Nikolay Ivanov Kolev

ourth Edition



Multiphase Flow Dynamics 1

Nikolay Ivanov Kolev

Multiphase Flow Dynamics 1

Fundamentals



Author

Dr. Nikolay Ivanov Kolev Möhrendorferstr. 7 91074 Herzogenaurach Germany E-mail: Nikolay.Kolev@herzovision.de

ISBN 978-3-642-20604-7

e-ISBN 978-3-642-20605-4

DOI 10.1007/978-3-642-20605-4

Library of Congress Control Number: 2002075843

© 2011 Springer-Verlag Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typeset & Cover Design: Scientific Publishing Services Pvt. Ltd., Chennai, India.

Printed on acid-free paper

987654321

springer.com

To Iva, Rali and Sonja with love!



Rügen, July, 2004, Nikolay Ivanov Kolev, 48×36 cm, oil on linen



Nikolay Ivanov Kolev, PhD, DrSc Born 1.8.1951, Gabrowo, Bulgaria

A Few Words about the New Editions of Volumes 1 through 5

The present content and format of the fourth, improved, and extended edition of Volumes 1, 2, and 3, and the second improved and extended edition of Volumes 4 and 5 were achieved after I received many communications from all over the world from colleagues and friends commenting on different aspects or requesting additional information. Of course, misprints and some layout deficiencies in the previous editions, for which I apologize very much, have also been removed, as is usual for subsequent editions of such voluminous 3000-page monographs. I thank everyone who contributed in this way to improving the five volumes!

The new editions contain my experiences in different subjects, collected during my daily work in this field since 1975. They include my own new results and the new information collected by colleagues since the previous editions. The overwhelming literature in multiphase fluid dynamics that has appeared in the last 40 years practically prohibits a complete overview by a single person. This is the reason why, inevitably, one or other colleague may feel that his personal scientific achievements are not reflected in this book, for which I apologize very much. However, it is the responsibility of transferring knowledge to the next generation that drove me to write these, definitely not perfect, books. I hope that they will help young scientists and engineers to design better facilities than those created by my generation.

29.12.2010 Herzogenaurach Nikolay Ivanov Kolev

Introduction

Multiphase flows, such as rainy or snowy winds, tornadoes, typhoons, air and water pollution, volcanic activities, etc., see Fig.1, are not only part of our natural environment but also are working processes in a variety of conventional and nuclear power plants, combustion engines, propulsion systems, flows inside the human body, oil and gas production, and transport, chemical industry, biological industry, process technology in the metallurgical industry or in food production, etc.



Fig. 1 The fascinating picture of the start of a discovery, a piece of universe, a tornado, a volcano, flows in the human heart, or even the "pure" water or the sky in Van Gogh's painting are, in fact, different forms of multiphase flows

The list is by far not exhaustive. For instance, everything to do with phase changes is associated with multiphase flows. The industrial use of multiphase systems requires methods for predicting their behavior. This explains the "explosion" of scientific publications in this field in the last 50 years. Some countries, such as Japan, have declared this field to be of strategic importance for future technological development.

Probably the first known systematic study on two-phase flow was done during the Second World War by the Soviet scientist *Teletov* and published later in 1958 with the title "On the problem of fluid dynamics of two-phase mixtures". Two books that appeared in Russia and the USA in 1969 by *Mamaev* et al. and by *Wallis* played an important role in educating a generation of scientists in this discipline, including me. Both books contain valuable information, mainly on steady state flows in pipes. In 1974 *Hewitt* and *Hall-Taylor* published the book "Annular two-phase flow", which also considers steady state pipe flows. The usefulness of the idea of a three-fluid description of two-phase flows was clearly demonstrated on annular flows with entrainment and deposition. In 1975 Ishii published the book "Thermo-fluid dynamic theory of two-phase flow", which contained a rigorous derivation of time-averaged conservation equations for the so called two-fluid separated and diffusion momentum equations models. This book founded the basics for new measurement methods appearing on the market later. The book was updated in 2006 by Ishii and Hibiki who included new information about the interfacial area density modeling in one-dimensional flows, which had been developed by the authors for several years. R. Nigmatulin published "Fundamentals of mechanics of heterogeneous media" in Russian in 1978. The book mainly considers one-dimensional two-phase flows. Interesting particular wave dynamics solutions are obtained for specific sets of assumptions for dispersed systems. The book was extended mainly with mechanical interaction constitutive relations and translated into English in 1991. The next important book for two-phase steam-water flow in turbines was published by *Deich* and *Philipoff* in 1981, in Russian. Again, mainly steady state, one-dimensional flows are considered. In the same year Delhaye et al. published "Thermohydraulics of two-phase systems for industrial design and nuclear engineering". The book contains the main ideas of local volume averaging and considers mainly many steady state one-dimensional flows. One year later, in 1982, Hetsroni edited the "Handbook of multiphase systems", which contained the state of the art of constitutive interfacial relationships for practical use. The book is still a valuable source of empirical information for different disciplines dealing with multiphase flows. In 2006 Crowe 2006 edited the "Multiphase flow handbook", which contained an updated state of the art of constitutive interfacial relationships for practical use. In the monograph "Interfacial transport phenomena" published by *Slattery* in 1990 complete, rigorous derivations of the local volumeaveraged two-fluid conservation equations are presented together with a variety of aspects of the fundamentals of the interfacial processes based on his long years of work. *Slattery*'s first edition appeared in 1978. Some aspects of the heat and mass transfer theory of two-phase flow are now included in modern textbooks such as "Thermodynamics" by Baer (1996) and "Technical thermodynamics" by Stephan and Mayinger (1998).

It is noticeable that none of the above mentioned books is devoted in particular to *numerical methods* of solution of the fundamental systems of partial differential equations describing multiphase flows. Analytical methods still do not exist. In 1986 I published the book "Transient two-phase flows" with Springer-Verlag, in German, and discussed several engineering methods and practical examples for integrating systems of partial differential equations describing two- and three-fluid flows in pipes.

Since 1984 I have worked intensively on creating numerical algorithms for describing complicated multiphase multicomponent flows in pipe networks and complex *three-dimensional* geometries mainly for nuclear safety applications. Note that the mathematical description of multidimensional two-phase and multiphase flows is a scientific discipline that has seen considerable activity in the last 30 years. In addition, for yeas thousands of scientists have collected experimental information in this field. However, there is still a lack of systematic presentation of the theory and practice of *numerical multiphase fluid dynamics*. This book is intended to fill this gap. Numerical multiphase fluid dynamics is the science of the derivation and the numerical integration of the conservation equations reflecting the mass momentum and energy conservation for multiphase processes in nature and technology at different scales in time and space. The emphasis of this book is on the generic links within computational predictive models between

- fundamentals,
- numerical methods,
- empirical information about constitutive interfacial phenomena, and
- a comparison with experimental data at different levels of complexity.

The reader will realize how strong the mutual influence of the four model constituencies is. There are still many attempts to attack these problems using singlephase fluid mechanics by simply extending existing single-phase computer codes with additional fields and linking with differential terms outside the code without increasing the strength of the feedback in the numerical integration methods. The success of this approach in describing low concentration suspensions and dispersed systems without strong thermal interactions should not confuse the engineer about the real limitations of this method.

This monograph can also be considered as a handbook on the numerical modeling of three strongly interacting fluids with dynamic fragmentation and coalescence representing multiphase multicomponent systems. Some aspects of the author's ideas, such us the three-fluid entropy concept with dynamic fragmentation and coalescence for describing multiphase, multicomponent flows by local volumeaveraged and time-averaged conservation equations, were published previously in separate papers but are collected here in a single context for the first time. An important contribution of this book to the state of the art is also the rigorous thermodynamic treatment of multiphase systems consisting of different mixtures. It is also the first time that the basics of the boundary fitted description of multiphase flows and an appropriate numerical method for integrating them with proven convergence has been published. It is well known in engineering practice that "the devil is hidden in the details". This book gives many hints and details on how to design computational methods for multiphase flow analysis and demonstrates the power of the method in the attached compact disc and in the last chapter in Volume 2 by presenting successful comparisons between predictions and experimental data or analytical benchmarks for a class of problems with a complexity not known in the multiphase literature until now. It starts with the single-phase U-tube problem and ends with explosive interaction between molten melt and cold water in complicated 3D geometry and condensation shocks in complicated pipe networks containing acoustically interacting valves and other components.

Volume 3 is devoted to selected subjects in multiphase fluid dynamics that are very important for practical applications but could not find place in the first two volumes of this work.

The state of the art of turbulence modeling in multiphase flows is also presented. As an introduction, some basics of the single-phase boundary layer theory, including some important scales and flow oscillation characteristics in pipes and rod bundles are presented. Then the scales characterizing the dispersed flow systems are presented. The description of the turbulence is provided at different levels of complexity: simple algebraic models for eddy viscosity, algebraic models based on the *Boussinesg* hypothesis, modification of the boundary layer share due to modification of the bulk turbulence, and modification of the boundary layer share due to nucleate boiling. Then the role of the following forces on the matematical description of turbulent flows is discussed: the lift force, the lubrication force in the wall boundary layer, and the dispersion force. A pragmatic generalization of the k-eps models for continuous velocity fields is proposed, which contains flows in large volumes and flows in porous structures. Its large eddy simulation variant is also presented. A method to derive source and sinks terms for multiphase k-eps models is presented. A set of 13 single- and two-phase benchmarks for verification of k-eps models in system computer codes are provided and reproduced with the IVA computer code as an example of the application of the theory. This methodology is intended to help engineers and scientists to introduce this technology step by step in their own engineering practice.

In many practical applications gases are dissolved in liquids under given conditions, released under other conditions, and therefore affect technical processes for good or for bad. There is almost no systematic description of this subject in the literature. That is why I decided to collect in Volume 3 useful information on the solubility of oxygen, nitrogen, hydrogen, and carbon dioxide in water, valid within large intervals of pressures and temperatures, provide appropriate mathematical approximation functions, and validate them. In addition, methods for computation of the diffusion coefficients are described. With this information solution and dissolution dynamics in multiphase fluid flows can be analyzed. For this purpose, the nonequilibrium absorption and release on bubble, droplet, and film surfaces under different conditions are mathematically described.

Volume 4 is devoted to nuclear thermal hydraulics, which is a substantial part of nuclear reactor safety. It provides knowledge and mathematical tools for the adequate description of the process of transferring the fission heat released in materials due to nuclear reactions into its environment. The heat release inside the fuel, the temperature fields in the fuels, and the "simple" boiling flow in a pipe, are introduced step by step, using ideas of different complexity like equilibrium, nonequilibrium, homogeneity, and nonhomogeneity. Then the "simple" three-fluid boiling flow in a pipe is described by gradually involving mechanisms like entrainment and deposition, dynamic fragmentation, collisions, and coalescence, turbulence. All heat transfer mechanisms are introduced gradually and their uncertainty is discussed. Different techniques like boundary layer treatments or integral methods are introduced. Comparisons with experimental data at each step demonstrate the success of the different ideas and models. After an introduction into the design of reactor pressure vessels for pressurized and boiling water reactors, the accuracy of modern methods is demonstrated using a large number of experimental data sets for steady and transient flows in heated bundles. Starting with single

pipe boiling going through boiling in a rod bundles the analysis of the complete vessel, including the reactor, is finally demonstrated. Then a powerful method for nonlinear stability analysis of flow boiling and condensation is introduced. Models are presented and their accuracies in describing critical multiphase flow at different level of complexity are investigated. The basics of the design of steam generators, moisture separators, and emergency condensers are presented. Methods for analyzing a complex pipe network flows with components like pumps, valves, etc., are also presented. Methods for the analysis of important aspects of severe accidents like melt-water interactions and external cooling and cooling of layers of molten nuclear reactor material are presented. Valuable sets of thermophysical and transport properties for severe accident analysis are presented for the following materials: uranium dioxide, zirconium dioxide, stainless steel, zirconium, aluminum, aluminum oxide, silicon dioxide, iron oxide, molybdenum, boron oxide, reactor corium, sodium, lead, bismuth, and lead-bismuth eutectic alloy. The emphasis is on the complete and consistent thermodynamical sets of analytical approximations appropriate for computational analysis. Thus the book presents a complete coverage of modern nuclear thermal hydrodynamics.





Fig. 2 Examples of multiphase flows in nuclear technology. See http://www.herzovision.de/kolev-nikolay/

Herzogenaurach, Winter 2010

Nikolay Ivanov Kolev

References

Baer HD (1996) Thermodynamik, Springer, Berlin Heidelberg New York

- Crowe CT ed. (2006) Multiphase flow handbook, Taylor & Francis, Boca Raton, London, New York
- Deich ME, Philipoff GA (1981) Gas dynamics of two phase flows. Energoisdat, Moscow (in Russian)
- Delhaye JM, Giot M, Reithmuller ML (1981) Thermohydraulics of two-phase systems for industrial design and nuclear engineering, Hemisphere, New York, McGraw Hill, New York
- Hetstroni G (1982) Handbook of multi phase systems. Hemisphere, Washington, McGraw-Hill, New York
- Hewitt GF, Hall-Taylor NS (1974) Annular two-phase flow, Pergamon, Oxford

Ishii M (1975) Thermo-fluid dynamic theory of two-phase flow, Eyrolles, Paris

Ishii M, Hibiki T (2006) Thermo-fluid dynamics of two-phase flow, Springer, New York

- Mamaev WA, Odicharia GS, Semeonov NI, Tociging AA (1969) Gidrodinamika gasogidkostnych smesey w trubach, Moskva
- Nigmatulin RI (1978) Fundamentals of mechanics of heterogeneous media, Nauka, Moscow, 336 pp (in Russian)
- Nigmatulin RI (1991) Dynamics of multi-phase media, revised and augmented edition, Hemisphere, New York
- Slattery JC (1990) Interfacial transport phenomena, Springer, Berlin Heidelberg New York
- Stephan K and Mayinger F (1998) Technische Thermodynamik, Bd.1, Springer, 15. Auflage
- Teletov SG (1958) On the problem of fluid dynamics of two-phase mixtures, I. Hydrodynamic and energy equations, Bulletin of the Moscow University, no 2 p 15

Wallis GB (1969) One-dimensional two-phase flow, McGraw-Hill, New York

Summary

This monograph contains theory, methods, and practical experience for describing complex transient multiphase processes in arbitrary geometrical configurations. It is intended to help applied scientists and practicing engineers to better understand natural and industrial processes containing dynamic evolutions of complex multiphase flows. The book is also intended to be a useful source of information for students in higher semesters and in PhD programs.

This monograph consists of five volumes:

Vol. 1 Fundamentals, 4th ed. (14 chapters and 2 appendixes), 782 pages.

Vol. 2 Mechanical interactions, 4th ed. (11 chapters), 364 pages,

Vol. 3 Thermal interactions, 4th ed. (16 chapters), 678 pages.

Vol. 4 Turbulence, gas absorption and release by liquid, diesel fuel properties, 2^{nd} ed. (13 chapters), 328 pages.

Vol. 5 Nuclear thermal hydraulics, 2nd ed. (17 chapters), 848 pages.

In Volume 1 the concept of three-fluid modeling is presented in detail "from the origin to the applications". This includes the derivation of local volume- and time-averaged equations and their working forms, the development of methods for their numerical integration, and finally a variety of solutions for different problems of practical interest.

Special attention is paid in Volume 1 to the link between partial differential equations and constitutive relations, called closure laws, without providing any information on the closure laws.

Volumes 2 and 3 are devoted to these important constitutive relations for the mathematical description of the mechanical and thermal interactions. The structure of the volumes is, in fact, a state of the art review and a selection of the best available approaches for describing interfacial transfer processes. In many cases, the original contribution of the author is incorporated in the overall presentation. The most important aspects of the presentation are that they stem from the author's long years of experience in developing computer codes. The emphasis is on the practical use of these relationships: either as stand-alone estimation methods or within a framework of computer codes.

Volume 4 is devoted to turbulence in multiphase flows.

Nuclear thermal hydraulics is the science providing knowledge about the physical processes occurring during the transfer of the fission heat released in structural materials due to nuclear reactions into its environment. Along its way to the environment thermal energy is organized to provide useful mechanical work or useful heat, or both. Volume 5 is devoted to the nuclear thermal hydraulics. In a way this is the most essential application of multiphase fluid dynamics in analyzing steady and transient processes in nuclear power plants.

In particular in Volume 1, Chapters 1, 2, 3, and 5, the concept of three-fluid modeling is introduced. Each field consists of multicomponents grouped into an inert and a noninert components group. Each field has its own velocity in space and its own temperature, allowing mechanical and thermodynamic nonequilibrium among the fields. The idea of dynamic fragmentation and coalescence is introduced. Using the Slattery-Whitaker local spatial averaging theorem and the Leibnitz rule, the local volume-averaged mass, momentum and energy conservation equations are rigorously derived for heterogeneous porous structures. Successively time averaging is performed. A discussion is provided on particle size spectra and averaging, cutting off the lower part of the spectrum due to mass transfer, the effect of the averaging on the effective velocity difference, etc. Chapter 1 also contains brief remarks on the kinematic velocity of density wave propagation in porous structures and on the diffusion term of void propagation in the case of pooling all the mechanical interactions in this kind of formalism. In the derivation of the momentum equations special attention is paid to rearranging the pressure surface integrals in order to demonstrate the physical meaning of the originating source terms in the averaged systems and their link to hyperbolicity. The Reynolds stress concept is introduced for multiphase flows. Chapter 2 also contains a collection of constitutive relations for lift- and virtual mass forces, for wall boundary layer forces, for forces causing turbulent diffusion, and for forces forcing the rejection of droplet deposition on a wall with evaporation.

Before deriving the energy conservation in Chapter 5, I provide Chapter 3 in which it is shown how to generate thermodynamic properties and the substantial derivatives for different kinds of mixtures by knowing the properties of the particular constituents. It contains the generalization of the theory of the equations of states for arbitrary real mixtures. With one and the same formalism a mixture of miscible and immiscible components in arbitrary solid, liquid, or gaseous states mixed and/or dissolved can be treated. This is a powerful method for creating a universal flow analyzer. Chapter 3 contains additional information on the construction of the saturation line by knowing pressure or temperature. An application of the material given in Chapter 3 is given in the new Volume 3 of this work to diesel fuel, where an inherently consistent set of equations of state for both gas and liquid is formulated. In addition, a section defining the equilibrium of gases dissolved in liquids is provided. These basics are then used in Volume 3 to construct approximations for the equilibrium solution concentrations of H₂, O₂, N₂ and CO₂ in water and to describe the nonequilibrium solution and dissolution at bubble, droplet, and film interfaces, which extend the applicability of the methods of multiphase fluid dynamics to flows with nonequilibrium solution and dissolution of gases. The generalizations of Chapter 3 are also used in Chapter 17 of Volume 4 to represent a variety of thermal properties including sodium vapor properties. An additional appendix to Chapter 3 shows a table where the partial derivatives of different forms of the equation of state is provided. This chapter provides the information necessary to understand the entropy concept, which is presented in Chapter 5.

In the author's experience understanding the complex energy conservation for multiphase systems and especially the entropy concept is very difficult for most students and practicing engineers. This is why Chapter 4 is provided as an introduction, showing the variety of notations of the energy conservation principle for single-phase multicomponent flows. Chapter 4 further contains a careful state of the art review for the application of the method of characteristics for modeling 1D and 2D flows in engineering practice.

The local volume-averaged and time-averaged energy conservation equation is derived in Chapter 5 in different notational forms in terms of specific internal energy, specific enthalpy, specific entropy, and temperatures. The introduction of the entropy principle for such complex systems is given in detail in order to enable the practical use of the entropy concept. The useful "conservation of volume" equation is also derived. Chapter 5 contains an additional example of the computation additional sections have been added to Chapter 5, which contain the different notations of energy conservation for lumped parameter volumes and steady state flows. The limiting case with gas flow in a pipe is considered in order to show the important difference to the existing gas dynamics solution where the irreversible heat dissipation due to friction is correctly taken into account.

Examples for a better understanding are given for the simple cases of lumped parameters – Chapter 6, infinite heat exchange without interfacial mass transfer, discharge of gas from a volume, injection of inert gas in a closed volume initially filled with inert gas, heat input in a gas in a closed volume, steam injection in a steam–air mixture, chemical reaction in a gas mixture in a closed volume, and hydrogen combustion in an inert atmosphere. Chapter 6 has been extended with cases including details of the modeling of combustion and detonation of hydrogen by taking into account the equilibrium dissociation.

The exergy for a multiphase, multicomponent system is introduced in Chapter 7 and discussed for the example of judging the efficiency of a heat pump.

Simplification of the resulting system of PDEs to the case of one-dimensional flow is presented in Chapter 8. Some interesting aspects of fluid structure coupling, such as pipe deformation due to temporal pressure change in the flow and forces acting on the internal pipe walls are discussed. The idea of algebraic slip is presented. From the system thus obtained the next step of simplification leads to the system of ordinary differential equations describing the critical multiphase, multicomponent flow by means of three velocity fields. Modeling of valves and pumps is discussed in the context of the modeling of networks consisting of pipes, valves, pumps, and other different components.

Another case of simplification of the theory of multiphase flows is presented in Chapter 9, where the theory of continuum sound waves and discontinuous shock waves for melt-water interaction is presented. In order to easily understand it, the corresponding theory for single- and two-phase flows is reviewed as an introduction. Finally, an interesting application for the interaction of molten uranium and aluminum oxides with water, as well of the interaction of molten iron with water is presented. Chapter 9 also deals with detonation during melt-water interaction. To better put this information into the context of the detonation theory, additional introductory information is given for the detonation of hydrogen in closed pipes, taking into account the dissociation of the generated steam.

Chapter 10 is devoted to the derivation of the conservation equations for multiphase, multicomponent, multivelocity field flow in general curvilinear coordinate systems. For a better understanding of the mathematical basics used in this chapter two appendixes are provided: Appendix 1 in which a brief introduction to vector analysis is given and Appendix 2 in which the basics of the coordinate transformation theory are summarized.

A new Chapter 11 gives the mathematical tools for computing eigenvalues and eigenvectors and for determination of the type of systems of partial differential equations. The procedure for the transformation of a hyperbolic system into canonical form is also provided. Then the relations between eigenvalues and critical flow and between eigenvalues and the propagation velocity of small perturbations are briefly defined. This is, in fact, a translation of one chapter of my first book published in German by Springer in 1986. This completes the basics of the multiphase, multicomponent flow dynamics.

Chapter 12 describes numerical solution methods for different multiphase flow problems. The first-order donor-cell method is presented in detail by discretizing the governing equations, creating a strong interfacial velocity coupling, and strong pressure-velocity coupling. Different approximations for the pressure equations are derived and three different solution methods are discussed in detail. One of them is based on the Newton iterations for minimizing the residuals by using the conjugate gradients. A method for temperature inversion is presented. Several details are given, which enables scientists and engineers to use this chapter for their own computer code development, such as the integration procedure (implicit method), the time step, and accuracy control. Finally, some high-order discretization schemes for convection-diffusion terms such as space exponential scheme and other high-order up-winding schemas are presented. Different analytical derivations are provided in Appendixes 12.1-12.8, including the analytical derivatives of the residual error of each equation with respect to the dependent variables. Some important basic definitions that are required for describing pipe networks are introduced. In addition, the variation of volumeporosity with time is systematically incorporated into the numerical formalism.

Chapter 13 presents a numerical solution method for multiphase flow problems in multiple blocks of curvilinear coordinate systems, generalizing, in fact, the experience gained in Chapter 12. Several important details of how to derive explicit pressure equations are provided. The advantage of using orthogonal grids also is easily derived from this chapter. Appendixes 1 and 2 of Volume I contain some additional information about orthogonal grid generation.

Chapter 14 provides several numerical simulations as illustrations of the power of the methods presented in this monograph. A compact disc that contains films corresponding to particular cases discussed in this chapter is attached. The films can be played with any tool capable of accepting *avi*- or animated *gif*-files.

As has already been mentioned, Volumes 2 and 3 are devoted to the so called closure laws: the important constitutive relations for mechanical and thermal interactions. The structure of the volume has the character of a state of the art review and a selection of the best available approaches for describing interfacial

processes. In many cases, the original contribution of the author is incorporated into the overall presentation. The most important aspects of the presentation are that they stem from the author's long years of experience in developing computer codes. The emphasis is on the practical use of these relationships: either as stand alone estimation methods or within a framework of computer codes.

Volume 4 is devoted to selected chapters of the multiphase fluid dynamics that are important for practical applications: The state of the art of the turbulence modeling in multiphase flows is presented. As an introduction, some basics of singlephase boundary layer theory, including some important scales and flow oscillation characteristics in pipes and rod bundles are presented. Then the scales characterizing dispersed flow systems are presented. The description of turbulence is provided at different level of complexity: simple algebraic models for eddy viscosity, algebraic models based on the *Boussinesq* hypothesis, modification of the boundary layer share due to modification of the bulk turbulence, and modification of the boundary layer share due to nucleate boiling. Then the role of the following forces on the matematical description of turbulent flows is discussed: the lift force, the lubrication force in the wall boundary layer, and the dispersion force. A pragmatic generalization of the k-eps models for continuous velocity fields, which contains flows in large volumes and flows in porous structures, is proposed. A method of how to derive source and sink terms for multiphase k-eps models is presented. A set of 13 single- and two phase benchmarks for the verification of k-eps models in system computer codes is provided and reproduced with the IVA computer code as an example of the application of the theory. This methodology is intended to help other engineers and scientists to introduce this technology step by step in their own engineering practice.

In many practical application gases are solved in liquids under given conditions, released under other conditions, and therefore affect technical processes for good of for bad. There is almost no systematical description of this subject in the literature. This is why I decided to collect useful information on the solubility of oxygen, nitrogen, hydrogen, and carbon dioxide in water under large intervals of pressures and temperatures, and provide appropriate mathematical approximation functions and validate them. In addition, methods for computation of the diffusion coefficients are described. With this information solution and dissolution dynamics in multiphase fluid flows can be analyzed. For this purpose, the nonequilibrium absorption and release on bubble, droplet, and film surfaces under different conditions is mathematically described.

In order to allow the application of the theory from the first three volumes also to processes in combustion engines, a systematic set of internally consistent state equations for diesel fuel gas and liquid valid in a broad range of changing pressures and temperatures is provided.

Volume 5 is devoted to nuclear thermal hydraulics, which is a substantial part of nuclear reactor safety. It provides knowledge and mathematical tools for an adequate description of the process of the transfer of the fission heat released in materials due to nuclear reactions into its environment. It step by step introduces the reader into the understanding of the "simple" boiling flow in a pipe described mathematically using ideas of different complexity like equilibrium, nonequilibrium, homogeneity, and nonhomogeneity. Then the mathematical description of the heat release inside the fuel, the resulting temperature distribution inside the fuels, and the interaction of the fuel with the cooling fluid are introduced. Next, the "simple" three-fluid boiling flow in a pipe is described by gradually involving the mechanisms like entrainment and deposition, dynamic fragmentation, collisions, coalescence, and turbulence. All heat transfer mechanisms are introduced gradually by discussing their uncertainty. Different techniques are introduced, like boundary layer treatments or integral methods. Comparisons with experimental data at each step demonstrate the success of the different ideas and models. After an introduction into the design of the reactor pressure vessels for pressurized and boiling water reactors the accuracy of modern methods is demonstrated using a large number of experimental data sets for steady and transient flows in heated bundles. Starting with single pipe boiling going through to boiling in rod bundles the analysis of the complete vessel including the reactor is finally demonstrated. Then a powerful method for nonlinear stability analysis of flow boiling and condensation is introduced. Models are presented and their accuracies for describing critical multiphase flow at different level of complexity are investigated. The basics of the design of steam generators, moisture separators, and emergency condensers are presented. Methods for analyzing complex pipe network flows with components like pumps, valves, etc., are also presented. Methods for the analysis of important aspects of severe accidents like melt-water interactions, external cooling, and cooling of layers of molten nuclear reactor material are presented. Valuable sets of thermophysical and transport properties for severe accident analysis are presented for the following materials: uranium dioxide, zirconium dioxide, stainless steel, zirconium, aluminum, aluminum oxide, silicon dioxide, iron oxide, molybdenum, boron oxide, reactor corium, sodium, lead, bismuth, and lead-bismuth eutectic alloy. The emphasis is on the complete and consistent thermodynamical sets of analytical approximations appropriate for computational analysis. Thus, the book presents a complete coverage of modern nuclear thermal hydrodynamics.

29.12.2010 Herzogenaurach

Nomenclature

cross-section, m²

Latin

А

Α	surface vector
а	speed of sound, m/s
a_{lw}	surface of the field <i>l</i> wetting the wall <i>w</i> per unit flow volume $\sum_{l=1}^{l_{max}} Vol_l$
	belonging to control volume <i>Vol</i> (local volume interface area density of the structure w) m^{-1}
$a_{l\sigma}$	surface of the velocity field l contacting the neighboring fields per unit
	flow volume $\sum_{l=1}^{l_{max}} Vol_l$ belonging to control volume <i>Vol</i> (local volume
	interface area density of the velocity field l), m^{-1}
a_l	total surface of the velocity field l per unit flow volume $\sum_{l=1}^{l_{max}} Vol_l$ belong-
	ing to control volume Vol (local volume interface area density of the velocity field l), m^{-1}
Cu_i	Courant criterion corresponding to each eigenvalue, <i>dimensionless</i>
C_{il}	mass concentration of the inert component i in the velocity field l
с" С	coefficients, dimensionless
C_m	mass concentration of the component m in the velocity field,
	dimensionless
C_i	mass concentration of the component <i>i</i> in the velocity field, <i>dimensionless</i>
C_p	specific heat at constant pressure, $J/(kgK)$
c^{vm}	virtual mass force coefficient, dimensionless
c^{d}	drag force coefficient, dimensionless
$c^{\scriptscriptstyle L}$	lift force coefficient, dimensionless
D_{hy}	hydraulic diameter (4 times cross-sectional area / perimeter), m
D_{3E}	diameter of the entrained droplets, m
D_{ld}	size of the bubbles produced after one nucleation cycle on the solid
	structure, bubble departure diameter, <i>m</i>

size of bubbles produced after one nucleation cycle on the inert solid D_{1dm} particles of field m = 2, 3critical size for homogeneous nucleation, m D_{lch} critical size in presence of dissolved gases, m D_{lcd} D'_l most probable particle size, m characteristic length of the velocity field l, particle size in case of D_{l} fragmented field, m D_{il}^l coefficient of molecular diffusion for species i into the field l, m^2/s coefficient of turbulent diffusion. m^2/s D_{il}^t D_{il}^* total diffusion coefficient, m^2/s DC_{ii} right-hand side of the nonconservative conservation equation for the inert component, $kg/(sm^3)$ diffusivity, m^2/s D total differential d Ε total energy, J specific internal energy, J/kg e function introduced first in Eq. (42), Chapter 2 $F(\xi)$ $F, f(\dots$ function of $(\dots$ force per unit flow volume, N/m^3 f f fraction of entrained melt or water in the detonation theory surfaces separating the velocity field l from the neighboring structure F_{lw} within Vol. m^2 surfaces separating the velocity field *l* from the neighboring velocity field $F_{l\sigma}$ within Vol. m^2 surface defining the control volume Vol. m^2 F frequency of the nuclei generated from one activated seed on the particle f_{im} belonging to the donor velocity field m, s^{-1} frequency of the bubble generation from one activated seed on the f_{lw} channel wall, s^{-1} coalescence frequency, s^{-1} $f_{l,coal}$ acceleration due to gravity, m/s^2 g Η height, m h specific enthalpy, J/kg eigenvectors corresponding to each eigenvalue h_i I unit matrix, dimensionless i unit vector along the x-axis J matrix, Jacobian j unit vector along the y-axis

k k	unit vector along the <i>k</i> -axis				
k k	kinetic energy of turbulent pulsation m^2/s^2				
k^T	coefficient of thermodiffusion, <i>dimensionless</i>				
k^p	coefficient of harodiffusion dimensionless				
\mathbf{L}_{il}	length m				
M_i	kg-mole mass of the species <i>i</i> , <i>kg/mole</i>				
m	total mass, kg				
$\mathbf{n}_{\Delta \mathbf{V}}$	unit vector pointing along ΔV_{ml} , dimensionless				
n	unit vector pointing outwards from the control volume Vol, dimensionless				
\mathbf{n}_{le}	unit surface vector pointing outwards from the control volume Vol				
$\mathbf{n}_{l\sigma}$	unit interface vector pointing outwards from the velocity field <i>l</i>				
n_{il}	number of the particle from species i per unit flow volume, m^{-3}				
n_l	number of particles of field <i>i</i> per unit flow volume, particle number				
	density of the velocity field l, m^{-3}				
n_{coal}	number of particles disappearing due to coalescence per unit time and				
'n	unit volume, m ²				
$n_{l,kin}$	particle production rate due to indecation during evaporation of $(1, 2)$				
	condensation, $1/(m's)$				
n''_{lw}	number of the activated seeds on unit area of the wall, m^{-2}				
\dot{n}_{lh}	number of the nuclei generated by homogeneous nucleation in the donor				
	velocity field per unit time and unit volume of the flow, $1/(m^3s)$				
$\dot{n}_{l,dis}$	number of the nuclei generated from dissolved gases in the donor velocity				
	field per unit time and unit volume of the flow, $1/(m^3 s)$				
$\dot{n}_{l,sp}$	number of particles of the velocity field l arising due to hydrodynamic				
	disintegration per unit time and unit volume of the flow, $1/(m^3s)$				
Р	probability				
Р	irreversibly dissipated power from the viscous forces due to deformation				
Dor	berimeter m				
пет п	l = 1: partial pressure inside the velocity field l				
r ∼li	l = 2.3: pressure of the velocity field l				
р	pressure, Pa				
<i>ġ</i> ‴	thermal power per unit flow volume introduced into the fluid, W/m^3				
$\dot{q}_{\sigma l}^{\prime\prime\prime}$	l = 1,2,3. Thermal power per unit flow volume introduced from the				
	interface into the velocity field l , W/m^3				

$\dot{q}'''_{w\sigma l}$	thermal power per unit flow volume introduced from the structure						
	interface into the velocity field l , W/m^3						
R	mean radius of the interface curvature, m						
$\mathbf{r}(x,y,z)$	position vector, m						
K S	(with indexes) gas constant, $J/(kgK)$						
s S	total entropy <i>I/K</i>						
s S	specific entropy, J/(kgK)						
Sc^{t}	turbulent Schmidt number, dimensionless						
Sc^{tn}	is the turbulent Schmidt number for particle diffusion, dimensionless						
Т	temperature, K						
T_l	temperature of the velocity field <i>l</i> , <i>K</i>						
Т	shear stress tensor, N/m^2						
t	unit tangent vector						
U	dependent variables vector						
Vol	control volume, m^3						
Vol	size of the control volume, <i>m</i>						
Vol_l	volume available for the field l inside the control volume, m^3						
$\sum_{l=1}^{l_{\max}} Vol_l$	volume available for the flow inside the control volume, m^3						
V	instantaneous fluid velocity with components, u , v , w in r , θ , and z						
	direction, <i>m/s</i>						
$\mathbf{V}_l^{ au}$	instantaneous field velocity with components, $u_l^{\vartheta}, v_l^{\tau}, w_l^{\tau}$ in r, θ , and z direction, m/s						
\mathbf{V}_l	time-averaged velocity, <i>m/s</i>						
\mathbf{V}_{l}^{\prime}	pulsation component of the instantaneous velocity field, m/s						
$\Delta \mathbf{V}_{lm}$	$\mathbf{V}_l - \mathbf{V}_m$, velocity difference, disperse phase <i>l</i> , continuous phase <i>m</i>						
	carrying $l, m/s$						
$\delta_i V_l^{ au}$	diffusion velocity, <i>m/s</i>						
$\mathbf{V}_{l\sigma}^{ au}$	interface velocity vector, <i>m/s</i>						
$\mathbf{V}_{l}^{\tau} \boldsymbol{\gamma}$	instantaneous vector with components, $u_l^{\vartheta} \gamma_r, v_l^{\tau} \gamma_{\theta}, w_l^{\tau} \gamma_z$ in r, θ , and z						
	directions, <i>m/s</i>						
ν	specific volume, m^3 / kg						
x	mass fraction, dimensionless						
У	distance between the bottom of the pipe and the center of mass of the						
	liquid, <i>m</i>						
×	vector product						

Greek

α_l	part of $\gamma_{\nu} Vol$ available to the velocity field <i>l</i> , local instantaneous volume					
	fraction of the velocity field <i>l</i> , <i>dimensionless</i>					
$lpha_{_{il}}$	the same as α_l in the case of gas mixtures; in the case of mixtures					
	consisting of liquid and macroscopic solid particles, the part of $\gamma_{v} Vol$					
	available to the inert component <i>i</i> of the velocity field <i>l</i> local instantane-					
	ous volume fraction of the inert component i of the velocity field l .					
	dimensionless					
$lpha_{l,\max}$	≈ 0.62 , limit for the closest possible packing of particles, dimensionless					
γ_{v}	the part of <i>dVol</i> available for the flow, volumetric porosity, <i>dimensionless</i>					
γ	surface permeability, dimensionless					
$\vec{\gamma}$	directional surface permeability with components $\gamma_r, \gamma_{\theta}, \gamma_z$,					
	dimensionless					
Δ	finite difference					
δ	small deviation with respect to a given value					
δ_l	= 1 for continuous field;					
	= 0 for disperse field, <i>dimensionless</i>					
д	partial differential					
Е	dissipation rate for kinetic energy from turbulent fluctuation, power					
	irreversibly dissipated by the viscous forces due to turbulent fluctuations,					
	W / kg					
η	dynamic viscosity, kg/(ms)					
θ	θ -coordinate in the cylindrical or spherical coordinate systems, <i>rad</i>					
K	= 0 for Cartesian coordinates,					
	= 1 for cylindrical coordinates					
K	isentropic exponent					
κ_l	curvature of the surface of the velocity field <i>l</i> , <i>m</i>					
λ	thermal conductivity, <i>W</i> /(<i>mK</i>)					
λ	eigenvalue					
$\mu_l^{ au}$	local volume-averaged mass transferred into the velocity field l per unit					
	time and unit mixture flow volume, local volume-averaged instantaneous					
	mass source density of the velocity field <i>l</i> , $kg/(m^3s)$					
μ_l	time average of μ_l^r , $kg/(m^3s)$					
$\mu_{\scriptscriptstyle wl}$	mass transport from exterior source into the velocity field <i>l</i> , $kg / (m^3 s)$					

 μ_{il}^{τ} local volume-averaged inert mass from species *i* transferred into the velocity field *l* per unit time and unit mixture flow volume, local volume-averaged

instantaneous mass source density of the inert component *i* of the velocity field *l*, $kg/(m^3s)$

- μ_{il} time average of μ_{il}^{τ} , $kg/(m^3s)$
- μ_{iml}^{τ} local volume-averaged instantaneous mass source density of the inert component *i* of the velocity field *l* due to mass transfer from field *m*, $kg/(m^3s)$

$$\mu_{iml}$$
 time average of μ_{iml}^{τ} , $kg/(m^3s)$

 μ_{ilm}^{t} local volume-averaged instantaneous mass source density of the inert component *i* of the velocity field *l* due to mass transfer from field *l* into velocity field *m*, $kg/(m^{3}s)$

$$\mu_{ilm}$$
 time average of μ_{ilm}^{τ} , kg / $(m^3 s)$

- v cinematic viscosity, m^2 / s
- V_1^t coefficient of turbulent cinematic viscosity, m^2/s

$$V_1^m$$
 coefficient of turbulent particles diffusion, m^2/s

$$\xi$$
 angle between $\mathbf{n}_{l\sigma}$ and $\Delta \mathbf{V}_{lm}$, rad

- ρ density, kg/m^3
- ρ instantaneous density, density; without indexes, mixture density, kg/m^3
- ρ_l instantaneous field density, kg/m^3
- ρ_{il} instantaneous inert component density of the velocity field l, kg/m^3
- $\langle \rho_l \rangle^l$ intrinsic local volume-averaged phase density, kg/m^3
- $(\rho w)_{23}$ entrainment mass flow rate, $kg/(m^2 s)$
- $(\rho w)_{32}$ deposition mass flow rate, $kg/(m^2 s)$
- $\left(\rho_{l}\mathbf{V}_{l}^{\tau}\right)^{le}$ local intrinsic surface mass flow rate, $kg/(m^{2}s)$

 σ , σ_{12} surface tension between phases 1 and 2, *N/m*

- τ time, s
- φ angle giving the projection of the position of the surface point in the plane normal to ΔV_{im} , *rad*
- $\chi_l^{m\sigma}$ the product of the effective heat transfer coefficient and the interfacial area density, $W/(m^3K)$. The subscript *l* denotes inside the velocity field *l*. The superscript $m\sigma$ denotes location at the interface σ dividing field *m* from field *l*. The superscript is only used if the interfacial heat transfer is associated with mass transfer. If there is heat transfer only, the

linearized interaction coefficient is assigned the subscript ml only, indicating the interface at which the heat transfer takes place.

Subscripts

с	continuous				
d	disperse				
lm	from <i>l</i> to <i>m</i> or <i>l</i> acting on <i>m</i>				
W	region "outside of the flow"				
е	entrances and exits for control volume Vol				
l	velocity field <i>l</i> , intrinsic field average				
i	inert components inside the field l , noncondensable gases in the gas field $l = 1$, or microscopic particles in water in field 2 or 3				
i	corresponding to the eigenvalue λ_i in Chapter 4				
М	noninert component				
т	mixture of entrained coolant and entrained melt debris that is in thermal and mechanical equilibrium behind the shock front				
ml	from <i>m</i> into <i>l</i>				
iml	from <i>im</i> into <i>il</i>				
max	maximum number of points				
n	inert component				
0	at the beginning of the time step				
Ε	entrainment				
coal	coalescence				
sp	splitting, fragmentation				
σ	interface				
au	old time level				
$\tau + \Delta \tau$	new time level				
*	initial				
0	reference conditions				
<i>p</i> , <i>v</i> , <i>s</i>	at constant <i>p</i> , <i>v</i> , <i>s</i> , respectively				
L	left				
R	right				
1	vapor or in front of the shock wave				
2	water or behind the shock wave				
3	melt				
4	entrained coolant behind the front - entrained coolant				
5	microparticles after the thermal interaction – entrained melt				

Superscripts

- í time fluctuation
- ' saturated steam
- " saturated liquid

	saturated solid phase
Α	air
d	drag
е	heterogeneous
i	component (either gas or solid particles) of the velocity field
$i_{\rm max}$	maximum for the number of the components inside the velocity field
L	lift
l	intrinsic field average
le	intrinsic surface average
$l\sigma$	averaged over the surface of the sphere
т	component
n	normal
n	old iteration
<i>n</i> +1	new iteration
t	turbulent, tangential
vm	virtual mass
τ	temporal, instantaneous
_	averaging sign

Operators

$\nabla \cdot$	divergence
∇	gradient
∇_n	normal component of the gradient
∇_t	tangential component of the gradient
∇_l	surface gradient operator, 1/m
∇^2	Laplacian
$\langle \rangle$	local volume average
$\langle \rangle'$	local intrinsic volume average
$\langle \rangle^{le}$	local intrinsic surface average

Nomenclature required for coordinate transformations

- (x, y, z) coordinates of a Cartesian, left oriented coordinate system (*Euclidean* space). Another notation which is simultaneously used is x_i (i = 1, 2, 3): x_1, x_2, x_3
- (ξ, η, ζ) coordinates of the curvilinear coordinate system called transformed coordinate system. Another notation which is simultaneously used is ξ^i $(i = 1, 2, 3): \xi^1, \xi^2, \xi^3$



 \sqrt{g} Jacobian determinant or Jacobian of the coordinate transformation

$$x = f(\xi, \eta, \zeta), y = g(\xi, \eta, \zeta), z = h(\xi, \eta, \zeta)$$

- *a_{ii}* elements of the *Jacobian* determinant
- a^{ij} elements of the determinant transferring the partial derivatives with respect to the transformed coordinates into partial derivatives with respect to the physical coordinates. The second superscript indicates the Cartesian components of the contravariant vectors
- $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ covariant base vectors of the curvilinear coordinate system tangent vectors to the three curvilinear coordinate lines represented by (ξ, η, ζ)
- $(\mathbf{a}^1, \mathbf{a}^2, \mathbf{a}^3)$ contravariant base vectors, normal to a coordinate surface on which the coordinates ξ , η and ζ are constant, respectively

$$\begin{array}{ll} g_{ij} & \text{covariant metric tensor (symmetric)} \\ g^{ij} & \text{contravariant metric tensor (symmetric)} \\ \left(\mathbf{e}^{1}, \mathbf{e}^{2}, \mathbf{e}^{3}\right) & \text{unit vectors normal to a coordinate surface on which the coordinates} \\ & \boldsymbol{\xi}, \ \boldsymbol{\eta} \ \text{and} \ \boldsymbol{\zeta} \ \text{are constant, respectively} \\ \mathbf{V}^{i} &= \mathbf{a}^{i} \cdot \mathbf{V}, \ \text{contravariant components of the vector } \mathbf{V} \\ \mathbf{V}_{i} &= \mathbf{a}_{i} \cdot \mathbf{V}, \ \text{covariant components of the vector } \mathbf{V} \\ \left(\boldsymbol{\gamma}_{\boldsymbol{\xi}}, \boldsymbol{\gamma}_{\boldsymbol{\eta}}, \boldsymbol{\gamma}_{\boldsymbol{\zeta}}\right) \ \text{permeabilities of coordinate surfaces on which the coordinates} \ \boldsymbol{\xi}, \ \boldsymbol{\eta} \\ & \text{and} \ \boldsymbol{\zeta} \ \text{are constant, respectively} \end{array}$$

Greek

Α,α	Alpha	Ι, ι	Iota	Σ, σ	Sigma
B, β	Beta	Κ, κ	Kappa	Τ, τ	Tau
Γ,γ	Gamma	Λ,λ	Lambda	Φ , φ	Phi
Δ, δ	Delta	Μ, μ	Mu	Χ, χ	Chi
Ε, ε	Epsilon	Ν, ν	Nu	Υ, υ	Ypsilon
Ζ, ζ	Zeta	Ξ, ξ	Xi	Ψ,ψ	Psi
Η,η	Eta	0, <i>o</i>	Omikron	Ω,ω	Omega
Θ. 19	Theta	Π, π	Pi		
-, 0		Ρ, ρ	Rho		

Table of Contents

1	Ma	ss conservation	1
	1.1	Introduction	1
	1.2	Basic definitions	2
	1.3	Nonstructured and structured fields	9
	1.4	The Slattery and Whitaker local spatial averaging theorem	10
	1.5	General transport equation (<i>Leibnitz rule</i>)	12
	1.6	Local volume-averaged mass conservation equation	13
	1.7	Time average	16
	1.8	Local volume-averaged component conservation equations	18
	1.9	Local volume- and time-averaged conservation equations	20
	1.10	Conservation equations for the number density of particles	24
	1.11	Implication of the assumption of monodispersity in a cell	30
		1.11.1 Particle size spectrum and averaging	30
		1.11.2 Cutting of the lower part of the spectrum due to	
		mass transfer	31
		1.11.3 The effect of averaging on the effective velocity	
		difference	33
	1.12	Stratified structure	35
	1.13	Final remarks and conclusions	35
Refe	erence	es	37
2	Con	servation of Momentum	41
	2.1	Introduction	41
	2.2	Local volume-averaged momentum equations	42
		2.2.1 Single-phase momentum equations	42
		2.2.2 Interface force balance (momentum jump condition)	42
		2.2.3 Local volume averaging of the single-phase	
		momentum equation	49
	2.3	Rearrangement of the surface integrals	51
	2.4	Local volume average and time average	55
	2.5	Dispersed phase in a laminar continuum – pseudo turbulence	56
	2.6	Viscous and Reynolds stresses	57
	2.7	Nonequal bulk and boundary layer pressures	61
		2.7.1 Continuous interface	61
		2.7.2 Dispersed interface	76
	2.8	Working form for the dispersed and continuous phase	93
	2.9	General working form for dispersed and continuous phases	97

	2.10) So	me practical simplifications	99
	2.11	Co	nclusion	103
Appe	endix	2.1		103
Appe	endix	2.2		105
Appe	endix	2.3		105
Refe	rence	es		110
3	Der	ivati	ves for the equations of state	117
0	3.1	Intro	oduction	117
	3.2	Mul	ti-component mixtures of miscible and non-miscible	
		com	ponents	119
		3.2.	1 Computation of partial pressures for known mass	
			concentrations, system pressure and temperature	120
		3.2.	2 Partial derivatives of the equation of state	
			$\rho = \rho(p, T, C_{2,\dots,i_{\max}})$	127
		3.2.	3 Partial derivatives in the equation of state	
			$T = T(\varphi, p, C_2)$, where $\varphi = s, h, e$	132
		37	A Chemical notential	141
		3.2.	5 Partial derivatives in the equation of state	141
		5.2.	$\rho = \rho(p, \sigma(C))$ where $\sigma = \sigma(b, \sigma)$	152
			$p - p(p; \varphi; \mathcal{C}_{2,\dots,i_{\max}})$, where $\varphi = s, h, e$	132
	3.3	Mix	ture of liquid and microscopic solid particles of	1.5.4
		diffe	trent chemical substances	154
		3.3.	1 Partial derivatives in the equation of state $(, -)$	
			$\rho = \rho(p,T,C_{2,\dots,i_{\max}})$	155
		3.3.	2 Partial derivatives in the equation of state	
			$T = T(p, \varphi, C_{2,\dots,i_{\max}})$ where $\varphi = h, e, s$	155
	3.4	Sing	le-component equilibrium fluid	156
		3.4.1	Superheated vapor	157
		3.4.2	Reconstruction of equation of state by using a limited	
			amount of data available	158
		3.4.3	3 Vapor-liquid mixture in thermodynamic equilibrium	165
		3.4.4	Liquid-solid mixture in thermodynamic equilibrium	166
		3.4.5	Solid phase	166
	3.5	Exte	nsion state of liquids	167
Appe	endix	3.1	Application of the theory to steam-air mixtures	167
Appe	endix	3.2	Useful references for computing properties of single	1.00
A		2.2	constituents.	169
Арре	enuix	. 3.3	distributions and relations between merinodynamic	171
Pafa	ronce		quantities	1/1
Kele	icite			172
4	On	the v	ariety of notations of the energy conservation for	
	sing	gle-pl	nase flow	175
	4.1	Intro	duction	175