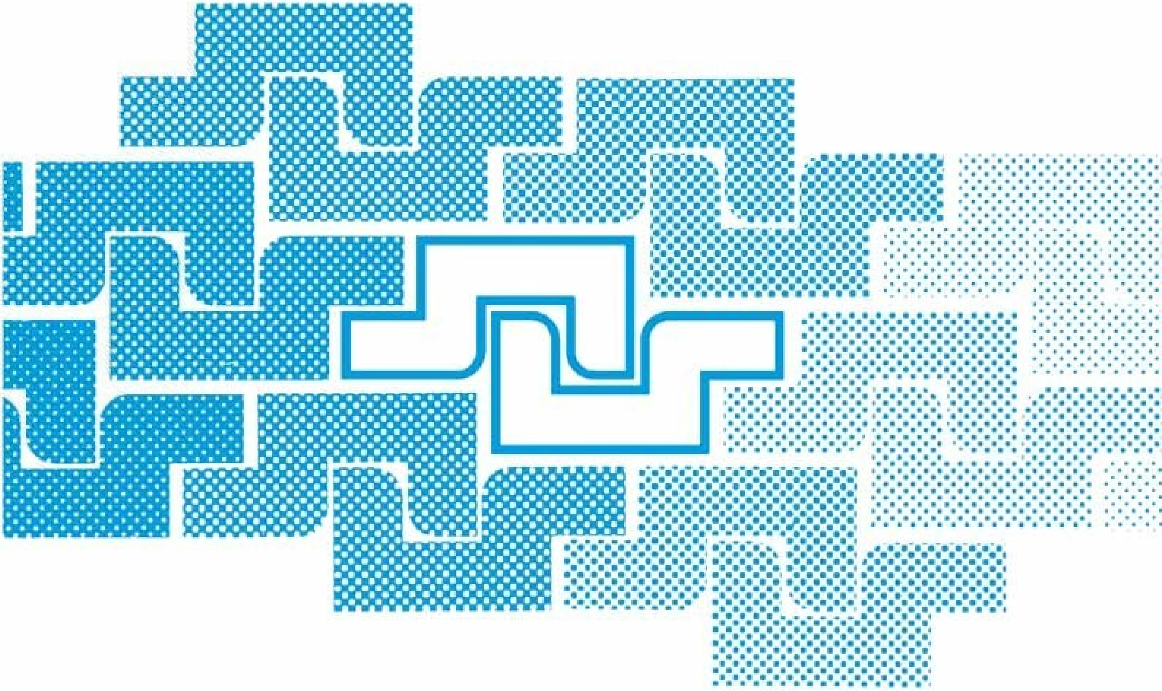

String Theory: From Gauge Interactions to Cosmology

Edited by

Laurent Baulieu, Jan de Boer,
Boris Pioline and Eliezer Rabinovici

NATO Science Series

II. Mathematics, Physics and Chemistry – Vol. 208

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Preface

The present volume is a collection of the lecture and seminar notes delivered at the Summer School “ String Theory : from Gauge Interactions to Cosmology”, which was held in Cargèse, France, from June 7 to 19, 2004.

The main focus of the school was on the inter-relations between the fields of superstring theory, cosmology and particle physics, which have been the subject of very active research recently. Much of the recent progress on topical problems in the field was covered, including the duality between gravity and gauge theories, strings and cosmology, critical phenomena in statistical mechanics, topological string theory, physics beyond the standard model, and the landscape of vacua of string theory.

The School was an excellent opportunity for the youngest researchers to establish close relationship with the lecturers and among each other, be it during the vibrant Gong Show session, under the shade of the Wisdom Tree or gazing at the beautiful Corsican coastline. We hope that these proceedings will further serve in fixing the acquired knowledge, and hopefully become a valuable reference for anyone working in this fascinating domain of physics.

It is a pleasure to extend our warm thanks to the NATO Division for Scientific Affairs, under the Advanced Study Institute grant PST.ASI 97 85 17, and to the the Human Factor-Mobility & Marie Curie Actions Division of the European Commission, under the High-Level Scientific Conference grant HPCFCT-2001-00298, whose generous funding have allowed this meeting to take place. NATO’s announced discontinuation of its support will be sorely felt in forthcoming events. We also wish to thank all the people who have contributed to the organization of this Conference, especially the staff of the Institut d’Etudes Scientifiques de Cargèse its Director, Elisabeth Dubois-Violette. We are also much indebted to Josette Durin, for her invaluable help in preparing this volume.

Finally, we are very grateful to all the participants of the School for a creating a wonderful stimulating atmosphere, and to the contributors of this volume for their dedication in preparing their notes.

Laurent Baulieu
Jan de Boer
Boris Pioline
Eliezer Rabinovici

Part I

Lectures

STRINGS IN A LANDSCAPE

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Abstract. This is a summary of some of the topics covered at the Cargese Summer School on String Theory in June of 2004. Most space is devoted to a discussion of ideas about the String Landscape, which formed the background for some of the talks and much of the discussion at the School.

1. Introduction

This paper is a synthesis of my summary remarks at the Cargese School, and my contribution to the wisdom tree discussion with Arkani-Hamed and Polchinski. The topics of the school were varied. There was a discussion of the interface between cosmology and Planck/String scale physics, in the talks of Brandenberger, Polchinski, Pioline, Elitzur, and Rabinovici. Polchinski and Arkani-Hamed devoted some of their time to discussion of the string landscape and expectations for fine tuning. Arkani-Hamed discussed his beautiful model of spontaneous breakdown of general coordinate invariance (also known as ghost condensation), as well as the split supersymmetry model motivated by the string theory landscape. Polchinski also talked about the emergence of cosmologically stable cosmic strings in one of the regions of moduli space that are being explored in the landscape. Tseytlin's talks were devoted to the string/gauge theory connection, and in particular to the emergence of integrable structures on both sides of the duality between maximally supersymmetric planar Yang-Mills theory and tree level string theory on $AdS_5 \times S^5$. Dijkgraaf and Ooguri discussed the emergent non-perturbative formulation of topological string theory. Morozov's lectures reviewed some of the older connections between exactly soluble string theories, and matrix models. Okun and Pokorski gave us detailed updates on neutrino and collider phenomenology. Finally, D. Bernard reminded us that conformal field theory still has interesting things to say about real problems in condensed matter physics.

I will confine these paragraphs to brief summaries of cosmology, ghost condensation, and a somewhat more detailed discussion of the debate over the landscape. I begin with cosmology.

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2. Cosmology

Robert Brandberger gave us a review of the calculation of fluctuations in inflationary cosmologies, with emphasis on the possibility of seeing Transplanckian effects in the cosmic microwave background. I will give you my understanding of this issue, which may not coincide with Robert's. The usual argument for the universality and independence of microscopic physics of the inflationary fluctuation spectrum relies on the adiabatic theorem. Given the assumption that the adiabatic theorem applies, effective field theory arguments show that the corrections to the leading order predictions for fluctuations are of order $(H/M_P)^2$, which usually means they are unobservable. Attempts to produce effects of order H/M_P rely on non-adiabatic initial conditions. Brandenberger showed us models of initial states which do produce such effects, as well as models which don't. String theory does not yet have the tools to express results of stringy cosmologies in terms of the language of field theory in which these results are presented. It is not clear which class of models string theory prefers.

What is clear is that models with many e-folds of inflation, which incorporate violations of the adiabatic theorem (and avoid making the UV more singular than field theory), must be very fine tuned in order to make an observational effect. Such models make large modifications to conventional predictions, only over a limited range of scales. It requires fine tuning to make this range of scales coincide with the range we observe in the CMB.

Elitzur, Pioline and Rabinovici talked about models of string cosmology which use the machinery of world-sheet conformal field theory to investigate singular cosmologies. These models fall into two types: those where an infinite universe undergoes a Big Crunch, which may or may not be followed by a Big Bang (with string theory providing the tool to remove and glue together the singularities) and those with closed universes which start with a Big Bang and end with a Big Crunch. Both types of models are generalized orbifold CFTs, *i.e.* they are gauged σ models, where the gauge group may be continuous or discrete.

In the Crunch-Bang models the correlation functions of string vertex operators have the conventional interpretation of a scattering theory. It has been shown pretty definitively that conventional string perturbation theory breaks down near the singularity. Hopes for progress depend on the notion that the production of string winding modes, which are concentrated near the singularity will smooth out the transition without recourse to Planck scale physics. It is hoped that some modified perturbation expansion will give a clear picture of what is going on near the singularity.

In Bang-Crunch models there are no asymptotic regions in which to define scattering amplitudes. Mathematical computations rely on infinite space-like regions with closed time-like curves, which have been dubbed "whiskers" (actually the Crunch-Bang orbifolds have similar whiskers). I do not understand the physical interpretation of the world sheet correlators in this context, or how they are related to physics in the actual cosmological part of the space-time.

3. Ghost condensation

Arkani-Hamed discussed the theory[1] of ghost condensation, which is amusing and beautiful. He and his collaborators have shown that theories which do not have a stable Lorentz

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invariant solution can nonetheless have stable solutions which have the symmetries of homogeneous isotropic cosmology. Moreover, because these Lagrangians describe a partial Higgsing of gravity, their low energy behavior is almost universal. It is described by a single new massive parameter f, analogous to the pion decay constant. The theory has a new field ϕ, whose low energy excitations have a non-Lorentz invariant dispersion relation. They can modify known theories in ways which are not yet ruled out, but are potentially observable. In particular, they can produce interesting non-Gaussian fluctuations in the Cosmic Microwave Background. The theory can also account for the acceleration of the universe, without a cosmological constant, although it does not solve the cosmological constant problem.

In order to explain the acceleration, f must be taken very small. The new field in the theory *couples only to gravity*, and for small f its effects on ordinary physics are almost negligible.

There are two problems with this model. The first is that we cannot derive it from string theory in any known way. The second is that the assumption that the new physics couples to the old only through gravity is artificial. Standard effective field theory reasoning leads us to expect non-renormalizable couplings of ϕ to standard model fields, scaled by f. For small values of f this is inconsistent with experiment. It appears to be technically natural to omit the direct couplings of the Goldstone field to matter. If we postulate that it couples only to gravity, then the small value of f/M_P ensures that radiative corrections do not induce large direct couplings of the Goldstone field to the standard model. Nonetheless, one would like to have a principle that explained the absence of direct couplings to the standard model, rather than a simple postulate, however self-consistent.

Neither of these criticisms is grounds for abandoning this very interesting new model. In particular, string theory as we know it is built to handle asymptotically flat and AdS backgrounds, and has notorious problems with any kind of real cosmology. Ghost condensation is an intrinsically cosmological model. It does not have any stable, Lorentz invariant background solutions. So, the inability to find it among solutions to the effective field equations of a known string theory, may be more a problem of the limitations of the present formulation of string theory, than an indication that ghost condensation is an incorrect idea.

The wonderful thing about ghost condensation is that it makes predictions for novel phenomena, which are just beyond the current reach of experiment. It should be studied and tested to the best of our ability.

4. Strings in a landscape

Although there were no talks devoted explicitly to the string landscape at Cargese, it pervaded many of the discussions. Arkani-Hamed's split supersymmetry model was motivated by the landscape, as was Polchinski's discussion of cosmic strings in string theory. And our discussion under the wisdom tree was a debate about the landscape.

Part of the confusion in the debate about the landscape is the conflation of two notions of effective action which occur in quantum field theory. The first is the 1PI effective action, which is an exact summary of the entire content of a field theory. Knowledge of it enables us to construct all of the correlation functions of the theory, in any of its vacuum states. In perturbation theory, we can compute the 1PI action around any vacuum state, and get the same result. It is important to realize that there is NO known analog of the 1PI action in string theory, which applies to all solutions of the theory. For example, both $AdS_5 \times S^5$ and ten dimensional Minkowski space, are solutions of the tree level effective action of Type IIB SUGRA, but higher order corrections (not to mention non-perturbative corrections) to the action which generates the S-matrix in Minkowski space will not be the same as those which generate the correlation functions on the boundary of AdS space. There is no single quantum effective action which we can use to get both the S-matrix and the AdS correlation functions by "shifting the field".

The other concept of effective action in field theory is the low energy or Wilsonian action. This is defined, either in a single vacuum, or in a set of quasi-degenerate vacua whose energy density differences (as well as the heights of the barriers between them) are small compared to some cutoff scale. The Wilsonian action only contains degrees of freedom whose fluctuations are significant at these low energy scales. It is important to avoid using it when the conditions of its validity are violated. For example, in minimally SUSic QCD with $N_F \leq N_C - 1$, the low energy degrees of freedom consist of a meson superfield M and one can compute the exact low energy superpotential of M. The low energy Kahler potential is canonical. If one uses this low energy Lagrangian to compute the energy density of states with a given expectation value of M, this Lagrangian gives a divergent answer at the symmetric point of vanishing eigenvalue. This is not the correct physical answer. The true Kahler potential (in the 1PI sense) of M is modified at the origin of moduli space, and the energy density there is really of order the QCD scale. We should stop paying attention to the predictions of the Wilsonian action when they are outside its range of validity.

In string theory, the effective actions we compute are analogous to Wilsonian actions, but their range of validity is even more constrained. In particular, the stringy derivation of the effective action views it as a tool for calculating boundary correlation functions in a *fixed asymptotic space-time background.* We tend to forget this because, particularly in situations with a lot of SUSY, the leading low energy term in the effective action is independent of the background. This fosters the illusion that different backgrounds can be viewed as *vacuum states of the same theory*. In fact, as emphasized in [2], the italicized phrase is borrowed from quantum field theory, and refers to concepts which depend entirely on the separation between IR and UV physics of that formalism. In string theory/quantum gravity, UV and IR physics are much more intimately entangled, and the concept of different vacuum states of the same underlying string theory Hamiltonian is much more circumscribed. When we have a continuous moduli space of super-Poincare invariant S-matrices, we can do experiments at one value of the moduli, which are sensitive to the S-matrix at other values of the moduli[1]. Note however, that the only Hamiltonian form we have for such models is in light cone frame, where different values of the moduli correspond to different Hamiltonians. Similarly, a moduli space of correlation functions on the boundary of Anti-deSitter space, corresponds to a one parameter set of different Hamiltonians, rather than different superselection sectors of the same Hamiltonian.

The recognition that changes in background asymptotics correspond to changes in the Hamiltonian, rather than changes of superselection sectors for a given Hamiltonian

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[1] These experiments are much more difficult than they would be in a SUSic quantum field theory, without gravity.

goes back to [3], and has become commonplace with the advent of AdS/CFT. Changes in non-normalizable modes of bulk fields[2] add relevant terms to the Hamiltonian. Changes in the (negative) cosmological constant correspond to changes in the fixed point which defines the boundary CFT, and thus to a completely different set of high energy degrees of freedom.

These facts lead us[3] to be suspicious of attempts to find new string theory models by patching together an effective potential using the degrees of freedom of *e.g.* Type II string theory in flat space-time. Indeed, once we include gravitational effects, the low energy action itself gives us reason to be suspicious of meta-stable de Sitter vacua constructed in this way.

Under the wisdom tree, Arkani-Hamed gave us an example of a situation where we feel pretty confident that we can reliably compute a Wilsonian effective potential with minima that have positive vacuum energy. Simply take the supersymmetric standard model, with cubic (A-term) soft supersymmetry breaking terms. We get a host of minima with energy density differences of order the SUSY breaking scale. There is no doubt that a TeV scale experimenter could verify the existence of these minima by exciting localized coherent excitations of the squark and slepton fields. Let us however assume that we are living in a Poincare invariant state[4], and see what this potential implies about the geometry of space-time when gravitational effects are taken into account. In order to exhibit a meta-stable dS space, we have to excite a region of order the putative Hubble radius into the meta-stable minimum. Old results of Guth and Farhi[4] show that the external observer can never verify the existence of the inflating region. A black hole forms around it. Any observer in the asymptotically flat region, who tries to jump into the black hole to find inflation, first encounters a singularity. So effective field theory tells us that we can find and explore meta-stable positive energy density minima of an effective potential, but not the meta-stable dS spaces that these minima have been thought to imply. That is, the Guth-Farhi results suggest that *the stable asymptotically flat vacuum state does not have excitations which correspond to meta-stable dS vacua, even when it has an effective potential with positive energy meta-stable minima*. Rather, it has excitations in which fields are excited into meta-stable minima only over regions small compared to the Hubble radius at those minima. The attempt to create larger regions succeeds only in creating black holes.

We see that in gravitational theories, the criterion for the validity of non-gravitational effective field theory reasoning depends on more than just the value of the energy density in Planck units. When the Schwarzschild radius of a region exceeds its physical size in the approximation in which gravity is neglected, a black hole forms. Effective field theory remains valid outside the black hole horizon (if it is large enough), but not inside. In the above example, no external observer can probe the putative dS region, without first encountering a singularity.

We also see that the solutions of the same effective equations of motion may not reside in the same quantum theory.

---

[2] Note that if we want to make finite rather than infinitesimal changes we must restrict our attention to Breitenlohner-Freedman allowed tachyon fields.

[3] well, at least they lead me

[4] In other words, fine tune the c.c. to be zero in one preferred vacuum state.

The results of Coleman and De Lucia[5], on vacuum tunneling in the presence of gravity, give us a sort of converse to this result. Given the same effective Lagrangian we used in the previous paragraph, we can assume the existence of the meta-stable dS space, and ask what it decays into. Here there is a surprise, particularly for those who constantly repeat the mantra "dS space decays into flat space". In fact, the analytic continuation of the CDL instanton is a negatively curved Friedmann Robertson Walker cosmology, which (if the potential has a stable zero cosmological constant minimum), is asymptotically matter dominated. Although it locally resembles flat space on slices of large cosmological time, its global structure is completely different. In particular, an attempt to set up an asymptotically Minkowskian coordinate system, starting from the local Minkowski frame of some late time observer, inevitably penetrates into regions where the energy density is high (the energy density is constant on slices of constant negative spatial curvature, and is of order the energy density of the false vacuum at early FRW times) .

Thus, both analysis of creation and decay of meta-stable dS states, suggests that if a potential has a stable minimum with vanishing cosmological constant, and another with positive energy density, the Minkowski solution and the meta-stable dS solution are simply not part of the same theory. There remains a possibility of the existence of a theory with a stable, matter dominated, FRW cosmological solution with a meta-stable dS excitation. The problem with this is that it is very unlikely that we can make a reliable exploration of this scenario within the realm of low energy effective field theory. The CDL instanton solution is non-singular. The $a = 0$ point of the FRW coordinate system is just a coordinate singularity marking the boundary between the FRW region and a region of the space-time which continues to inflate. However, arbitrary homogeneous perturbations of the CDL solution have curvature singularities. Further, there is a large class of localized perturbations which evolve to Big Crunch singularities, rather than passing smoothly through the $a = 0$ point. For example, consider a localized perturbation on some hyperbolic time slice a finite proper time prior to $a = 0$. Let it be homogeneous in a large enough region that signals from its inhomogeneous tail cannot propagate to the $a = 0$ point. Then we will have a singularity. I will mention a more generic example below.

If we re-examine the Guth-Farhi argument in the FRW context, we see that it continues to hold until we go back in time to a point where the cosmic energy density is of order the barrier to the meta-stable minimum. At high enough energy density, there are FRW solutions in which the field classically evolves into the meta-stable minimum. It will then decay by tunneling, into the FRW continuation of the CDL instanton. Generic FRW solutions (including arbitrary homogeneous perturbations of the CDL instanton) have curvature singularities at a finite cosmic time in the past. To establish their existence as genuine theories of quantum gravity one must go beyond effective field theory, and probably beyond perturbative string theory.

Freivogel and Susskind[6] have suggested a scattering theory in which asymptotic states are associated with incoming and outgoing wave perturbations of the nonsingular, time symmetric Lorentzian continuations of the, CDL instantons for the various meta-stable vacua of string theory. They claim that in this framework, the breakdown of effective field theory is avoided, as long as the effective potential is everywhere smaller than Planck scale. I find this suggestion interesting, but it is not based on reliable calculations. If one considers black holes with radius larger than the dS radius, formed in the remote past of the FRW part of the time symmetric Lorentzian CDL geometry, it is hard to see how these solutions asymptote to the future CDL geometry without encountering a singularity. Near the $a = 0$ point where the FRW coordinates on the CDL solution become singular, most of the space-time is isometric to dS space at its minimal radius. If we have formed a black hole in the past, with radius much larger than this, then the entire space-time must end in a singularity.

Thus, I would claim that unlike asymptotically flat or AdS space-times, we have no reliable effective field theory argument that there are an infinite number of states in space-times that asymptote to the time symmetric Lorentzian CDL instanton. An infinite number of states is a minimal requirement for the existence of a quantum theory that can make precise mathematical predictions, which can be self consistently measured in the theory. We must understand the nature of the singularities in these perturbations of the CDL instanton geometry before we can conclude that the framework makes sense.

I think it is more likely, that meta-stable dS vacua exist only in the context of a Big Bang cosmology. If we consider the problem of accessing a meta-stable dS minimum of an effective potential, at times when the cosmic energy density is of order the barrier height of the potential, then the Guth-Farhi problem does not appear to exist. Starting from a Big Bang singularity, one can find homogeneous solutions where a scalar field wanders over its potential surface at a time when the energy is higher than the barriers between minima, and then settles in to a meta-stable minimum with positive cosmological constant. One must understand the Big Bang to make a reliable theory of such a situation, but apart from that the solution is non-singular. In particular, the problem of large black holes in the initial state, is not present for this situation. The difficulties are all associated with understanding the Big Bang singularity.

To summarize, it is clear from semi-classical calculations alone, that the concept of a vacuum state associated with a point in scalar field space is not a valid one in theories of quantum gravity. A given low energy effective field theory may have different solutions which do not have anything to do with each other in the quantum theory. One solution may be a classical approximation to a well defined quantum theory, while the other is not. It seems likely that the context in which we will have to investigate the existence of non-existence of the landscape is Big Bang cosmology. This is the only situation in which we can reliably construct a universe which gets stuck in a meta-stable dS minimum.

Thus, I claim that if string theory really has a multitude of meta-stable dS states, then exact theory into which they fit is a theory of a Big Bang universe which temporarily gets stuck, with some probability, in each of these states. This is ultimately followed by decay to negatively curved FRW universes. These FRW universes have four infinite dimensions and 6 or 7 large compact dimensions, which are expanding to infinity. It is clear that the probability for finding a particular dS vacuum is partly determined by the density matrix at the Big Bang and not just by counting arguments. In the next subsection I will describe existing proposals for the cosmological distribution of vacua. My main point here is that the nature of the Big Bang will have to be addressed before we can hope to understand the correct statistics of stringy vacua.

4.1. ETERNAL INFLATION

The string landscape seems to fit in well with older ideas which go under the name of *eternal inflation*, *the self reproducing inflationary universe*, *etc.*. The simplest model which exhibits this sort of behavior is one with a single scalar field with two minima, one with positive vacuum energy and the other with vanishing energy. One considers the expanding branch of the meta-stable dS universe with the field in the false minimum. In the quantum field theory approximation this seems to produce an ever-expanding region of space and one allows the dS space to decay by CDL bubble formation independently in each horizon volume. In the eternal inflation picture, one tries to interpret the result as a single classical space-time. One obtains a Penrose diagram with a future space-like boundary. The future space-like boundary is fractal, with regions corresponding to singularities[5] as well as FRW asymptotics, interspersed in a causally disconnected way. In the landscape there will be many different singular regions of the boundary, as well as many different FRW regions. Advocates of the landscape/eternal inflation picture then make the analogy to maps of the observable universe, with different causally disconnected regions being the analogs of different planets. Physics it is said, depends on "where you live" and organisms like ourselves can only live in certain regions of the map. A key difference between different minima of the effective potential in eternal inflation, and different planets is that *we cannot, even in principle, communicate with causally disconnected regions of the universe.*

How is one to interpret such a picture in terms of conventional quantum mechanics? I believe that the fundamental clue comes from the principle of Black Hole Complementarity[7]. Black holes also present us with two regions of space-time which are causally disconnected. Hawking showed long ago that this was an artifact of the semi-classical approximation, and that black holes return their energy to the external space-time in which they are embedded. If we assume that the region behind the horizon has independent degrees of freedom, commuting with those in the external space, then we are confronted by the information loss paradox.

String theorists have believed for some years now, that this is not the case. The principle of Black Hole Complementarity is the statement that the observables behind the horizon do not commute with those in the external space-time. For a large black hole, (and for a long but finite time as measured by the infalling observer), these two sets of observables are both individually well described by semiclassical approximations, but the two descriptions are not compatible with each other.

Fischler and I tried to relate this principle to the *Problem of Time*[8]. In the semi-classical quantization of gravity one attempts to solve the Wheeler-DeWitt equation

$$\mathcal{H}\Psi = 0$$

with an ansatz

$$\Psi = e^{iS}\chi(t, \phi),$$

where S is the action of some classical space-time background solution, and χ is the wave functional of a quantum field theory in this space-time

$$i\partial_t \chi = H(t)\chi.$$

---

[5] The singular regions correspond to decays to parts of the potential where the vacuum energy is negative. They would exist in the landscape context, but not in the simple model we are discussing.

The (generally time dependent) Hamiltonian $H(t)$ depends both on the choice of classical background, and on the particular time slicing chosen for that background. For example, for the Schwarzschild background we could choose H_{Sch}, the Schwarzschild Hamiltonian, or some time dependent $H(t)$ where t is the proper time of a family of in-falling observers. Even ignoring subtle questions of whether these two Hamiltonian evolutions act on the same Hilbert space, it is clear that they are different and that $[H(t), H_{Schw}] \neq 0$ at *any* time t. It is therefore not surprising that the semi-classical observables of different observers do not commute with each other.

Given this description of black holes, it is natural to conjecture that a similar phenomenon occurs for any space-time with horizons. In [8] this was called Cosmological Complementarity for asymptotically dS spaces. E. Verlinde has suggested the name Observer Complementarity for the general case.

If we apply this logic to Eternal Inflation, we obtain a picture quite different from the original description of these space-times. We simply associate a single Hilbert space and many different (generally time dependent) Hamiltonians to the fractal Penrose diagram. Each Hamiltonian is associated with the causal patch of a given observer. Mathematically, the situation can be equally well described by saying we have a collection of different theories of the universe. There is a philosophical cachet, associated with the phrase, "physics depends on where you live in the multiverse", which is absent from this alternative way of describing the physics.

In the formalism described in [6] this is almost precisely what is conjectured. For each meta-stable dS point L in the landscape, which can decay into the Dine-Seiberg region of moduli space, and for each (typically 10 or 11 dimensional) Super-Poincare invariant solution V of string theory into which L can decay there is a different unitary S-matrix $S_{L,V}$[6]. It is claimed that each of these S-matrices contains all the physics of the landscape, because there is a canonical way to compute the unitary equivalence U in the formula $S_{L,V} = U_{L,V,L',V'} S_{L'V'} U^{\dagger}_{L,V,L',V'}$. The statement that there is a theory of eternal inflation in which all of the points L are meta-stable states is really the statement that the theory contains a canonical algorithm for computing the "gauge transformations" U[7]. The authors of [6] claim that all of the meta-stable dS states will show up as resonances in every S-matrix, $S_{V,L}$. This claim is plausible if the S-matrices are indeed related by unitary conjugation. The spectrum of the S-matrix is then gauge invariant. In ordinary scattering theory, time delays, which are related to resonance lifetimes, are related to the spectrum of the S-matrix. In the eternal inflation context, there is no universal notion of time for the different asymptotic states, so more work is necessary to understand these concepts.

If indeed the information about each meta-stable dS vacuum can be extracted from the spectral density of a given S-matrix $S_{V,L}$, and if we can find a reliable framework, for defining and calculating these S-matrices, then the landscape will have a mathematical

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[6] We can also consider initial and final states corresponding to different CDL instantons. In[6] these are claimed to be different gauge copies of the same information in the S matrices. I will mention a different interpretation below.

[7] For the moment, no approximate statement of what this algorithm is has been proposed. I would conjecture that, if the formalism makes sense, the transition amplitudes between two different FRW spacetimes, mentioned in the previous footnote, provide the algorithm for calculating the U mappings.

definition. From a practical point of view however, we just say that string theory gives us an algorithm for constructing models of the world (in the landscape context this means choosing particular stable or meta-stable minima) which is not unique and that we are trying to use data to constrain which model we choose. In the next section, I will describe the situation in the language of the approximate Hamiltonian of a given meta-stable dS observer, rather than that of eternal inflation.

I have emphasized the problems with this S-matrix point of view and suggested the alternative notion that the landscape could only make sense in the context of Big Bang Cosmology. One must thus understand how to describe the initial states. There are two possibilities, either there is some principle which picks out a fixed initial state[8] at the Big Bang, or there is a generalized S-matrix in which we relate a particular state at the Big Bang to a particular linear combination of final scattering states in one of the FRW backgrounds defined by a decaying meta-stable dS space. In a manner analogous to Freivogel and Susskind, one would conjecture that the descriptions in terms of different future FRW backgrounds would be unitarily equivalent to each other, by unitary transformations which do not respect locality. Since we are unlikely to have much control over the initial conditions at the Big Bang, one should choose the in-state at the Big Bang to be a high entropy density matrix. Thus, the practical difference between the two proposals is the entropy of the initial state.

I know of two proposals for the initial density matrix at the Big Bang, which might lead to a set of cosmological selection rules for meta-stable points in the landscape. The first, *modular cosmology*[9] postulates an early era in which the universe can be described semi-classically, but the potential on moduli space is smaller than the total energy density. The metric on moduli space then provides a finite volume measure. Furthermore, the motion of the moduli is chaotic. These facts suggest that the probability of finding the universe in a given meta-stable minimum of the potential is the volume of the basin of attraction of that minimum, divided by the volume of moduli space.

Holographic cosmology[10] gives an alternative view of the initial state of the universe, as a “dense black hole fluid” where standard notions of local field theory do not apply. The model contains two phenomenological parameters, which govern the transition between this phase of the universe and a normal phase in which the field theory description is valid. There are indications that the transition occurs at an energy density well below the unification scale of standard model couplings. We might then expect a transition directly into a state with most of the moduli frozen. In order for the model to provide an adequate account of the fluctuations in the CMB, one must have at least one “active” modulus at these low energies, which can provide for a modest number of e-folds of inflation. In such a model, minima of the potential on moduli space with energy higher than the scale at which a field theoretic description of the universe is possible, cannot make any sense. At best a small class of low energy minima could be compatible with holographic cosmology[9]. If holographic cosmology is compatible with landscape ideas, the probability of accessing a particular minimum will be determined by quantum grav-

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[8] *e.g.* the Hartle Hawking Wave Function of the Universe.

[9] The potentials calculated in the landscape have no indication of a cut-off at energy scales far below the unification scale. This suggests that the two theoretical frameworks are not compatible, but I am trying to avoid jumping to conclusions.

itational considerations, far removed from effective field theory. In this framework, the dense black hole fluid is stable and is the most probable state of the universe. A normal universe like our own is determined by a somewhat improbable initial condition, but one expects the maximum entropy initial state that does not collapse to a dense black hole fluid. The survival probability of a given normal state depends on both the properties of the black hole fluid and the low energy physics of the normal state, so the determination of the most probable meta-stable minimum would be a complicated quantum gravitational calculation.

In both of these classes of models, simple enumeration of meta-stable minimum is not a good account of the physical probability distributions.

5. Phenomenology of the landscape

For practical purposes , the landscape gives us a large set of alternative effective Lagrangians for describing the physics we have observed or will observe in our universe. These are parametrized by a collection of numbers, which include the dimension of space-time, the name, rank and representation content of the low energy gauge theory, the value of the cosmological constant, and the values of all the coupling constants and masses of fields in the Lagrangian[10]. These numbers can be collected together and viewed as a multidimensional probability space. In the supergravity approximation, we have a way of calculating an *a priori* distribution for these numbers. Proponents of the landscape would claim that this is an approximation to some more exact distribution, though no-one has suggested a procedure for calculating the corrections. In the previous section I have suggested that early universe cosmology may make important modifications of the distribution of metastable minima. If it turned out that the distribution predicted the Lagrangian we observe with high probability, it would be a great triumph for string theory.

The value of the cosmological constant tells us that this is not the case. Weinberg's bound[11],which constrains cosmological parameters by insisting on the existence of galaxies, has the form (in Planck units).

$$\Lambda \leq K \rho_0 Q^3,$$

where ρ_0 is the dark matter density at the beginning of the matter dominated era, Q is the amplitude of primordial density perturbations at horizon crossing, and K is a pure number of order 1.. For any reasonable values of the other parameters, this means Λ is much smaller than the typical value found in the landscape.

It is clear then that we must supply additional data from experiment in order to fix our description of the world. The landscape framework supplies some theoretical guidance - it tells us that there are a finite number of possibilities, of order 10^{10^2} (to order of magnitude accuracy in the logarithm). Various authors [12] have begun to investigate the *a priori* distribution of properties like the gauge group and number of generations, the scale of supersymmetry breaking, the existence of large warp factors which give rise to large hierarchies of energy scales *etc.*, assuming a uniform distribution on the space of

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[10] In principle we could also have non-trivial conformal field theories in the low energy world, at least in some approximation.

minima. The hope is that correlations will become evident which will tell us that a small number of inputs is enough to extract the Lagrangian of the world we live in from the ensemble of Lagrangians the landscape presents us with.

The anthropic principle has also been invoked as an input datum to impose on the ensemble of Lagrangians. For its proponents, the attraction of this principle is that the answer to a single yes/no question, "Is there carbon based life?" puts strong constraints on a collection of parameters in the Lagrangian (assuming all others fixed at their real world values). This attraction may be an illusion. In a probability space, the characteristic function of any subset of data points is a single yes/no question. So physicists must ask if there is any special merit to the particular characteristic function chosen by the anthropic principle. This is a hard question to answer, because we do not have much theoretical understanding of life and intelligence, and we have no experimental evidence about other forms of life in the universe we inhabit.

If the typical life form resembles the great red spot on Jupiter rather than us, then this life form would think that the criterion that is most appropriate to apply in our universe is the Redspotthropic principle. To put this in a more positive manner: if considerations of carbon based life lead to explanations of the values of the fundamental parameters, then we are making a prediction about the typical form of life that our descendants will find when they explore the universe. It should look just like us, or at least be sufficiently similar that the criteria for its existence are close to the criteria for ours. If instead, our descendants' explorations show that the typical life form could tolerate much larger variations in the fundamental constants, then our so-called explanations would really be a fine-tuning puzzle. The Red Spot People could calculate and understand that *we* wouldn't be there if the up quark mass were a little bigger, but they might reasonably ask "Who ordered them?".

Of all *soi disant* anthropic arguments, Weinberg's bound on cosmological parameters is the least susceptible to this kind of criticism. If there are no galaxies, there are no planets, no Red Spots, no Black Clouds, perhaps no conceivable form of life. Polchinski and others at the school cited the numerical success of this bound as evidence that anthropic reasoning may have relevance to the real world. It is important to realize that this numerical success depends on keeping all other parameters fixed. Arkani-Hamed reported on unpublished work with Dimopoulos, which showed that if both Λ and M_P (really the ratio of these parameters to particle physics scales, which are held fixed) are allowed to vary subject only to anthropic constraints, then the preferred value of Λ is larger than experimental bounds by many orders of magnitude. Similarly the authors of [31] following [14] argued that if both Λ and Q are allowed to vary then the probability of finding a universe like our own is of order 10^{-4}. A contrary result was reported in [15], but only by assuming an *a priori* probability distribution that favored small values of Q.

In inflationary models of primordial fluctuations, the value of Q depends on details of the inflaton potential at the end of slow roll. We would certainly expect this parameter to vary as we jump around the landscape. Similar remarks apply to ρ_0. Allowing ρ_0 to vary would further reduce the probability that the anthropic distribution favors the real world. Thus, at least with our current knowledge of the landscape, it seems likely that the numerical success of Weinberg's bound in the landscape context is not terribly impressive[11].

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[11] I cannot resist remarking that in the context of Cosmological Supersymmetry Breaking[16],

The greatest challenge to all methods of dealing with the landscape is the large number of parameters in the standard model which have to be finely tuned to satisfy experimental constraints. These include the strength of baryon, lepton and flavor violating couplings, θ_{QCD} and the values of many quark and lepton masses. Anthropic reasoning helps with some of these parameters, but not all, and is insufficient to explain the lifetime of the proton, the value of θ_{QCD} and many parameters involving the second and third generation quarks and leptons. From the landscape point of view, the best way to deal with this (in my opinion) is to find classes of vacua in which all of these fine tuning problems are solved, perhaps by symmetries. One can then ask whether there are enough vacua left to solve the cosmological constant problem. This might be a relatively easy task. One could then go on to see whether other features of this class of vacua are in concordance with the real world.

It is clear that at a certain point in this process, if we don't falsify the landscape easily[12], we will run into the problem that current technology does not allow one to calculate the low energy parameters with any degree of precision. Indeed, the error estimates are only guesses because we don't even know in principle how to calculate the next term in the expansion in large fluxes. A more fundamental framework for the discussion of the landscape is a practical necessity as well as a question of principle. I have suggested that if a rigorous framework for the landscape exists, it is probably to be found in the context of a theory of a Big Bang universe with Eternal Inflation, and future FRW asymptotics for any given observer. It is likely that we will be unable to define more precise calculations of the properties of the landscape without finding a rigorous mathematical definition of such a space-time.

The fundamental object in such a space-time would be a scattering matrix[8] relating a complete set of states at the Big Bang to states in Lorentzian CDL bubble space-times corresponding to decays of meta-stable dS landscape states into the Dine-Seiberg region of moduli space. The final states, in addition to particle labels, would carry indices ,(V, L) describing a particular dS minimum L and a particular "asymptotic vacuum", V, into which it decays. Thus, we would have matrix elements

$$S(I|V, L, p_i),$$

where I labels an initial state at the Big Bang and p_i a set of "particle" labels for localized scattering states in a given CDL bubble. An important unanswered question is whether the S-matrix is unitary for each (L, V)[13] or only when all (L, V) sectors are taken into account. The first alternative is analogous to the proposal of [6].

The following argument has a bearing on this question. I have stated above that there was no problem with an infinite number of final states for fixed L. This is not necessarily the case. If I extrapolate scattering data on $\mathcal{I}_+$ backwards, using the classical equations of motion, and assuming a minimal finite energy for each particle, then all but a finite number of states will encounter a space-like singularity before transition to the metastable

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only Λ varies, and Weinberg's bound retains its original numerical status.

[12] *e.g.* by showing that the number of vacua left after all the other fine tuning problems are solved is too small to solve the cosmological constant problem.

[13] One would then invoke the existence of unitary mappings taking the different unitary S-matrices into each other.

dS regime. This is the time reverse of the argument about black holes in the initial state of the time symmetric CDL bubble. This suggests that for fixed L, the matrix $S(I|V.L, p_i)$ has finite rank: only a finite subspace of the space of all out states on the CDL geometry labeled by L, V would be allowed. This leads to a modification of the proposal of [6] in which only the S-matrix for fixed V, keeping all possible values of L, is unitary. However, there is no clear reason now to assume that V should be fixed, so perhaps only the full S-matrix is unitary.

A more disturbing conclusion is reached if one combines the claim of [17] that the number of L sectors is finite, with the above argument. One then concludes that the whole S-matrix has finite rank, and that the entire landscape fits into a Hilbert space with a finite number of states. One of the supposed virtues of the landscape picture of metastable dS was that, unlike a stable dS space, the landscape was part of a system with an infinite number of states, which could make infinitely precise quantum measurements on itself. If both Douglas' claim, and that of the last paragraph are true, this is no longer obvious. All but a finite number of the final states in a given CDL instanton geometry, would not connect to a tunneling process from a meta-stable dS vacuum, but instead would evolve directly from a Big Bang. The part of the scattering matrix that involved meta-stable dS resonances would be of finite rank. The whole issue of a rigorous framework for the landscape remains as murky as ever.

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QUANTIZATION OF HOLOMORPHIC FORMS AND $\mathcal{N}=1$ SUPERSYMMETRY ON SPECIAL MANIFOLDS

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Abstract: We study the quantization of a holomorphic two–form coupled to a Yang–Mills field on special manifolds in various dimensions, and we show that it yields twisted supersymmetric theories. For Kähler manifolds in four dimensions, our topological model is related to $\mathcal{N}=1$ Super Yang–Mills theory. Extended supersymmetries are recovered by considering the coupling with chiral multiplets.

1. Introduction

The idea that Poincaré supersymmetry is a ”phase” of a more fundamental symmetry is appealing. For instance, it was shown that Poincaré supersymmetry and topological symmetry are deeply related, and that the field spectrum of dimensionally reduced $\mathcal{N}=1$ $D=11$ supergravity can be determined in the context of an 8-dimensional gravitational Topological Quantum Field Theory (TQFT) [1]. One can foresee that many models, which are dimensional reductions and truncations of maximal supergravity, might be possibly related by twist to topological models [2]. The BRST operator that characterizes a topological symmetry is a scalar operator which can be defined in any given curved space, while Poincaré supersymmetry is a delicate concept in curved space. Therefore, topological symmetry could be a more fundamental concept than Poincaré supersymmetry. On the other hand, in order to perform the twist operation that relates Poincaré supersymmetry and topological symmetry, one often needs to use manifolds with special holonomy.

We will discuss the quantization of (holomorphic) two–forms coupled to a Yang-Mills field on special manifolds in various dimensions. These theories are basically ATQFT's (Almost Topological Quantum Field Theories), in the sense that they are defined in terms of a classical action and a set of observables which are invariant under changes of coordinates belonging to restricted classes, for instance, reparametrizations that respect a complex structure. This is to be compared to genuine TQFT that contain observables invariant under all possible changes of metrics. Interesting cases that we will analyse in detail are Kähler manifolds in four dimensions and special manifolds in higher (6,7 and

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8) dimensions. In particular in seven dimensions we will analyse G_2 manifolds of the kind recently studied by Hitchin [15].

One of the original motivations for this work was to try to understand how (twisted) $N = 1$ supersymmetric theories can be directly constructed as TQFT. As we will see in the next section, this immediately leads to the introduction in the classical action of a "charged" 2-form B, valued in the adjoint representation of a Lie algebra. In these models one can also consider the coupling to chiral multiplets. If these transforms in the adjoint representation, one recover in this way also the extended supersymmetry in a twisted form.

In Sect.2 we introduce the holomorphic BF model and discuss its relationship with $N = 1$ (twisted) supersymmetry. Notice that the quantization of this model requires the use of the Batalin–Vilkoviski formalism. In Sect.3 the four dimensional case is considered, including a detailed discussion on the coupling with a chiral multiplet. In Sect.4 we discuss the six–dimensional case on a Calabi–Yau three–fold and show how two different quantizations yields respectively to theories related to the B model and A model of topological string. In Sect.5 we discuss the eight–dimensional theory on a Calabi–Yau four–fold and its dimensional reduction to $\mathrm{CY}_3 \times S^1$. The eight–dimensional model is discussed also for manifolds with $SU(4)$ structure. This theory can be regarded as a generalization of the four–dimensional self–dual Yang–Mills model [22].

2. $N = 1$ supersymmetry and the holomorphic BF theory

The standard construction of a TQFT leads to models with $N = 2$ supersymmetry. To see this, let us consider the "prototype" case of Topological Yang–Mills theory in four and eight dimensions. The relevant BRST transformations read

$$\begin{array}{rclrcl} \delta A_\mu & = & \Psi_\mu + D_\mu c & \delta\Psi_\mu & = & D_\mu\Phi - [c,\Psi_\mu] \\ \delta c & = & -\Phi - \frac{1}{2}[c,c] & \delta\Phi & = & -[c,\Phi] \end{array} \tag{2.1}$$

These equations stand for the geometrical identity $(\delta + d)(A + c) + \frac{1}{2}[A + c, A + c] = F + \Psi + \Phi$ [23]. There are as many components in the topological ghosts as in the gauge fields, and to gauge fix the topological freedom, one must also introduce as many antighosts as topological ghosts. The antighosts are an anticommuting antiself dual 2-form $\kappa_{\mu\nu}$ and an anticommuting scalar η. For each one of the antighosts, there is an associated Lagrange multiplier field, and their BRST equations are :

$$\begin{array}{rclrcl} \delta\kappa_{\mu\nu} & = & b_{\mu\nu} - [c,\kappa_{\mu\nu}] & \delta b_{\mu\nu} & = & -[c,b_{\mu\nu}] \\ \delta\bar\Phi & = & \eta - [c,\bar\Phi] & \delta\eta & = & [c,\eta] \end{array} \tag{2.2}$$

The twist operation is a mapping from these ghost and antighost fermionic degrees of freedom on a pair of spinors, which leads one to reconstruct the spinor spectrum of $N = 2$ supersymmetry, both in 4 and 8 dimensions. The scalar BRST operator δ can then be identified as a Lorentz scalar combination of the $N = 2$ Poincaré supersymmetry generators. However, this "twist" operation has different geometrical interpretation in 4 and 8 dimensions. In the former case, it is a redefinition of the Euclidean Lorentz group contained in the global $SU_L(2) \times SU_R(2) \times SU(2)$ invariance of the supersymmetric theory. In the latter case, it uses the triality of 8-dimensional space. In the previous works