SOIL MECHANICS AND TRANSPORT IN POROUS MEDIA

Theory and Applications of Transport in Porous Media

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Volume 19

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Soil Mechanics and Transport in Porous Media

Selected Works of G. de Josselin de Jong

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Preface

Each topical area in science has its own pioneers. Pioneers in science are typically people with unorthodox and original ideas, ideas that change our way of thinking about the world that surrounds us. In the fields of geomechanics and geohydrology, Gerard De Josselin De Jong is a typical example of such a pioneer. His scientific career started in the late fifties of the previous century, and from that time he produced a number of highly significant papers that contributed to the basic understanding of the aforementioned topical areas. He could achieve these results because of his rather unusual and unorthodox way of solving scientific problems. First, he "visualized" the problem in his mind. He always said: "I need to see a "picture" of what's going on". Then he translated this virtual "picture" into a mathematical model, and subsequently tried to solve the resulting mathematical problem. In many cases, his strategy was successful.

Visualization is maybe the key-word in De Josselin De Jong's life. Not only visualization of complex scientific problems, but also visualization of the world surrounding him: as an graphical artist. He is able to capture the real world in beautiful paintings, drawings, litho's, etchings, etc. The real world brought back to its basics: beautiful, exciting, and maybe most important: recognizable and understandable. Abstract art is not his game, neither abstract science! 'If I am not able "see" what's going on, I am not interested'.

Almost all graphs in his scientific papers were hand-drawn. No ruler was ever used. Looking at these graphs is a pleasure, almost works of art. No computer graphics tool is or will ever be able to produce such eye catching and beautiful scientific graphs. Remarkable, but true.

In this volume we present a selection of Gerard De Josselin De Jong's scientific papers. The papers are reproduced in their original form: in the original format (as they appeared in the journals or reports), including typo's, errors, and misprints. The volume consists of two parts. The first part is devoted to his scientific contributions to the topical field of soil mechanics, his main field of interest as full-professor of Soil Mechanics (Geo-technics) at Delft University of Technology. Although the subject of subsurface flow and transport processes did not belong to the chair he hold as a full-professor, he was very interested in these subjects. This interest resulted in a series of highly original papers, which are still relevant for our basic understanding of flow and transport processes in porous media. A selection of these papers can be found in the second part of this volume.

The editors,
Ruud J. Schotting, Hans (C.J.) van Duijn and Arnold Verruijt

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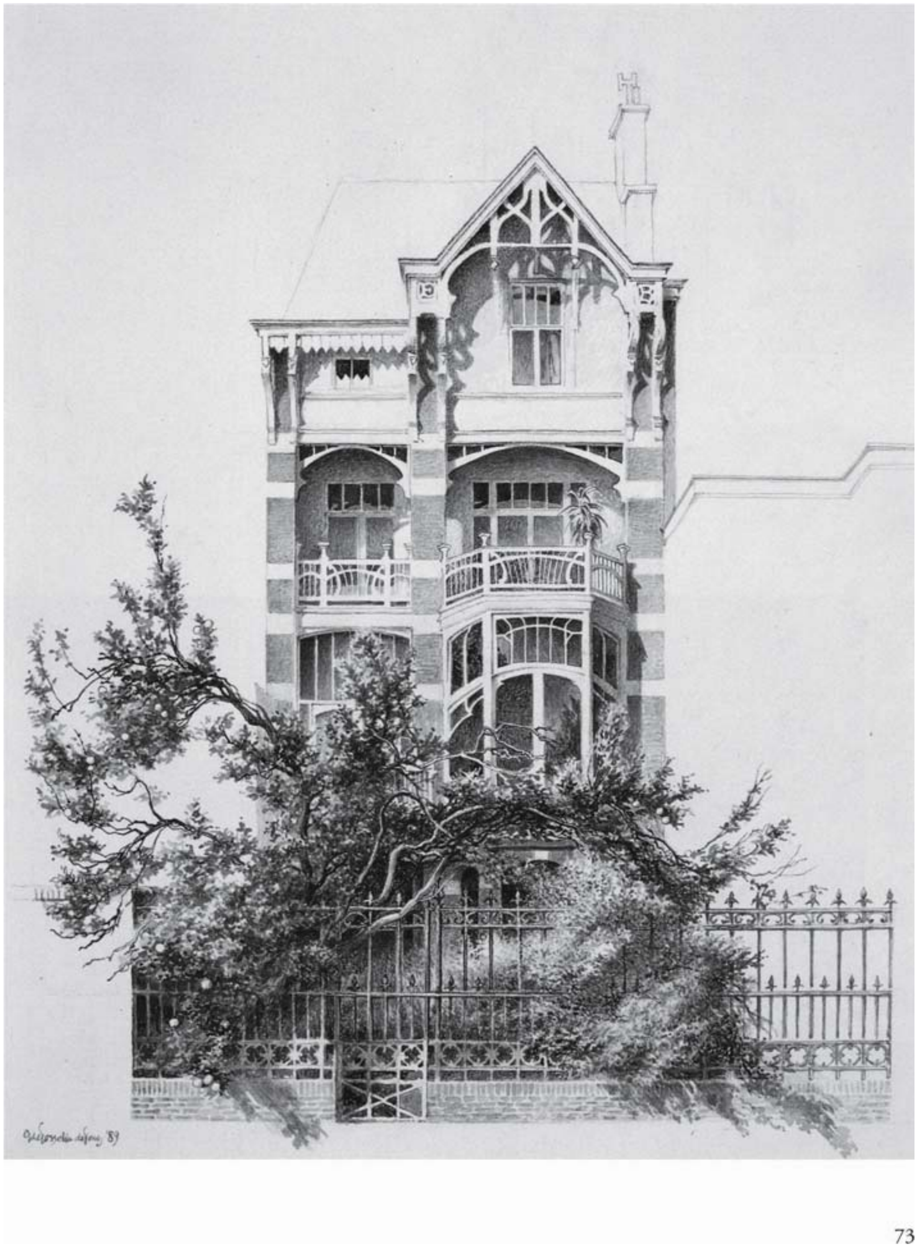
Short Curriculum Vitae of G. de Josselin de Jong

1915 Born in Amsterdam
1934 Gymnasium-β in Haarlem
1941 Civil Engineering degree at Delft University of Technology
May 1941 - Sept. 1942 Engineer at Delft Soil Mechanics Laboratory (currently Geo Delft)
Sept. 1942 Arrested by the German Navy during an attempt to escape to England
Nov 1942 Sentenced to 15 years imprisonment in Germany
May 1945 Liberated by English troups in the northern part of Germany
1945 - 1947 Lived in Amsterdam, main activities drawing and painting
1947 - 1949 Lived in Paris, worked with different architects and for Bureau d'Etude de Béton Précontraint
Nov. 1947 Marriage with Cara Waller
1949 - 1959 Researcher at Delft Soil Mechanics Laboratory
Febr. 1959 Ph.D. degree at Delft University of Technology
1959 - 1960 Visiting Research Assistant, University of California, Berkeley, USA
1960 - 1980 Full Professor of Soil Mechanics, Delft University of Technology
1980 Retirement



"Jugendstil house at the Hooistraat seen from the Nieuwe Uitleg, The Hague",
by G. de Josselin de Jong, 1982. *Washed pen, 38cm x 27,5 cm*.
Property of Mrs. Lagaay-Govers.

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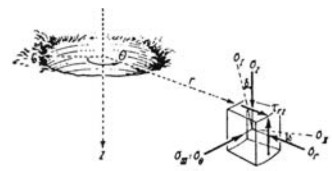

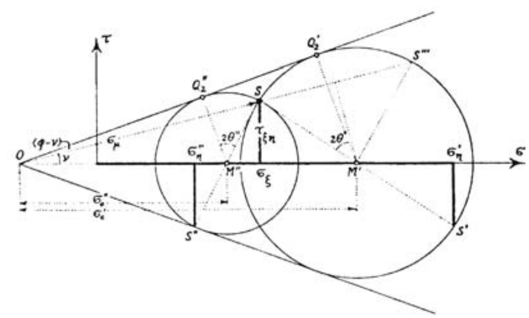

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Soil Mechanics

3.1 Introduction to Soil Mechanics

The most important papers by Professor G. de Josselin de Jong on soil mechanics can be subdivided into three main topics: consolidation of soils, the stability of a vertical cut off, and the kinematics of granular soils in the plastic zone. This last topic contains his main contribution to theoretical soil mechanics, and has been rather controversial for some time, before being recognized as an important fundamental frame work for the analysis of soil behaviour. He also made significant contributions to the development of measuring techniques in the laboratory and in the field. Some of these can be found in his theoretical papers, some were published separately.

The first basic assumption of De Josselin de Jong's model for plastic flow is that plastic deformations are generated when the stresses satisfy the Mohr-Coulomb yield criterion, which is a certain condition on the stresses, namely that no plastic flow occurs if in all directions (i.e. on all planes) the shear stress τ and the normal stress σ satisfy the condition $\tau < c + \sigma \tan\phi$, and that plastic flow may occur if on any plane $\tau = c + \sigma \tan\phi$. This is generally accepted as a good basis for the description of the behaviour of isotropic materials with internal friction.

The essential part of De Josselin de Jong's model of plastic flow, the *double sliding free rotating model*, is that the plastic deformation consists of three components: sliding deformations on the two planes on which the Coulomb criterion is satisfied, plus an arbitrary rotation. Essential in the model is that the intensities of the two sliding deformations are unrelated, the only restriction being that their signs must be such that positive amounts of energy are dissipated. This indepence of the two sliding deformations leads to an important consequence of the model, namely that the tensor of plastic strain rates need not be coaxial with the tensor of stress. Another property of the original version of the model is that it assumes that during plastic flow the volume remains constant, which constitutes another form of non-coaxiality. A third essential property of the model is that the two sliding deformations do not completely describe the displacement field, but that the displacements may contain an arbitrary additional (free) rotation.

This model, and in particular some of its consequences, initially met with considerable opposition, although it seems that all of this has now vanished.

The constant volume assumption, or, to be more precise, the possibility of constant volume plastic deformation, violates an assumption derived from work of Prager and Drucker that was used with great success in metal plasticity, namely the assumption that the plastic potential, which governs the direction of plastic flow, and the yield surface, are identical. This is now called the assumption of an *associated flow rule*. It took some time before it was realized that this is not a physical necessity, but simply a convenient property of certain materials. It is now generally accepted that for frictional materials, such as soils, a *non-associated flow rule* describes reality much better, and that the constant volume assumption often applies, especially for large strains. In modern models a volumetric component of plastic flow is often incorporated as a possibility, depending upon the density of the material, but always with the constant volume case as the limiting situation for large deformations, or even the default condition.

The independence of the two sliding components of the double sliding free rotating model also met with some opposition, because it means that there may also be a deviation of the principal direction of plastic strain with the principal direction of stress in the plane of shear deformation. This may seem strange, because it may be surmised that the coaxiality of the plastic strains and the stresses is a necessary consequence of the isotropy of the material. The proof of that property presupposes the existence of a unique relation between stresses and strains, however, and this is just what De Josselin de Jong denies, at least for a rigid plastic material. Although it is now widely acknowledged that this type of non-coaxiality may indeed occur, in many modern numerical models that include plastic flow, the coaxiality of stresses and incremental plastic strains is still assumed, for definiteness or for simplicity. That there may indeed be a deviation of these two pricipal directions was proved experimentally by Drescher and De Josselin de Jong in 1971. In a contribution to the discussions at a conference in Oslo De Josselin de Jong presented some interesting results from large scale shear tests on sand, which also seem to indicate non-coaxiality.

Another important property of his model is the free rotation, which states that while the deformations may be determined by the stresses, the displacement field may include an arbitrary additional rotation. This may now seem rather trivial, but at the time of the presentation of the model, which were the days of analytical solutions of elementary problems, it gave rise to considerable controversy. This was particularly evident in the analysis of the results of simple shear tests. The classical interpretation of this type of test is that the critical ratio of shear stress to normal stress is reached on horizontal planes, so that the friction angle can immediately be determined from this ratio. De Josselin de Jong realized that the uniform shear deformation is also consistent with shearing along vertical planes, plus a rotation (the toppling book row mechanism), and that this failure mode is much more likely to occur if the horizontal normal stress is smaller than the vertical normal stress. It gave him great satisfaction when one of the leading English scientists, Peter Wroth, appeared to support his views. De Josselin de Jongs model could be used to explain the highly variable results of shear tests. In modern finite element models that include plastic flow the free rotation usually is automatically ensured, but it seems that the assumption of coaxiality of stresses and strain rates may be an unsafe constraint in many of these models.

Another somewhat controversial topic was the derivation of lower limits for the maximum height of a vertical cut off in a uniform cohesive material, without internal friction. An upper limit, on the basis of a circular slip surface, was obtained by Fellenius in 1927: $h < 3.83c/\gamma$. Simple lower limits can be obatined from equibrium fields as $h > 2c/\gamma$ and $h > 2,82c/\gamma$. Using his graphical technique of constructing stress fields that satisfy the two equilibrium equations and the yield condition De Josselin de Jong succeeded in gradually raising this lower limit, reaching a value $h > 3.39c/\gamma$ in 1978. Unfortunately, in the same year Pastor obtained an even higher lower limit, $h > 3.64c/\gamma$, using a completely different method. It has been conjectured that perhaps the existing upper limit, $h < 3.83c/\gamma$, is also a lower limit, and it may seem that certain variational techniques can be used to prove that. In the early 1980's this lead to considerable controversy in the pages of Géotechnique. De Josselin de Jong (and others) argued, rather convincingly, that it is extremely difficult to avoid certain hidden fallacies in the variational approach, and the hope on a breakthrough seems to have vanished.

Among soil engineers De Josselin de Jong was one of the first to realize that the three dimensional consolidation theory of Biot (and not the much simpler heat conduction analogy) was the proper generalization of Terzaghi's one dimensional theory. The theoretical proof is elementary, as Biots theory incorporates elasticity theory as a special case, in the absence of pore water pressures. Experimental support came from laboratory tests at the Delft University on spherical samples, although his friend Robert Gibson preceded him in that respect by a few months. He published a series of papers on three dimensional consolidation, in Dutch, with some of his collaborators, presenting analytical solutions to a variety of problėms. Plans to expand this into a book, together with Gibson and Robert Schiffman never materialized, perhaps because the subject matter expanded faster than solutions could be derived, and perhaps also because the development of numerical methods made analytical solution methods somewhat obsolete. On the subject of consolidation it may also be mentioned that his admiration of the pioneer of Dutch soil mechanics, Professor A.S. Keverling Buisman, led him to try to generalize Buismans theory of secular (or secondary) consolidation to a beautiful model including viscoelastic deformation and an early version of a multiple porosity.

1.6 Lower Bound Collapse Theorem and Lack of Normality of Strainrate to Yield Surface for Soils

By

G. de Josselin de Jong

In soil mechanics practice there is a need for a lower bound collapse theorem, which permits an analysis with a result on the safe side. The usual analysis of slip surfaces may give unsafe results for a purely cohesive soil, since it is based upon a kinematically admissable collapse system and therefore constitutes an upper bound. It is therefore necessary to investigate a great number of slip surfaces and the smallest load is an approximation to the actual load which will produce collapse, but it is never known how much the computed load exceeds the actual one.

Upper bound theorems for a material possessing COULOMB friction have been treated by DRUCKER (1954, 1961), but it is still necessary to establish a lower bound theorem. Indeed a lower bound theorem would seem to be of more practical value since it would lead to a result on the safe side. Unfortunately the virtual work proofs of lower bound theorems break down if the material does not obey the postulate of DRUCKER: that additional loads cannot extract useful net energy from the body and any system of initial stresses.

Now in soils there are two possible ways of extracting work, since soils in general are friction systems. The first possibility was mentioned by DRUCKER [1954] and is obtained by changing the isotropic stress in the body with internal friction. The second way to extract work is a consequence of the possible deviation angle between the principal directions of strain rate and stress tensors. This can be shown by considering the extreme case of deviation corresponding to the sliding of the upperleft block in Fig. 1 along a slip surface at $\left(45° - \frac{1}{2}\varphi\right)$ to the direction of the major principal stress. The slip occurs under constant volume conditions. Initially the stress state is represented by the points AA in the stress diagram of Fig. 2, lying just inside the limit circle. The additional forces are the stresses AB which bring

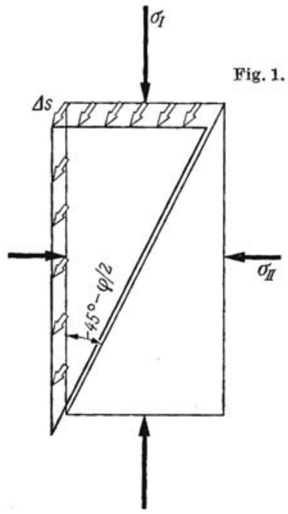

Fig. 1.

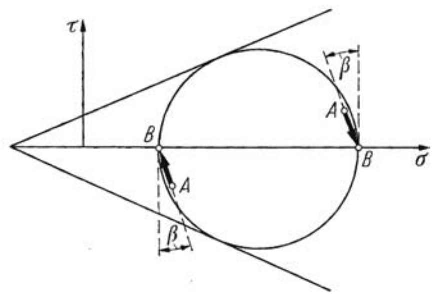

Fig. 2.

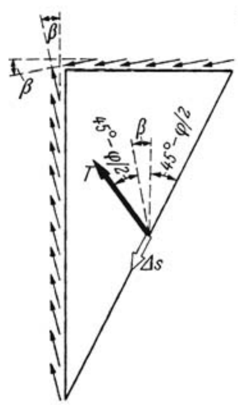

Fig. 3.

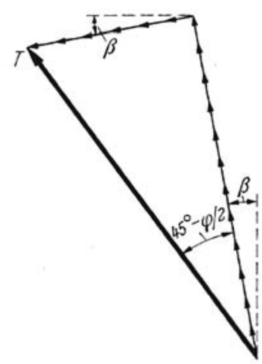

Fig. 4.

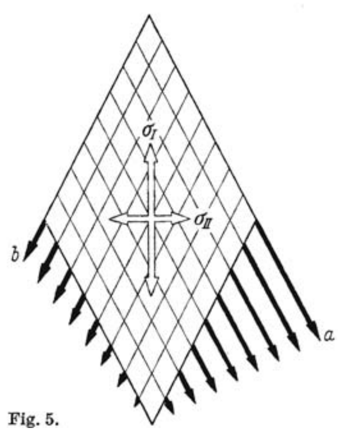

Fig. 5.

the system to a failure condition at BB. Let us consider the case when the vectors AB make an angle β with the τ-axis. The angle β can be made as small as we please by letting A approach B.

The additional loads on the moving upper left block then consist of stresses uniformly distributed along the vertical and horizontal faces and acting at an inclination β to these faces, Fig. 3. The resultant T of the additional forces on the upper left block is shown in Fig. 4 to make an angle of $\left(45° - \frac{1}{2}\varphi + \beta\right)$ with the vertical.

Under the influence of the existing stresses the block slides in a direction, at $\left(45° - \frac{1}{2}\varphi\right)$ downwards. If the displacement of the block is ΔS in that direction, then the work done by the body and the system of initial stresses on the added stress resultant T is equal to ΔS times the component of T in the direction opposite to ΔS. The work is therefore.

$$\Delta S \cdot T \cos (90° - \varphi + \beta) = \Delta S \cdot T \sin (\varphi - \beta).$$

This is positive if β is smaller than φ, thus positive work can be extracted.

Work can be extracted from a yielding system if the plastic strain rate tensor plotted as a vector in the corresponding generalised stress space is not normal to the yield surface.

In order to show the lack of normality in the case of soil explicitly, it is convenient to consider a stack of parallel cylinders which form a two dimensional analogy of a grain system with internal friction. Then the generalised stresses are the 4 stresses σ_x, σ_y, τ_{xy}, τ_{yx}, and the generalised stress space is therefore 4-dimensional. Fortunately τ_{xy} is equal to τ_{yx} and only the diagonal of length $\tau\sqrt{2}$ is a relevant coordinate. Therefore the generalised stress space can be reduced to the 3-dimensional space of Fig. 6, with coordinates σ_x, σ_y, $\tau\sqrt{2}$.

Let the material obey a Coulomb friction law, such that the yield criterion is:

$$(\sigma_x - \sigma_y)^2 + 4\tau^2 = [\sin\varphi(\sigma_x + \sigma_y + 2c \operatorname{cotg}\varphi)]^2. \tag{1}$$

To obtain a simpler expression for the yield surface, the coordinates are changed in the orthogonal system p, q, t according to

$$p = \frac{1}{2}\sqrt{2}(\sigma_x - \sigma_y),$$

$$q = \frac{1}{2}\sqrt{2}(\sigma_x + \sigma_y + 2c \operatorname{cotg}\varphi),$$

$$t = \tau\sqrt{2}$$

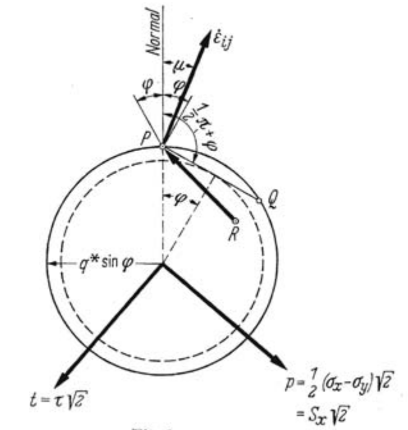

Fig. 8.

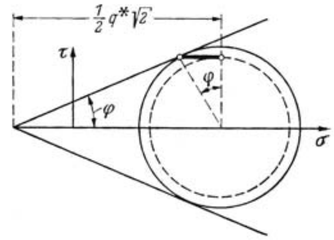

Fig. 9.

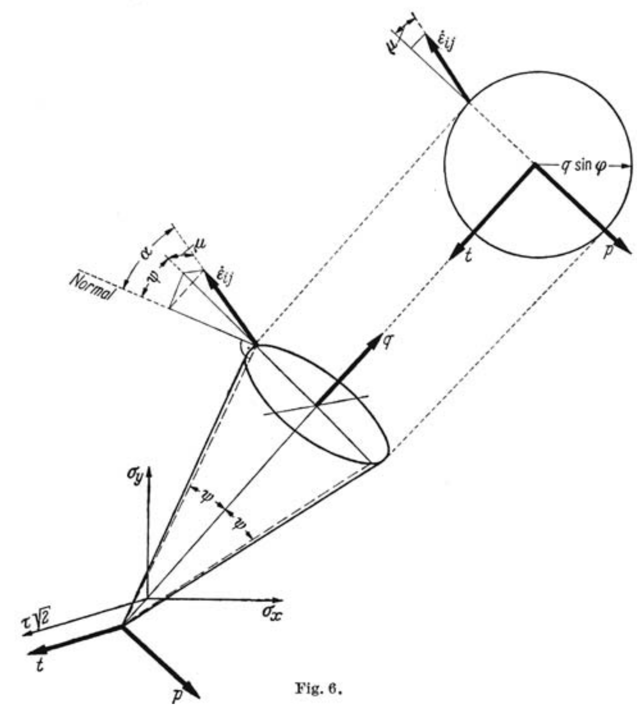

Fig. 7.

Fig. 6.

Then q is the bisectrix of σ_x and σ_y, and p is a coordinate in the σ_x, σ_y-plane perpendicular to q. In these coordinates the yield criterion is

$$2p^2 + 2t^2 = 2q^2 \sin^2 \varphi. \tag{2}$$

This shows that the yield surface is a cone with q as axis and which intersects the planes for q = constant by a circle with radius $q \sin \varphi$. The angle ψ is then related to φ by

$$\tan \psi = \sin \varphi. \tag{3}$$

If the rod material is assumed to behave as the mechanical model proposed by the author (1958, 1959) plastic shear strain rates consist of volume conserving slip in the directions at $\left(45° - \frac{1}{2}\varphi\right)$ with the major principal stress. The two conjugate shear strain rates need not be equal. If they are a and b respectively as shown in Fig. 5, then the deviation angle μ between principal directions of strain rate tensor is given by

$$\tan \mu = \frac{a-b}{a+b} \tan \varphi. \tag{4}$$

Since a and b can only be positive, this relation implies

$$-\varphi \leq \mu \leq \varphi. \tag{5}$$

It can be shown by a straightforward but somewhat tedious computation that the deviation angle α between the strain rate vector and the normal to the yield surface is then given by:

$$\cos \alpha = \cos \psi \cos \mu. \tag{6}$$

Since the sliding motion is considered to take place at constant volume the strain rate vector $\mathring{\varepsilon}_{ij}$, plotted in a coordinate system corresponding to the generalised stresses, lies in the q = constant plane. This plane makes an angle ψ with the normal to the yield surface as shown in Fig. 6. In order that the angle α between $\mathring{\varepsilon}_{ij}$ and the normal obeys (6) it is necessary that $\dot{\varepsilon}_{ij}$ is not normal to the circle in the q = constant plane of Fig. 7, but makes an angle μ with the radius of that circle.

According to the first collapse theorem a body is capable of supporting the external loads in any loading program, if it is possible to find a safe statically admissable stress distribution $\sigma_{ij}^{*(s)}$. A stress distribution is called statically admissable if it obeys the equilibrium conditions inside the body, if it satisfies boundary conditions on the part of the boundary where surface tractions are given and if a yield inequality is nowhere violated. For perfectly plastic materials the yield inequality simply requires that $\sigma_{ij}^{*(s)}$ lies inside the yield surface. This requirement is clearly necessary and is also sufficient because convexity of the yield surface and normality of the strain rate vector to that surface ensure that the real collapse stress state σ_{ij} is such that the quantity

$$[\sigma_{ij} - \sigma_{ij}^{*(s)}]\,\mathring{\varepsilon}_{ij}$$

is always positive. The proof of the first collapse theorem follows then by use of virtual work considerations [for a comprehensive description of this theorem and related matter see f.i. KOITER (1960)].

Since there is not always normality in the case of soils the yield inequality condition has to be modified. The modification necessary to take care of the angle μ is only small if by some other means it is possible to prove that q cannot decrease below a certain value q^*.

If the mechanical model of Fig. 5 is applicable, Eqs. (4) and (5) say that the absolute value of μ cannot exceed φ. Now let P represent a real collapse stress state $\sigma_{ij}^{(P)}$, then P lies on the circle with radius $q^* \sin \varphi$ in the plane $q = q^*$, Fig. 8. All stress states $\sigma_{ij}^{(R)}$ represented by a point R lying below PQ, the line at an angle $\left(\frac{1}{2}\pi + \varphi\right)$ to the normal in P, may be called statically admissable with respect to P, because the angle, between any line PR and the vector $\mathring{\varepsilon}_{ij}$ for $\mu = \varphi$, will be larger than $\frac{1}{2}\pi$. Therefore the quantity

$$[\sigma_{ij}^{(P)} - \sigma_{ij}^{(R)}]\,\mathring{\varepsilon}_{ij}$$

will always be positive for $\mu = \varphi$, and clearly this result is generally valid in the interval $0 \leq \mu \leq \varphi$.

Since the actual collapse stress will be everywhere on the circle, the statically admissable stress state $\sigma_{ij}^{*(s)}$ is limited by all lines PQ drawn from all points of the circumference. This means that the stress states are limited by the dotted circle in Fig. 8, with a radius of length $q^* \sin \varphi \cos \varphi$.

Since the coordinates p and t actually are $\sqrt{2}$ times the deviator-stresses s_x, τ_{xy}, the requirement of the dotted circle can be represented in the usual MOHR-diagram of Fig. 9 by the dotted circle whose radius is equal to the shear stress at the tangent point of MOHR-circle and COULOMB envelope line. This means that a safe statically admissable stress state is limited by the dotted circle (Fig. 9) which is equivalent to reducing the angle of shearing resistance to a value φ^* given by:

$$\sin \varphi^* = \sin \varphi \cos \varphi\,.$$

Although by this modification of the definition for a statically admissable stress state, the difficulties created by the uncertainty about the deviation angle between principal directions of stress tensor and strain rate tensor are circumvented, it must be emphasized that this only applies if by other means it is established that q cannot decrease below the value q^*. The region limiting the statically admissable stress states is therefore given by a circular cylinder starting

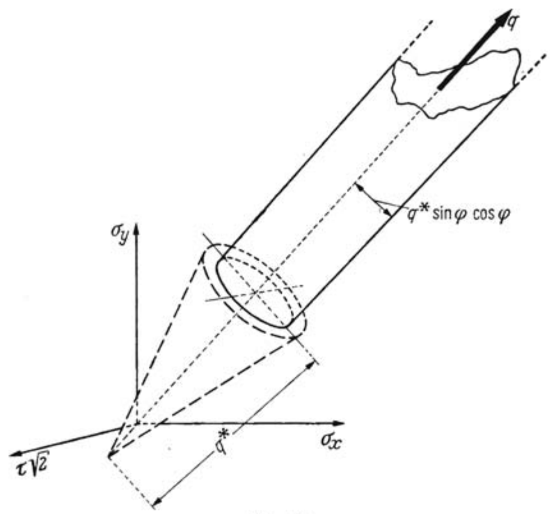

Fig. 10.

on the base of the cone with height q^* and running up to infinity with a radius $q^* \sin \varphi \cos \varphi$.

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Discussion

Contribution de K. H. ROSCOE: I would like to question the universal application of Professor DE JONG's statement that the normality condition does not apply to soils. The following remarks are very tentative since I have not had an opportunity to make a proper study of DE JONG's proposals. It does however seem that he is considering soil to be a non-dilatant material possessing constant cohesion and constant internal friction and he is concerned only with states of failure of such a medium. I wish to make two observations regarding these assumptions. Firstly soil is a dilatant medium and as it dilates the apparent cohesion and internal friction will change. Secondly the MOHR-COULOMB envelope is not a true yield surface for soils. If yield is defined as permanent irrecoverable deformation then soils yield, and of course dilate, at stress levels well below those required to satisfy the MOHR-COULOMB criterion of failure.

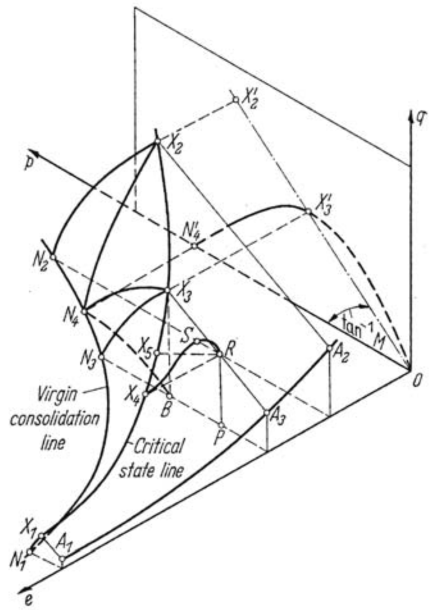

Fig. 1. Isometric view of idealised (p, q, e) yield surface for clays.

The position can be made clearer by referring to Fig. 1 which represents our concepts of the yield surface, obtained from triaxial tests on samples of a saturated remoulded clay, in p, q, e space; where $p = (\sigma_1' + 2\sigma_3')$, $q = (\sigma_1' - \sigma_3')$, e is the voids ratio and σ_1' and σ_3' are the major and minor principal effective compressive stresses respectively. In Fig. 1 the curve $N_1 N_2$ is the isotropic virgin consolidation curve and $X_1 X_2$ is the critical state line. The projection of the critical state line on the (p, q) plane is the straight line $O X_2'$. When a sample reaches a state corresponding to a point on the curve $X_1 X_2$ it will continue to distort in shear without further dilation and without change of stress.

The (p, q, e) yield surface for virgin and lightly over-consolidated clays is represented by the curved surface $N_1 N_2 X_2 X_1$ and in my paper to this symposium I have endeavoured to show that there is some experimental justification for such a surface. Its precise shape is open to some doubt as discussed by ROSCOE, SCHOFIELD and THURAIRAJAH (1963), and ROSCOE and SCHOFIELD (1963). Typical (p, q, e) state paths for undrained tests on normally consolidated samples are represented by curves $N_1 X_1$, $N_2 X_2$ and $N_3 X_3$, while a typical path for a drained test is $N_4 X_2$. It is important to notice that whenever a sample is at a state corresponding to a point on the surface $N_1 N_2 X_2 X_1$, and the deviator stress is increasing, it will be yielding. Consider for example a sample initially at state N_4. If it traverses any state path on the yield surface within the sector $N_2 N_4 X_3$ it will work harden as it yields but it will not fail until the critical state is attained. If the state change corresponds to the path $N_4 X_3$ which lies vertically above the elastic swelling curve $N_4 B$ then the sample will yield and not work harden. The relevant plastic potential curve is then $N_4' X_3'$. We have called the curve $N_4 X_3$ an elastic limit curve. As a sample work hardens the relevant plastic potential curve continuously grows in size but remains geometrically similar to curve $N_4' X_3'$. We have proposed that the form of the plastic potential curves is governed by the equation $q = M p \log_e \frac{p_0}{p}$ where p_0 is the initial consolidation pressure, and M is as shown in Fig. 1.

Let us now consider more heavily over-consolidated clays. The experimental data that is available for such clays is much less reliable than for lightly over-consolidated clays, hence the following remarks are extremely tentative. We suggest that the (p, q, e) yield surface for undrained tests is $A_1 A_2 X_2 X_1$ in Fig. 1. Consider an over-consolidated sample initially in a state represented by the point P. If it is subjected to an undrained test it will follow a state path which may be idealised by the path PRX_3 in Fig. 1. During the portion PR the sample behaves virtually elastically but it begins to yield at R and continues to yield and work harden until it reaches the peak deviator stress, as well as the critical state, at X_3. If the sample was allowed to dilate during a test then present evidence suggests that the state path comes above the undrained surface. For example an ideal representation of a $p =$ constant test is given by the path $PRSX_4$. In such a test yield begins at R but the sample continues to work harden over the range RS and attains the peak deviator stress at S. The sample then becomes unstable and subsequent successive states correspond to SX_4. I suggest that some path above a line such as RX_5 may be found in which this unstable portion is not present. For such a test the deviator stress would never diminish as the state changed from P to X_5. Hence as a sample, of initial state P, traverses any state path between RX_5 and RX_3, it will continually work harden until it attains the critical state when it fails. It is possible that a family of plastic potential curves of the type shown by OX_3' apply during all the work hardening processes undergone by over-consolidated samples. The curve OX_3' may have the same equation as $N_4'X_3'$, but adequate exerimental evidence is not available to be able to see how such plastic potentials relate to the yield surfaces for anything other than lightly over-consolidated clays. We have a little indirect evidence on the heavily overconsolidated or "dense" side from simple shear tests on steel balls. This medium appears, during any work hardening process, to have plastic potential curves of the type shown in Fig. 2. The equation of these curves is $\tau = M\sigma \log_e \frac{\sigma_0}{\sigma}$, where τ is the maximum shear stress and σ the mean normal stress under conditions of plane strain. This equation follows directly from the application of the normality condition to the boundary energy equation which was discussed by Poorooshasb and Roscoe (1961) for steel balls. Further work is still required to connect these potential curves with the observed yield surfaces.

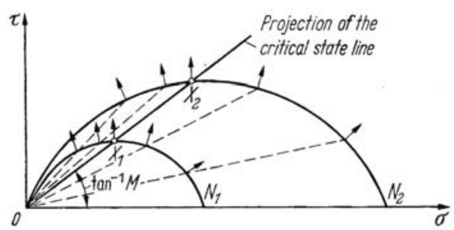

Fig. 2. Plastic potential curves for steel balls.

Finally I would like to make the point that far too much effort has been made in soil mechanics to study failure conditions. Engineers design, and hope their structures operate, at much lower stress levels. This is the region of yielding that should be studied in detail. The Mohr-Coulomb envelope may, or may not, be shown to be valid for the failure of soils but it is not a yield surface in the true sense of the word since the yielding of a sample cannot be related to a movement on the envelope. With such a theory yield does not occur until failure takes place.

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Réponse de G. DE JOSSELIN DE JONG: It was not my intention to say that for soils there never is normality, but that normality is not necessary. In the cases studied by M. ROSCOE normality may have been observed, but these are special cases, which are not representative for the situation in general.

That M. ROSCOE did not observe the deviation of the principal directions of stress and strain rate tensors, is due to the fact, that the stress coordinates p and q in his diagrams are not the complete set of generalised stresses. The samples were 3-dimensional, so the testresults require a representation in a 9 dimensional stress space. Since shearstresses on perpendicular faces are equal the amount of dimensions can be reduced to 6. The system I talked about this morning, is 2-dimensional and so there are 4 generalised stresses, from which τ_{xy} is τ_{yx}, reducing the system to 3 stress coordinates.

Since M. ROSCOE only considers the stress combinations p and q, his graphs correspond in a way to the σ_x, σ_y plane which intersects the cone enclosed by the yield surface along the axis. The deviation of the principal directions of the tensors is only visible in the plane perpendicular to the axis.

Cf. aussi, p. 46, la citation de D. C. DRUCKER.

PROCEEDINGS OF THE GEOTECHNICAL CONFERENCE OSLO 1968

on Shear Strength Properties
of Natural Soils and Rocks

COMPTES RENDUS DE LA CONFERENCE GEOTECHNIQUE OSLO 1968

sur les propriétés de résistance au
cisaillement des sols naturels
et des roches

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NORWEGIAN GEOTECHNICAL INSTITUTE
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PROF. G. DE JOSSELIN DE JONG (Netherlands):

In their paper (3/14) Roscoe, Bassett and Cole review concepts pertaining to the coincidence of principal directions of stress and strain. Besides the points mentioned, it must be noted that a case of non-coincidence is to be expected if rupture planes or rupture zones develop erratically throughout the soil mass. Such planes or zones originate if the material yields in these discrete regions before the rest of the soil mass deforms excessively.

Since planes at an angle of $\pm(\pi/4-\varphi/2)$ with the major principal stress direction have to transmit a stress combination which is most unfavourable to support, it is approximately in these directions that the rupture planes or zones develop. These directions coincide with the stress characteristics. Because the material is yielding in the rupture zones the shear strain rate is undetermined and may be unequal for the two conjugate directions. This inequality is not yet sufficient to create non-coincidence; it is also necessary that the angle φ is unequal to zero.

For an ideally isotropic material it can be expected that in a soil mass of sufficiently large dimensions the average shear strain rate in the two conjugate directions will be the same, thus resulting in coincidence of principal directions of stress and strain. However, even the slightest deviation from isotropy may result in a considerable difference between these principal directions. The mechanism is in a way similar to the instability of a rod under axial compression. If a perfectly straight cylindrical rod is compressed by forces exactly along its axis, then theoretically the rod should only reduce in length, but in reality it will always buckle in some unpredictable sidewise direction.

It may be difficult to visualize the strain rate tensor in this case where the strain rate is concentrated in discrete zones, because a tensor essentially only can be defined for a continuous deformation. The discrete rupture pattern can be replaced, however, by a continuous deformation which averages the discrete jumps and the tensor associated to this representative deformation is the one considered. That this concept represents a physical reality was demonstrated by the following test.

A sandblock ($60\times60\times15$ cm) schematically represented in Fig. 1 is enclosed on the sides by 12 loading elements provided with teeths, and two thin plastic sheets along the lower and upper plane. The air pressure in the pores was reduced by 0.9 atm. in order to create an allround pressure. Then forces were applied to the loading elements in the direction of the sides, such that a system of pure shear stress was added to the allround pressure. The principal stress directions are then parallel to the diagonals. The sand was deposited in layers parallel to one of the diagonals in order to prevent that anisotropy created by deposition would offer a preference for one of the two conjugate directions of imminent shear.

The difference with Roscoe's simple shear apparatus is that in his apparatus deformation is enforced and stresses are measured, whereas in our test the stresses are applied and the sandblock is left to move in the manner it pleases.

The deformation was measured by photographing the upper sheet, which being transparent showed the grains.

In Fig. 2 two photographs of the block representing a loading from $\sigma_1/\sigma_3=1.65$ to 3.15 are superimposed. The photographs are shifted and rotated in such a manner that the particle traces form a family of curves with orthogonal asymptotes. These asymptotes then have the direction of the principal strain rate directions. The smoothness of the curves which are approximately hyperbolas indicates that the deformation was practically uniform. A detailed survey of the deformation by use of a chartographic stereomicrometer at the International Institute for Aerial Survey and Earth Sciences at Delft showed an average deviation angle between principal stress and strain rate direction of 12°, with a spread of about 3°.

In order to investigate whether this homogeneous deformation field existed throughout the sample the sand was mixed with a small amount of cement, enough to solidify the sand mass by adding water after the test. The sand was deposited in black and white layers.

After the bloc had solidified it was abrased to show the successive sections at 2.5 cm intervals of the height. A typical view of such a section is given in Fig. 3. The deformation was concentrated in narrow zones which followed the direction of the stress characteristics. These rupture zones were found in the same location for every section, which indicates

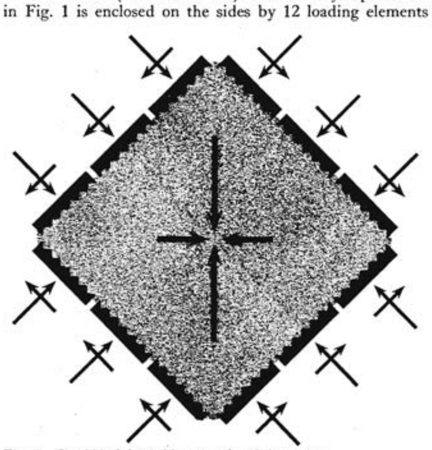

Fig. 1. *Sand block loaded by normal and shear stress.*

Fig. 2. *Superimposed photographs of sand block before and after loading.*

that vertical planes are formed throughout the sample. Some originated in the middle of a loading element. The shear-strain-rate differed for all the ruptive zones.

The reason that they were not observed in the photograph was the relative rigidity of the plastic envelopping sheet. The sheet averaged the deformation and showed therefore a homogeneous deformation with deviating principal directions because of the underlying mechanism of unequal strain-rate in the erratic rupture zones.

The mechanism observed in the test was very similar to the one proposed in ref. 1, p. 57, see Fig. 4 taken from that publication.

The indeterminancy of the deformation created by this mechanism needs not to be of too much concern for further use in predictions of soil behaviour. It will anyhow not be possible to predict deformation of soil masses in detail because the initial stress conditions are mostly impossible to ascertain. A more realistic approach to the determination of stability analysis is the use of a lower bound theorem, which gives an answer that is on the safe side and irrespective of previous loading history. It has been shown in ref 2, that the noncoincidence is taken care of by a small reduction of φ to φ^*, such that

$$\sin\varphi^* = \sin\varphi\cos\varphi$$

However, the noncoincidence is only a minor difficulty, the major one is the ignorance with respect to the isotropic stress. How a solution can be constructed which gives higher and therefore more attractive values than the solution mentioned in ref. 2, is beyond the scope of this discussion, but amounts to the construction of a field of stress characteristics taking the solution with the lowest isotropic stress everywhere.

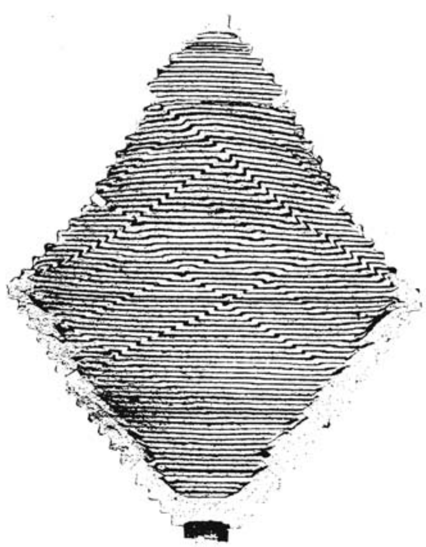

Fig. 3. *Discontinuities in interior of sand block.*

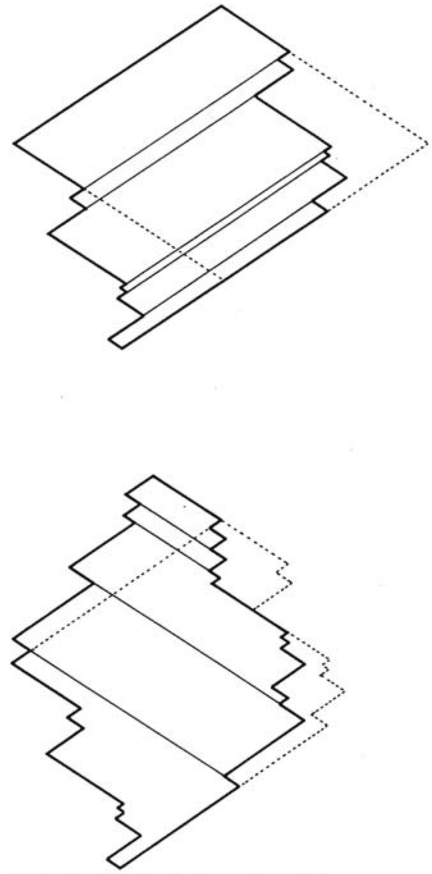

Fig. 4. *Failure mechanism for granular materials.*

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CHAIRMAN:

Thank you Professor de Josselin de Jong – I am sorry we are short of time. The following speaker is Professor Šuklje.

PROF. L. ŠUKLJE (Yugoslavia):

Monsieur le Président,

Je vous demande d'accorder l'hospitalité de la 3ème Section au sujet que j'ai traité dans mon rapport apparu

THE DOUBLE SLIDING, FREE ROTATING MODEL FOR GRANULAR ASSEMBLIES

G. DE JOSSELIN DE JONG*

INTRODUCTION

The sliding block model for the mechanism of deformation, in a body composed of grains, is based on the concept that movements of grains with respect to each other occur along planes that coincide preferably with the stress characteristic planes. In the case of plane strain these planes intersect the two-dimensional plane of consideration along two characteristic lines called S_1 and S_2.

The object of this Note is not to consider the probable veracity of such a model, but to establish the flow rule and the constitutive equations which follow from the special character of the model. The properties are taken to be those that were proposed by de Josselin de Jong (1958, 1959). In that model sliding can occur simultaneously in the S_1 and S_2 directions at different shear strain rates, but limited in sense, and in addition the sliding elements are free to rotate.

Geniev (1958) considered such a model, but restricted sliding to one of the two characteristic directions. Most investigators reject this restriction and agree that it is desirable to permit a double sliding motion. Mandl and Fernández Luque (1970) reconsidered the double sliding model and confirmed equations, obtained by Spencer (1964) and Zagainov (1967) for the stationary case, that principal directions of stress remain fixed. However, the equations refer to a model that is restricted in its rotation, as if the sliding elements are forced by an external agency to conserve their orientation in space. Therefore these equations refer to a different model from that of de Josselin de Jong. Their model is not free to rotate and so it cannot execute motions which are commonly accepted to have been observed in reality, e.g.

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the rotation of the soil mass separated from an embankment by a circular slip plane. Another kind of rotation was observed in a verification experiment by Drescher (1971).

Mandel (1966, p. 307) directed attention to this lack of freedom of rotation and re-established equations obtained previously (Mandel, 1947). However, he remarks that the concept of double sliding and rotation combined is void because every deformation without volume change can be decomposed in such a manner. This remark is correct if sliding is free to occur along each characteristic plane in both senses, i.e. either in the direction of the shear stress on that plane or against it.

By restricting the sliding sense as proposed by de Josselin de Jong the model is made to obey the thermodynamic requirement that energy is dissipated during sliding. The necessity of this requirement is a consequence of the frictional character of the mechanism. The grains of, say, a dry sand do not move with respect to each other because friction forces in the contact points between them prevent this. Sliding can only occur if the friction is surmounted and therefore shear strain will develop only in the direction of the shear stress in the plane o sliding and never against it. De Josselin de Jong (1959, p. 57) called this the requirement off direction and formulated it as

$$a \geqslant 0$$

$$b \geqslant 0$$

By this restricting requirement the concept of double sliding and rotation combined is no longer meaningless because, when introduced mathematically, a system of hyperbolic differential equations is obtained with a limited range of solutions. This hyperbolic system is unusual, because its coefficients, instead of being fixed for every point in the field, obey inequalities which determine, instead of unique characteristic directions at every point, a fan of possible directions for the characteristics. This has been shown graphically by de Josselin de Jong (1959, pp. 72–80). The pertinent differential equation (de Josselin de Jong, 1959, p. 92) was given in terms of the undetermined characteristics and their curvatures and is unattractive.

The object of this Note is to re-establish the constitutive equations as referred to a cartesian x, y co-ordinate system. These co-ordinates are straight and so the curvatures of the characteristics disappear from the equations, which simplifies their form. Since the constitutive equation contains coefficients that obey inequalities, it can be presented as an inequality.

A common objection against the double sliding mechanism is that the principal directions of stress and strain rate tensors can deviate. It is often proposed that such a deviation can occur only if the material is not isotropic. However, the reasoning to substantiate this starts with the assumption that an analytic functional relationship exists between the invariants of the two tensors (see e.g. Eringen, 1962, p. 158).

Since for the double sliding model the constitutive law contains an inequality, no such analytic function exists and therefore there is no need for coincidence of principal directions. Nevertheless the requirements of isotropy (see Eringen, 1962, p. 139) are fulfilled because the inequality is invariant for the full orthogonal group of co-ordinate transformations. This is also true for three dimensions.

Mandl and Fernández Luque (1970) tried to remove the objection to non-coaxiality in isotropic materials by mentioning that in two dimensions the co-ordinate transformations for reflexion cannot be obtained from those for rotation simply by taking the negative of all matrix components, as can be done in three dimensions. However, this only proves that a proof based on such a sign inversion cannot be applied in two dimensions; it does not mean that another proof might not exist. Another proof exists if there is a functional relationship between the two tensors. The functional relationship does not exist in this case and therefore non-coaxiality is acceptable in three as well as in two dimensions.