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Mathematical and Statistical Models and Methods in Reliability

Applications to Medicine, Finance, and Quality Control

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Contents

Part III Applications

Preface

The contributions in this volume were all presented as invited papers at the Sixth International Conference on Mathematical Methods in Reliability: Theory, Methods, Applications (MMR 2009), which was held at Gubkin Russian State University of Oil and Gas (Gubkin University, Moscow, Russia) during June 22–26, 2009. The International Organizing Committee of this conference included organizers of the previous conferences, namely, Professors Nikolaos Limnios (France), Mikhail Nikulin (France, Russia), Bo Lindqwist (Norway), Sally McNulty (USA), Tim Bedford (UK), and Vladimir Rykov (Russia). In addition to Gubkin University, the Peoples Friendship University of Russia (PFUR), and the University of Bordeaux-2 (France) participated in the meeting's organization.

Reliability theory is a multidisciplinary science aiming to provide complex technical, computer, and informational systems and processes that are resistant to failure. Catastrophic events of the recent past, such as the explosion of the 4th block of Chernobyl's nuclear power station in April 1986, the failure of the blocking system that switched out 21 United States electrical stations in August 2003, and the 2009 breakdown of a turbine in the Sayano-Shushenskaya electrical station, show the necessity for the scientific community to pay more serious attention to reliability problems. Although human error played an important role in most of these events, mathematical modeling and careful investigation into causes of failure are nevertheless very important.

During the early stages of research on reliability theory, the primary focus was on developing mathematical terminology and formalism. These rudiments were established in works such as B. V. Gnedenko, Yu. K. Belyaev, A. D. Solov'ev's *Mathematical Methods in Reliability Theory* and R. E. Barlow F. Proschan's *Mathematical Theory of Reliability.* More modern developments and practical problems began to receive attention at the end of the last century, when the First International Conference on Mathematical Methods in Reliability Theory (MMR 1997) was organized in Bucharest in 1997. Since then, six more conferences have been undertaken as part of the MMR series:

the second, in Bordeaux (France, 2000);
the third, in Trondheim (Norway, 2002);
the fourth, in Stanta Fe (New Mexico, USA, 2004);
the fifth, in Glasgow (Scotland, UK, 2007);

the sixth, in Moscow (Russia, 2009, based on which this volume is being prepared); and

the seventh, in Beijing (China, planned for 2011).

More than 200 people from 35 countries participated in the sixth conference and presented a total of 167 talks. Ten plenary talks (1 hour) were also presented; most appear in this volume. All the talks given at the conference were broadly classified according to the following topics: “Mathematical models and methods in reliability theory” (22 sessions), “Statistical methods in reliability theory” (10 sessions), “Computer tools and support of reliability problems solution” (3 sessions), “Applications of reliability theory in industry, medicine, power stations, transport and other spheres” (9 sessions). Accordingly, these topics are well represented in the present collection.

It is worth noting that the conference also included a “round table discussion” devoted to the memory of one of the field’s greatest pioneers, B. V. Gnedenko. Professors V. Korolyuk, Yu. Belyaev, I. Ushakov, I. Kovalenko, and D. B. Gnedenko (B.V.’s son) all discussed their memories and experiences with this remarkable scientist and man. Further information on this and other conference events can be found at http://mmr.gubkin.ru.

We would like to thank the Russian Foundation of Fundamental Investigation and the open joint-stock company “Gasprom. Promgas” for their financial support of the conference. Our final thanks go to Mr. Tom Grasso (Editor, Birkhäuser, Boston) for his support and encouragement in producing this book, and to Mrs. Debbie Iscoe for her fine work on the entire manuscript.

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List of Tables

List of Figures

Part I

Reliability Models, Methods, and Optimization

1

Reliability of Semi-Markov Systems with Asymptotic Merging Phase Space

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Abstract: The aim of this chapter is to present, under an unified framework, asymptotic merging of phase state space of complex systems in reliability. Such simplification methods are important in reliability since the most system have very large phase spaces and it is almost impossible to handle them by usual analytical methods. Results presented here are of averaging type and obtained by weak convergence techniques. Nevertheless, a result of diffusion approximation is also given.

Keywords and phrases: Reliability, Semi-Markov process, Renewal process, Markov process, Merging, Random evolution, Weak convergence, Singular perturbation

1.1 Introduction

Modeling reliability of real systems is often hard, or impossible to handle by direct analytical-numerical methods, due essentially to the very large number of components. For example, a system of twenty binary components (yet a small system for real problems) gives a phase (state) space of more than one million of states which is almost impossible to handle by the usual analytical-numerical methods.

A common point of methods developed in order to handle such systems is the reduction of the number of states called aggregation or merging of phase space methods. Of course, in such methods the merging system has to keep the essential characteristics of the original system in regards of reliability.

The exact aggregation is a natural candidate method to this end. This method, in the case of a system described by a Markov process, $x(t), t \geq 0$, with (a large) phase space, E, is to consider another Markov process, $\widehat{x}(t), t \geq 0$, with (a much smaller) state space, $\widehat{E}$. This means that there exists a (merging) function $v : E \to \widehat{E}$ such that

$$Q(f \circ v) = (\widehat{Q}f) \circ v,$$

for any bounded and measurable function f on $\widehat{E}$; where Q and $\widehat{Q}$ are the generators of the Markov processes x and $\widehat{x}$, respectively.

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Unfortunately, the exact aggregation method usually does not work for reliability problems, essentially, due to the fact that failure rate and repair rates values are of different magnitudes. So, there is need to provide approximation methods for aggregation. The method that seems to give interesting results for merging the phase space is the method of (functional) asymptotic merging by means of weak convergence techniques, (see, e.g., [KL05a,b, KT93, KS95, A08, SV79, SK89, SHS02, S04, LS]). The most known are average, diffusion and Poisson approximation methods (see, e.g., [KL05a,b, K90, LO01]).

We are proposed here to present some kind of asymptotic methods in the case where the temporal behavior of the system is described by a semi-Markov process which is the most general process encountered in the literature of reliability modeling. The problem here can be formulated as follows. Given a family of semi-Markov processes in series scheme, that is, for $\varepsilon > 0$, the process $x^\varepsilon(t), t \geq 0$, is a semi-Markov process, we have

$$v(x^\varepsilon(t)) \Longrightarrow \widehat{x}(t), \quad \varepsilon \to 0,$$

where the limit process $\widehat{x}(t)$ is a Markov process. So, in this way, we not only reduce the initial phase space to a simpler one, but also we get a Markov process instead of the initial semi-Markov one.

The asymptotic considered here is of functional type in series scheme where the considered systems are indexed by a series parameter $\varepsilon > 0$ in a weak convergence framework. Average and diffusion approximations are considered.

This article is a continuation of our work presented in the MMR2000 [KL00,KL04] where we presented asymptotic methods in the particular case of Markov switching stochastic systems. We present here, asymptotic results for semi-Markov systems, under a unified framework for reliability problems.

The particular systems studied here are: the semi-Markov process in merging phase space, the integral functional, the dynamical system, and an heuristic principle for superposed renewal processes.

This chapter is organized as follows. Section 1.2 presents a short review of some asymptotic results. Section 1.3 presents an asymptotic merging result for the supporting semi-Markov process and some additional results especially for the failure time. Section 1.4 presents reward functional asymptotic results. Section 1.5 presents dynamical system asymptotic merging results, and fluctuations of such functionals in Sect. 1.6. Section 1.7 presents a heuristic method for superposition of two renewal processes. Finally, Sect. 1.8 presents results of the stationary phase merging

---

1.2 Reliability of the Renewal System

Let us review here some results on repairable systems with two identical components where they are solved by a renewal process approach and a generalization to semi-Markov process approach (Gnedenko [G64a,b, GS74], Soloviev [SL64, SL71], and Korolyuk [K89]). In both cases, their solution is based on the solution of a singular perturbation problem for reducible-invertible operators.

1. Renewal duplicated system. Let us consider a two component cold standby system. The lifetimes of which are iid, with common distribution F; and their repair (or replacement) times are also iid with common distribution function G and moreover they are independent of the lifetimes. Denote by α the lifetime and by β the repair time. Thus, we have $F(t) = \mathbf{P}(\alpha \le t)$ and $G(t) = \mathbf{P}(\beta \le t)$.

The system fails at time τ when the working component fails while the repaired component is still under repair. So, the reliability problem consists to find the distribution function $\Phi(t) = \mathbf{P}(\tau \le t)$. The solution of this problem is given by Gnedenko [G64a] in terms of Laplace transforms as follows

$$\varphi(s) = \frac{\psi(s)}{1 - g(s)}, \tag{1.1}$$

where:

$$\varphi(s) := \mathbf{E}\mathrm{e}^{-s\tau} = \int_0^\infty \mathrm{e}^{-st}\mathrm{d}\Phi(t),$$
$$\psi(s) := \int_0^\infty \mathrm{e}^{-st}\overline{G}(t)\mathrm{d}F(t),$$
$$g(s) := \int_0^\infty \mathrm{e}^{-st}G(t)\mathrm{d}F(t),$$

with $\overline{G}(t) := 1 - G(t)$.

We denote by $q := \psi(0) = \int_0^\infty \overline{G}(t)\mathrm{d}F(t) = \mathbf{P}(\beta > \alpha)$ which is the probability of failure of the system on every working interval. This is also called the terminating probability in Feller [F66] in the case of a terminating renewal process, where q represents the defect of the distribution in the renewal process.

An obvious view of solution (1.1) gives possibility to get a limit result as $q \to 0$ (Soloviev [SL64]), that is

$$\lim_{q \to 0} \mathbf{P}(q\tau > t) = \mathrm{e}^{-t/a}, \quad a = \mathbf{E}\alpha. \tag{1.2}$$

The proof of the limit result (1.2) is based on the asymptotic representation of functions ψ and g in the formula (1.1), as $q \to 0$, namely:

$$\psi(qs) = q + o(q), \quad 1 - g(qs) = q(1 + as) + o(q). \tag{1.3}$$

The problem of reliability of the duplicated system is generalized in the case of different distributions of working times and renewal times (Gnedenko [G64a]).

Now working times α_k and renewal times β_k, $k = 1, 2$ have different distribution functions:

$$F_k(t) = \mathbf{P}(\alpha_k \le t), \quad G_k(t) = \mathbf{P}(\beta_k \le t), \quad k = 1, 2.$$

Working times to the first failure of the system τ_k, $k = 1, 2$ also depend on the number of initial working components. For the Laplace transform functions of working times up to failure, that is,

$$\varphi_k(s) = \mathbf{E}\mathrm{e}^{-s\tau_k} = \int_0^\infty \mathrm{e}^{-st}\mathrm{d}\Phi_k(t), \quad k = 1, 2$$

may be obtained a system of algebraic equations

$$Q(s)\varphi(s) = \psi(s), \tag{1.4}$$