, valid within a large range of pressures and temperatures, providing appropriate mathematical approximation functions and validating them. In addition, methods for computation of the diffusion coefficients are described. With this information, the solution and dissolution dynamics of multiphase fluid flows can be analyzed. For this purpose the non-equilibrium absorption and release on bubble, droplet and film surfaces under different conditions is mathematically described.

In order to allow the theory from all four volumes to be applied to processes in combustion engines, a systematic set of internally consistent state equations for diesel fuel, gas and liquid, valid over a broad range of pressures and temperatures, are also provided in Volume 4.

Nuclear thermal hydraulics provides a description of the physical processes that occur in structural materials during the release of fission heat from nuclear reactions. During its release to the environment, the thermal energy can be harnessed to provide useful mechanical work or heat, or both. Volume 5 is devoted to nuclear thermal hydraulics. In a way this is the most essential application of multiphase fluid dynamics in analyzing steady and transient processes within nuclear power plants.

December 2010 Nikolay Ivanov Kolev
Herzogenaurach

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1 Some single-phase boundary layer theory basics

Hundreds of very useful constitutive relations that describe the interactions in multiphase flows are based on the achievements of single-phase boundary layer theory. That is why it is important to recall at least some of them, before moving on to more complex interactions in multiphase flow theory. My favorite book to start learning the main ideas of single-phase boundary layer theory is the famous monograph by *Schlichting* (1982). This chapter gives only the basics required to understand the rest of the book.

1.1 Flow over plates, velocity profiles, shear forces, heat transfer

Consider continuum flow parallel to a plate along the *x*-axis having velocity far from the surface equal to u_∞. The shear force acting on the surface per unit flow volume is then

$$f_w = \frac{F_w \tau_w}{V_{flow}}, \tag{1.1}$$

where the wall shear stress is usually expressed as

$$\tau_w = c_w\left(x\right)\frac{1}{2}\rho u_\infty^2 . \tag{1.2}$$

Here the friction coefficient c_w is obtained from the solution of the mass and momentum conservation at the surface.

1.1.1 Laminar flow over one site of a plane

For laminar flow over one site of a plane, the solution of the momentum equation delivers the local shear stress as a function of the main flow velocity and of the distance from the beginning of the plate as follows

$$c_w(x) = \frac{\tau_w(x)}{\frac{1}{2}\rho u_\infty^2} = \frac{0.332}{\left(\frac{u_\infty x}{\nu}\right)^{1/2}}, \tag{1.3}$$

Eq. (7.32) *Schlichting* (1982) p.140. The averaged drag coefficient over Δx is then

$$\overline{c_{w,\Delta x}} = \frac{F_w/(\Delta y \Delta x)}{\frac{1}{2}\rho u_\infty^2} = \frac{1.328}{\left(\frac{u_\infty \Delta x}{\nu}\right)^{1/2}}, \ \mathrm{Re}_{\Delta x} < 5\times 10^5, \tag{1.4}$$

Eq. (7.34) *Schlichting* (1982) p. 141. The corresponding heat transfer coefficients *h* are reported to be

$$Nu_x = \frac{hx}{\lambda} = \frac{1}{\sqrt{\pi}}\left(\frac{u_\infty x}{\nu}\right)^{1/2} \mathrm{Pr}^{1/2} \ \text{for } \mathrm{Pr} \rightarrow 0 \ \text{for liquid metals,} \tag{1.5}$$

$$Nu_x = \frac{hx}{\lambda} = 0.332\left(\frac{u_\infty x}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} \ \text{for } 0.6 < \mathrm{Pr} < 10, \tag{1.6}$$

$$Nu_x = \frac{hx}{\lambda} = 0.339\left(\frac{u_\infty x}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} \ \text{for } \mathrm{Pr} \rightarrow \infty, \tag{1.7}$$

Schlichting (1982) p. 303. Averaging over Δx results in

$$\overline{Nu_{\Delta x}} = 2 Nu_{\Delta x}. \tag{1.8}$$

Note that *Jukauskas* and *Jyugja* (1969) reported the same result:

$$Nu_x = \frac{hx}{\lambda} = 0.33\left(\frac{u_\infty x}{\nu}\right)^{1/2} \mathrm{Pr}^{1/3} \left(\mathrm{Pr}/\mathrm{Pr}_w\right)^{1/4}, \tag{1.9}$$

also taking into account the wall influence (Pr_w is computed at wall temperature). *Miheev* and *Miheeva* (1973) reported a favorable comparison with the above correlation with experimental data in their Fig. 3-6, p. 69.

1.1.2 Turbulent flow parallel to plane

For turbulent flow over one site of a plane, the solution of the momentum equation gives the local shear stress as a function of the main flow velocity and the distance from the beginning of the plate, as follows

$$c_w\left(x\right)=\frac{\tau_w\left(x\right)}{\frac{1}{2}\rho u_\infty^2}=\frac{0.0296}{\left(\frac{u_\infty x}{\nu}\right)^{1/5}}\ , \tag{1.10}$$

Eq. (21.12) *Schlichting* (1982) p. 653. This equation is obtained by assuming the validity of the so-called 1/7-th velocity profile,

$$\frac{u\left(x,y\right)}{u_{\max}}=\left(\frac{y}{\delta\left(x\right)}\right)^{1/7}\ , \tag{1.11}$$

with the boundary layer thickness varying with the distance from the edge of the plate in accordance with

$$\delta\left(x\right)=0.37x\left(\frac{u_\infty x}{\nu}\right)^{1/5}\ . \tag{1.12}$$

At a distance from the wall given by

$$y=\delta_{99\%}\approx 5\sqrt{\frac{u_\infty x}{\nu}} \tag{1.13}$$

the velocity reaches 99% of the flow mean velocity. $\delta_{99\%}$ is called the *displacement thickness*. The averaged *steady state* drag coefficient over Δx is then

$$\overline{c_{w,\Delta x}}=\frac{F_w/\left(\Delta y\Delta x\right)}{\frac{1}{2}\rho u_\infty^2}=\frac{0.074}{\mathrm{Re}_{\Delta x}^{1/5}}\ ,\ 5\times10^5<\mathrm{Re}_{\Delta x}<10^7\ , \tag{1.14}$$

Eq. (21.11) *Schlichting* (1982) p. 652. Here $\mathrm{Re}_{\Delta x}=u_\infty\Delta x/\nu$. The corresponding steady state local and averaged heat transfer coefficients *h* are reported to be

$$Nu_x=\frac{hx}{\lambda}=0.0296\left(\frac{u_\infty x}{\nu}\right)^{0.8}\mathrm{Pr}^{1/3}\ , \tag{1.15}$$

$$\overline{Nu_{\Delta x}} = \frac{h\Delta x}{\lambda} = 0.037\left(\frac{u_\infty \Delta x}{\nu}\right)^{0.8} \Pr^{1/3}, \tag{1.16}$$

respectively. The influence of the wall properties in the last equation is proposed by *Knudsen* and *Katz* (1958) to be taken into account by computing the properties at the following effective temperature

$$T_{eff} = T + \frac{0.1\Pr + 40}{\Pr + 72}\left(T_w - T\right). \tag{1.17}$$

The only known information for the influence of the unsteadiness of the far field velocity is given by *Sidorov* (1959),

$$\overline{c_{w,\Delta x}} = \frac{0.0263}{\left\{\mathrm{Re}_{\Delta x}\left[1-\left(1-\frac{0.78}{\mathrm{Re}_{\Delta x}^{1/14}}\right)^{-1}\frac{1}{u_\infty^2}\frac{du_\infty}{d\tau}\right]\right\}^{1/7}}. \tag{1.18}$$

Note that *Miheev* (1966) reported very similar results:

$$Nu_x = \frac{hx}{\lambda} = 0.03\left(\frac{u_\infty x}{\nu}\right)^{0.8} \Pr^{0.43}\left(\Pr/\Pr_w\right)^{1/4}, \tag{1.19}$$

$$\overline{Nu_{\Delta x}} = \frac{h\Delta x}{\lambda} = 0.037\left(\frac{u_\infty \Delta x}{\nu}\right)^{0.8} \Pr^{0.43}\left(\Pr/\Pr_w\right)^{1/4}, \tag{1.20}$$

taking also into account the wall influence ($\Pr_w$ is computed at wall temperature). *Miheev* and *Miheeva* (1973) reported a favorable comparison with the above correlation with experimental data in their Fig. 3-7, p. 70.

1.2 Steady state flow in pipes with circular cross sections

Consider continuum flow along the x-axis of a circular pipe having velocity cross section averaged velocity equal to $\overline{w}$. The shear force acting on the surface per unit flow volume is then

$$f_w = \frac{F_w \tau_w}{V_{flow}}, \tag{1.21}$$

where the wall shear stress is usually expressed as

$$\tau_w = c_w \frac{1}{2}\rho \overline{w}^2 . \qquad (1.22)$$

Here the friction coefficient c_w, called the *Fanning* factor in the literature, is obtained from the solution of the developed steady state mass and momentum conservation in the pipe. Replacing the wall surface to pipe volume ratio with $4/D_h$, we have for the wall friction force per unit volume of the flow

$$f_w = \frac{F_w}{V_{flow}} c_w \frac{1}{2}\rho \overline{w}^2 = \frac{4}{D_h} c_w \frac{1}{2}\rho \overline{w}^2 = \frac{\lambda_{fr}}{D_h} \frac{1}{2}\rho \overline{w}^2 . \qquad (1.23)$$

where

$$\lambda_{fr} = 4c_w , \qquad (1.24)$$

called the friction coefficient, is usually used in Europe. Note the factor of 4 between the *Fanning* factor and the friction coefficient and

$$\tau_w = \frac{\lambda_{fr}}{8}\rho \overline{w}^2 . \qquad (1.25)$$

Note that for a steady developed single flow, the momentum equation reads $\frac{1}{r}\frac{d}{dr}(r\tau) - \frac{dp}{dx} = 0$. With $\frac{dp}{dx} = \frac{dp_w}{dx}$, and therefore $\frac{1}{r}\frac{d}{dr}(r\tau) - \frac{dp_w}{dx} = 0$, we have $d(r\tau) = \frac{dp_w}{dx} rdr$, or after integrating

$$\tau(r) = \frac{dp_w}{dx}\frac{r}{2} , \qquad (1.26)$$

which gives at the wall the relation between the wall shear stress and the pressure gradient due to friction

$$\tau_w = \frac{dp_w}{dx}\frac{R}{2} . \qquad (1.27)$$

Usually for describing turbulent flows in pipe the following dimensionless variables are used: the friction velocity

$$w^* = \sqrt{\frac{\tau_w}{\rho}} = \overline{w}\sqrt{\frac{\lambda_{fr}}{8}}\,, \tag{1.28}$$

the dimensionless cross section averaged velocity

$$w^+ = w/w^*\,, \tag{1.29}$$

and the dimensionless distance from the wall

$$y^+ = y\,w^*/\nu\,, \tag{1.30}$$

where y is the distance from the wall. Note that $y\,w^*/\nu$ is in fact the definition of a boundary layer *Reynolds* number. With this transformation the measured mean velocity distribution near the wall is not strongly dependent on the *Reynolds* number as shown in Fig. 1.1. *Hammond* (1985) approximated this dependency by a continuous function of the type $y^+ = y^+\left(u^+\right)$, which must be inverted iteratively if one needs $u^+ = u^+\left(y^+\right)$.

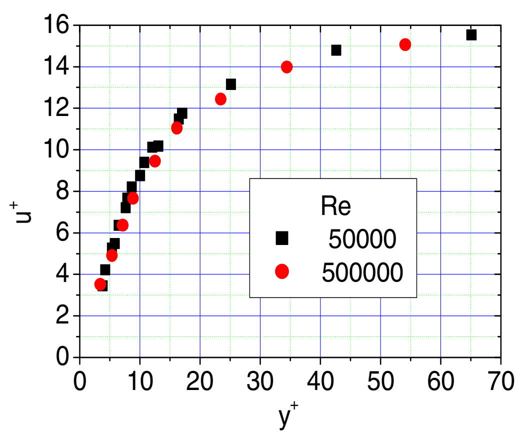

Fig. 1.1 Mean velocity distribution near the wall, *Laufer J* (1953)

The penetration of the wall roughness k into the boundary layer dictates different solutions of practical interest. Usually the dimensionless roughness of the surface $k^+ = kw^*/\nu$ is compared to the characteristic dimensionless sizes of the boundary layer to define the validity region of the specific solution of the momentum equation.

1.2.1 Hydraulically smooth wall surface

Hydraulically smooth surfaces are defined if

$$0 \le k^+ \le 5 . \qquad (1.31)$$

1.2.1.1 The Blasius solution

In 1911, *Blasius* obtained the following equation,

$$\boxed{\lambda_{fr} = 0.3164/\mathrm{Re}^{1/4} ,} \qquad (1.32)$$

where $\mathrm{Re} = \overline{w} D_h / \nu$. This result has been validated with his data and the data of other authors for $\mathrm{Re} < 10^5$. Later it was found that the velocity profile associated with this friction coefficient has the form

$$w(y) = \frac{(2n+1)(n+1)}{2n^2} \overline{w} \left(\frac{y}{R} \right)^{1/n} = w_{\max} \left(\frac{y}{R} \right)^{1/n} . \qquad (1.33)$$

It is known from the *Nikuradse* measurements that the exponent is a function of *Reynolds* number: for $\mathrm{Re} \le 1.1 \times 10^5$, $n = 7$

$$w(y) = w_{\max} \left(\frac{y}{R} \right)^{1/7} = \frac{60}{49} \overline{w} \left(\frac{y}{R} \right)^{1/7} , \qquad (1.34)$$

or

$$w^+(y) = 8.74 \left(y^+ \right)^{1/7} . \qquad (1.35)$$

and for $\mathrm{Re} \le 3.2 \times 10^6$, $n = 10$.

1.2.1.2 The Collins et al. solution

A more sophisticated solution than the *Blasius* profile, which depends on the *Reynolds* number, was proposed by *Collins* et al. (1978) and *Bendiksen* (1985):

$$\frac{w(r)}{w_{\max}} = 1 - \gamma r^2 - (1-\gamma) r^{2n} , \qquad (1.36)$$

$$\gamma = 7.5 / \left[4.12 + 4.95 \left(\log \mathrm{Re} - 0.743 \right) \right] , \qquad (1.37)$$

$$n = (\gamma - 1)\frac{\log \mathrm{Re} - 0.743}{\log \mathrm{Re} + 0.31} - \left(1 - \frac{1}{2}\gamma\right)^{-1} - 1. \quad (1.38)$$

The resulting friction coefficient is then

$$\frac{1}{\sqrt{\lambda_{fr}}} = 3.5 \log \mathrm{Re} - 2.6 . \quad (1.39)$$

1.2.1.3 The von Karman universal velocity profiles

Velocity profile: A more accurate mathematical representation of the velocity profiles in Fig. 1.1 was generalized by *von Karman* (1939), using the *Prandtl* mixing length theory

$$\sqrt{\overline{w'^2}} \approx \ell_w \frac{d\overline{w}}{dy},\ \sqrt{\overline{u'^2}} \approx \ell_u \frac{d\overline{w}}{dy},\ C_{wu}\sqrt{\overline{w'^2}}\sqrt{\overline{u'^2}} = C_{wu}\ell_w \ell_u \left(\frac{d\overline{w}}{dy}\right)^2 \approx \ell^2 \left(\frac{d\overline{w}}{dy}\right)^2 . \quad (1.40)$$

Here

$$C_{wu} = \frac{\overline{w'u'}}{\sqrt{\overline{w'^2}}\sqrt{\overline{u'^2}}} \quad (1.41)$$

is called *correlation coefficients between the fluctuations in both directions*. The effective turbulent cinematic viscosity is assumed to be proportional to the velocity gradient

$$\nu^t = \ell^2 \left|\frac{d\overline{w}}{dy}\right| \quad (1.42)$$

outside the laminar boundary layer. The *mixing length* is proposed to be proportional to the wall distance,

$$\ell = \kappa y , \quad (1.43)$$

a lucky abstract assumption which turns out to be useful. The constant $\kappa = 0.4$ is called the *von Karman* constant. The shear stress in the boundary layer is then

$$\tau = \rho\nu \frac{d\overline{w}}{dy} + \rho\overline{w'u'} = \rho\nu \frac{d\overline{w}}{dy} + \rho C_{wu}\sqrt{\overline{w'^2}}\sqrt{\overline{u'^2}} = \rho\nu \frac{d\overline{w}}{dy} + \rho\ell^2 \left(\frac{d\overline{w}}{dy}\right)^2 . \quad (1.44)$$

The dimensionless shear stress is divided by the wall shear stress to give

$$\frac{\tau}{\tau_w} = \rho \frac{\nu}{\tau_w} \frac{d\overline{w}}{dy} + \rho \frac{\ell^2}{\tau_w} \left(\frac{d\overline{w}}{dy} \right)^2 = \frac{dw^+}{dy^+} + \ell^{+^2} \left(\frac{dw^+}{dy^+} \right)^2 . \tag{1.45}$$

For a boundary layer flow with zero pressure gradient $\partial\tau/\partial y \approx 0$ at the wall and therefore $\tau \approx \tau_w$, the quadratic equation

$$\frac{dw^+}{dy^+} + \ell^{+^2} \left(\frac{dw^+}{dy^+} \right)^2 - 1 = 0 \tag{1.46}$$

can be solved with respect to the gradient and integrated over y. In this way a velocity profile can be generated. For negligible viscous stress $dw^+ = \frac{1}{\ell^+} dy^+$. For mixing length $\ell^+ = \kappa y^+$, the equation $dw^+ = \frac{1}{\kappa} d \ln y^+$ has the analytical solution

$$w^+ = \frac{1}{\kappa} \ln y^+ + const , \tag{1.47}$$

which is not dependent on the molecular viscosity. This is the *von Karman* law for fully turbulent flow. *Schlichting* found that the data of *Nikuradse* (1933) for $\mathrm{Re} < 3.4 \times 10^6$ are well reproduced by the velocity profile defined by

$$w_{\max} - w(y) = w^* 2.5 \ln(R/y) , \tag{1.48}$$

where

$$w = w_{\max} - 4.07 w^* . \tag{1.49}$$

Friction coefficient: The above profile dictates the following expression for the friction factor

$$\frac{1}{\sqrt{\lambda_{fr}}} = 2 \log\left(\mathrm{Re}\sqrt{\lambda_{fr}}\right) - 0.8 , \tag{1.50}$$

Schlichting (1982) p. 624. In fact this is Eq. (43) from *van Driest* (1955) p. 1011. Knowing the friction factor the friction velocity is

$$w^* = \overline{w}\sqrt{\lambda_{fr}/8} \tag{1.51}$$

and the profile is defined.

Von Karman (1939) introduced a buffer zone between the viscous and fully turbulent layers already introduced by *Prandtl* (1910). In the first zone close to the wall the flow is laminar. In the second and third the constants are computed so that they have smooth profiles. Table 1.1 summarizes the *Schlichting* velocity profiles.

Table 1.1 Velocity profiles in the boundary layer

Sub-layer	Defined by	Velocity profile
viscous	$y^+ \le 5$	$w^+ = y^+$
buffer	$5 \le y^+ \le 30$	$w^+ = 5\ln y^+ - 3.08$
fully turbulent	$30 < y^+$	$w^+ = 2.5\ln y^+ + 5.5$

Table 1.2 contains some important integrals of the widely used universal profile. Γ^+ is the volumetric flow rate per unit width of the wall.

Table 1.2 Dimensionless volumetric flow per unit width of the surface

Sub-layer	$\Gamma^+ = \int_0^{\delta^+} w^+ dy^+$	$\int_0^{\delta^+} \left(\frac{dw^+}{dy^+}\right)^2 dy^+$
viscous	$0.5\left(\delta^+\right)^2 \le 12.5$	δ^+
buffer	$5\delta^+ \ln \delta^+ - 8.08\delta^+ + 12.664$ ≤ 280.44	$5 + 25\left(\frac{1}{5} - \frac{1}{\delta^+}\right)$
fully turbulent	$2.5\delta^+ \ln \delta^+ + 3.5\delta^+ - 64.65$	$9.1667 + 6.25\left(\frac{1}{30} - \frac{1}{\delta^+}\right)$ for $\delta^+ \to \infty$, 9.735

Note that the boundary layer Reynolds number, defined as

$$Re_\delta = \overline{w}4\delta_2/\nu = \frac{\overline{w}}{w^*}\frac{4\delta_2 w^*}{\nu} = 4\Gamma^+, \tag{1.52}$$

is used for many applications in film flow theory. For film flow analysis it is interesting to find the inversed dependences from Table 1.2, namely $\delta^+ = \delta^+\left(\Gamma^+\right)$. Approximations for such dependences using profiles with a constant of 3.05 instead of 3.08 with an error of less than 4% was reported by *Traviss* et al. (1973):

$$\delta^+ = 0.707\,Re_\delta^{0.5} \text{ for } 0 < Re_\delta \le 50, \tag{1.53}$$

$$\delta^+ = 0.482\,Re_\delta^{0.585} \text{ for } 50 < Re_\delta \le 1125, \tag{1.54}$$

$$\delta^+ = 0.095\,\mathrm{Re}_\delta^{0.812} \text{ for } 1125 < \mathrm{Re}_\delta . \tag{1.55}$$

Note that these relations are implicit regarding the film thickness. For convenience some authors proposed explicit relations for the film thickness e.g. *Jaster* and *Kosky* (1976) for pipes:

$$\delta^+ = 0.7071\,\mathrm{Re}_2^{0.5} \text{ for } \mathrm{Re}_2 \le 1250 , \tag{1.56}$$

$$\delta^+ = 0.0504\,\mathrm{Re}_2^{0.875} \text{ for } 1250 < \mathrm{Re}_2 , \tag{1.57}$$

where $\mathrm{Re}_2 = (\rho w)_2 D_h / \eta_2$, D_h is the pipe diameter and the subscript 2 indicates a film. Observe the differences in constructing the *Reynolds* numbers Re_δ and Re_2: in the first case the characteristic length is $4\delta_2$, while in the second it is D_h.

After *Prandtl* and *von Karman* more complicated expressions for the mixing length were proposed: *van Driest* (1955) introduced the so-called damping function

$$\ell = \kappa y\left[1 - \exp\left(-y^+/26\right)\right], \tag{1.58}$$

which reflects the fact that the fluctuations are diminishing close to the wall, an observation already made by *Stokes* (1845). It is known that the constant actually depends on the Reynolds number and takes values between 20 and 30. The advantage of this function is that it gives a smooth velocity profile over the three layers discussed above.

Van Driest made the observation that roughness introduces additional turbulence in the boundary layer, so that for $k^+ > 60$ no damping is expected. Formally it is expressed by

$$\ell = \kappa y\left\{1 - \exp\left(-y^+/26\right) + \exp\left[-60\frac{y^+}{k^+}\Big/26\right]\right\} \text{ for } k^+ < 60 , \tag{1.59}$$

which goes to Eq. (1.58) if the roughness goes to zero. For rough walls defined with $k^+ > 60$ there is no longer any damping, and $\ell = \kappa y$.

For pipe flow, *Grötzbach* (2007) recommended combining the *Nikuradse* (1932) mixing length parabola with the *van Driest* damping factor:

$$\frac{\ell}{R} = \left[0.14 - 0.08\left(1-\frac{y}{R}\right)^2 - 0.06\left(1-\frac{y}{R}\right)^4\right]\left[1-\exp\left(-y^+/26\right)\right]. \tag{1.60}$$

Heterogeneous turbulence in the boundary layer: It was experimentally observed by *Laufer J* (1952, 1953) that the fluctuations of the velocity are equilateral in the central part of the pipe but heterogeneous close to the wall (Fig. 1.2). This is confirmed by many authors e.g. *Quarmby* and *Quirk* (1974). *Laufer*'s data indicate that the fluctuations of the axial velocities near the wall region are about three times larger than the fluctuations in the other directions – heterogeneous turbulence. The radial fluctuation velocity can be approximated by a *Boltzmann* function

$$u'^{+} = \frac{a_1 - a_2}{1 + e^{\left(y^+ - y_0^+\right)/dy^+}} + a_2 , \tag{1.61}$$

where $a_1 = -30.333\,65$, $a_2 = 0.89475$, $y_0^+ = -43.514\,54$ and $dy^+ = 12.72364$.

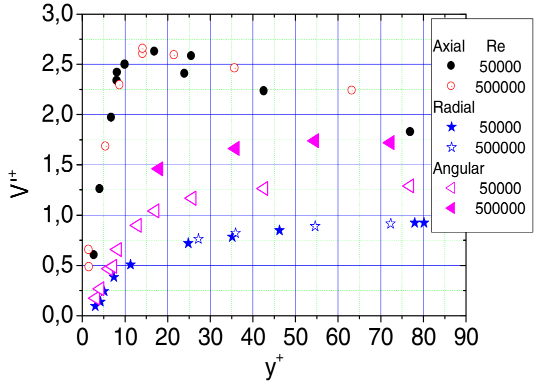

Fig. 1.2 Velocity fluctuations as a function of the distance from the wall measured by *Laufer J* (1953)

Observe that the dimensionless radial fluctuation velocity is almost independent of the *Reynolds* number and that for $y^+ > 30$ it is around 0.9 to 1. *Vames* and *Hanratty* (1988) reviewed turbulent measurements in a pipe and reported that close to the wall $r \to R$ the fluctuation velocity is

$$\boxed{v' \approx 0.9 w^* ,} \tag{1.62}$$

the characteristic time scale of turbulent pulsation is $\Delta\tau_e = 0.046\, D_h / w^*$ and the eddy diffusivity is $\nu^t = 0.037 w^* D$ (note that $\nu^t = v'^2 \Delta\tau_e = 0.0414 w^* D$). Knowing the fluctuation of the normal to the wall velocity in a pipe flow is important for analyzing deposition processes in particle-loaded flows. This is also essential for post-critical heat transfer description in annular flow with droplets in the gas core for pipes and rod bundles. The data obtained for the pipes can be used for bundles due to the systematical experimental observations reported by *Rehme* (1992) p. 572: "The experimental eddy viscosities normal to the wall are nearly independent of the relative gap width and are comparable to the data of circular tubes by *Reichardt* close to the walls …."

There are attempts to approximate the information presented in Fig. 1.2. *Matida* et al. (1998) proposed the following approximations for the pulsation of the velocity components close to the wall neglecting the dependence on the *Reynolds* number:

$$\frac{u'}{w^*} = \frac{0.5241 y^+}{1 + 0.0407 y^{+1.444}}, \tag{1.63}$$

$$\frac{v'}{w^*} = \frac{0.00313 y^{+2}}{1 + 0.00101 y^{+2.253}}, \tag{1.64}$$

$$\frac{w'}{w^*} = \frac{0.160 y^+}{1 + 0.0208 y^{+1.361}}. \tag{1.65}$$

In this region the fluctuation of the radial velocity measured for large *Reynolds* number by *Laufer SL* (1953) was approximated by *Lee* and *Durst* (1980) as follows

$$\frac{v'/w^*}{\ell/R} = 2.9 \left(\frac{R}{y} \right)^{0.4}. \tag{1.66}$$

Johansen (1991) reported the following approximation for pipe and channels flows with radius or half with *R* valid not only in the boundary layer but in the entire cross section:

a) The eddy viscosity

$$\frac{\nu^t}{\nu} = \left(\frac{y^+}{11.15} \right)^3, \; y^+ < 3, \tag{1.67}$$

$$\frac{\nu^t}{\nu} = \left(\frac{y^+}{11.4} \right)^2 - 0.049774, \; 3 \le y^+ < 52.108, \tag{1.68}$$

$$\frac{\nu^t}{\nu} = 0.4 y^+, \; 52.108 \le y^+, \tag{1.69}$$

is in agreement with the profiles computed with direct numerical simulation reported by *Kim* et al. (1987).
b) The profile of the time-averaged axial velocity in accordance with the above expressions is

$$\frac{w(y)}{w*} = 11.4 \tan^{-1}\left(\frac{y^+}{11.4}\right), \quad y^+ \le y_0^+, \tag{1.70}$$

$$\frac{w(y)}{w*} = 15.491 + 2.5 \ln\left(\frac{1+0.4y^+}{1+0.4y_0^+}\right), \quad y^+ > y_0^+, \tag{1.71}$$

$$y_0^+ = 52.984. \tag{1.72}$$

c) The fluctuation of the velocity normal to the wall

$$v'^+ = \frac{v'}{u*}, \tag{1.73}$$

approximated with

$$v'^+ = 0.033y^+\left[1 - \exp\left(-\frac{y^+}{3.837}\right)\right], \tag{1.74}$$

$$v'^+ = \exp\left[-\left(\frac{y^+}{30}\right)^{7.82} \Big/ 7.82\right], \quad y^+ \le 30, \tag{1.75}$$

$$v'^+ = v'^+(30) - \left[v'^+(30) - 0.65\right]\frac{y^+ - 30}{R^+ - 30}, \quad 30 \le y^+ \le R^+, \tag{1.76}$$

agree with the measurements of *Kutateladze* et al. (1979). The characteristic time scale of the fluctuation is then given by $\Delta\tau_e = \nu^t / v'^2$.

Wall boundary conditions for 3D-modeling: Using $k-\varepsilon$ models in computer codes with large scale discretization is very popular nowadays. In these codes the boundary layer cannot be resolved. For computing the bulk characteristics boundary conditions at the wall are required. Usually a point close to the wall e.g. $y_p^+ = 30$, is defined where the profile $w^+ = 2.5 \ln y^+ + 5.5$ starts to become valid. At this point the values of the turbulent kinetic energy per unit mass and its dissipation are

$$k_p = 2.5\, w^{*3} / y_p^+ , \tag{1.77}$$

$$\varepsilon_p = w^{*2} / \sqrt{c_\nu} , \tag{1.78}$$

with $c_\nu = 0.09$ coming from the definition equation of the turbulent cinematic viscosity $\nu^t = c_\nu k^2 / \varepsilon$, see for instance *Lee* et al. (1986). *Bradshaw* (1967) found experimentally a useful relationship $k \approx const\ \tau_w / \rho$ that can be used for this purpose as approximation. *Harsha* and *Lee* (1970) provided extensive measurements showing the correctness of the relation $k \approx 3.3\tau_w / \rho$ for wakes and jets. Computing the *Fanning* factor in $\tau_w = c_w \rho \overline{w}^2 / 2$ by using appropriate correlation we can approximate the specific turbulent energy at the wall region. Note that *Alshamani* (1978) reported that there is no linearity in the boundary layer $y^+ > 5$ but $k \approx \left(2.24 w'^+ - 1.13\right) \tau_w / \rho$, where w'^+ is the dimensionless fluctuation of the axial velocity.

For completeness, the *Yu* et al. (2001) velocity profile in a circular pipe is shown by direct numerical simulation to be adequate also for liquid metal flows with very low *Prandtl* number

$$w^+ = \frac{1}{0.436} \ln r^+ + 3.2 - 227 / r^+ + \left(50 / r^+\right)^2 , \tag{1.79}$$

where $r^+ = r\, w^* / \nu$.

1.2.1.4 The Reichardt solution

In looking for an appropriate pipe flow turbulence description that was valid not only up to the boundary layer but up to the axis of the pipe, *Reichardt* (1951) reproduced the available data for high *Reynolds* numbers in the form of turbulent cinematic viscosity as a function of the distance from the wall:

$$\frac{\nu^t}{w^* R} = \frac{\kappa}{3} \left[\frac{1}{2} + \left(1 - \frac{y}{R}\right)^2 \right] \left[1 - \left(1 - \frac{y}{R}\right)^2 \right] , \tag{1.80}$$

in

$$\tau = \rho \left(\nu + \nu^t\right) \frac{d\overline{w}}{dy} ,\ \tau / \tau_w = 1 - y / R . \tag{1.81}$$

Making a reasonable approximation *Reichardt* succeeded in obtaining a single equation for the velocity profile that covers all the regions from the wall to the axis:

$$\frac{w(y)}{w^*} = 2.5\ln\left[\left(1+0.4y^+\right)\frac{1.5\left(2-\frac{y}{R}\right)}{1+2\left(1-\frac{y}{R}\right)^2}\right]$$

$$+7.8\left[1-\exp\left(-y^+/11\right)-\frac{y^+}{11}\exp\left(-0.33y^+\right)\right]. \tag{1.82}$$

For $y << R$ the expression simplifies to

$$\frac{w(y)}{w^*} = 2.5\ln\left(1+0.4y^+\right) +7.8\left[1-\exp\left(-y^+/11\right)-\frac{y^+}{11}\exp\left(-0.33y^+\right)\right]. \tag{1.83}$$

The derivative

$$\frac{dw}{dy} = \frac{w^{*2}}{\nu}\left\{\frac{1}{1+0.4y^+}+\frac{7.8}{11}\left[\exp\left(-y^+/11\right)+\left(0.33y^+-1\right)\exp\left(-0.33y^+\right)\right]\right\}, \tag{1.84}$$

is very useful for computation of the lift force acting on small particles in the boundary layer at $y = R$. For upward bubbly flow it is directed from the wall into the bulk flow and for droplets from the boundary layer toward the wall.

Note that *Lee* et al. (1986) reported an alternative form of the cinematic viscosity as a function of the distance from the wall:

$$\frac{\nu^t}{\nu} = 0.4y^+\left[1-\frac{11}{6}\left(\frac{y}{R}\right)+\frac{4}{3}\left(\frac{y}{R}\right)^2-\frac{1}{3}\left(\frac{y}{R}\right)^3\right]\left[1-\exp\left(-\frac{y^+}{16}\right)\right]^2. \tag{1.85}$$

1.2.2 Transition region

The transition from the hydraulically smooth to completely rough region is defined by

$$5 \le k^+ \le 70. \tag{1.86}$$

In this case the velocity profile is

$$w_{max} - w(y) = w^* 2.5\ln\left(R/y\right), \tag{1.87}$$

$$\overline{w} = w_{max} - 3.75w^* . \tag{1.88}$$

The friction coefficient correlation proposed by *Colebrook* (1939),

$$\frac{1}{\sqrt{\lambda_{fr}}} = 1.74 - 2\log\left(\frac{k}{R} + \frac{18.7}{\mathrm{Re}\sqrt{\lambda_{fr}}}\right), \tag{1.89}$$

is valid for all roughness regimes. *Avdeev* (1982) proposed an explicit approximation of this equation which is more convenient

$$\frac{1}{\sqrt{\lambda_{fr}}} = 1.74 - 2\log\left(\frac{k}{R} + \frac{49}{\mathrm{Re}^{0.91}}\right). \tag{1.90}$$

Nikuradse (1933) proposed to describe the velocity profile close to the wall for sand roughness as follows

$$w^+ = \frac{1}{\kappa}\ln\left(y^+E\right), \tag{1.91}$$

where

$$E = 9.025 \text{ for } 0 \le k^+ \le 5 , \tag{1.92}$$

$$E = 41.35\Big/\left(k^+\right)^{0.945} \text{ for } 5 < k^+ \le 16 , \tag{1.93}$$

$$E = 115.8\Big/\left(k^+\right)^{1.318} \text{ for } 16 < k^+ \le 70 , \tag{1.94}$$

$$E = 30.03/k^+ \text{ for } k^+ > 70 . \tag{1.95}$$

1.2.3 Completely rough region

The completely rough region is defined by

$$70 \le k^+ . \tag{1.96}$$

The velocity profile is defined by

$$\frac{w(y)}{w^*} = 5.75\log\left(\frac{y}{k}\right) + 8.5 , \tag{1.97}$$

which results in

$$\overline{w}/w^* = \ln(R/k) + 4.75\,, \tag{1.98}$$

$$w_{\max} - w(y) = 3.75 w^*\,. \tag{1.99}$$

The friction coefficient found by *von Karman* is

$$\frac{1}{\sqrt{\lambda_{fr}}} = 2\log\left(\frac{R}{k}\right) + 1.60\,. \tag{1.100}$$

A slight change made by *Schlichting*

$$\frac{1}{\sqrt{\lambda_{fr}}} = 2\log\left(\frac{R}{k}\right) + 1.74\,, \tag{1.101}$$

gives the best fit to the *Nikuradse* data.

White (2006) proposed the following approximation of the velocity profile with an offset that depended on the type of the roughness

$$\frac{w(y)}{w^*} = \frac{1}{\kappa}\ln\left(\frac{yw^*}{\nu}\right) + B - \Delta B\,, \tag{1.102}$$

where

$$\Delta B = \frac{1}{\kappa}\ln\left(1 + c_B\frac{kw^*}{\nu}\right). \tag{1.103}$$

Here $B = 5$, $\kappa = 0.41$, $c_B \approx 0.3$ for sand roughness and $c_B \approx 0.8$ for stationary wavy wall data. The advantage of this approach is that it can also be used to describe the drag coefficient between the liquid and gas wavy interface. Comparing this with data, *Hulburt* et al. (2006) proposed to use $c_{B,2F,base} \approx 0.8$ for gas interaction with the *base* waves and $c_{B,2F,trav} \approx 4.7$ for gas interaction with the *traveling* waves.

1.2.4 Heat transfer to fluid in a pipe

Comparing the momentum and the energy conservation equation with steady developed flow in a pipe